Hyperbolic Knot Theory
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Chapter 3

- Definition 3.3: the *transition map* or *coordinate change map*
$$\gamma = \phi \circ \psi^{-1} \colon \psi(U \cap V) \to \phi(U \cap V)$$
restricts to an element of G on connected components of $\psi(U \cap V)$.
[That is, different elements of G may correspond to different components.]
- In the displayed equation above Definition 3.9: x_n should be x_{n-1}:
$$\Phi_{[\alpha]}(x) = \gamma_{0,1}(x_1)\gamma_{1,2}(x_2)\dots\gamma_{(n-2),(n-1)}(x_{n-1})\phi_{n-1}(x).$$
- Example 3.18: ... To simplify things, let's keep the length of the edge between horocycles at 0 and 1 constant as we extend horocycles, and assume that the cusps at 0 and 1 are complete. Choose horocycles at 0 and 1 of (Euclidean) radius 1/2, so that these horocycles are tangent along the edge between 0 and 1, hence the distance between horocycles is 0. This distance will remain equal to 0 under each holonomy element, and because we are assuming the cusp at 1 is complete, such a holonomy element will preserve the horocycle about 1. Thus there will be a horocycle at $x = 3/2$ tangent to the horocycle about 1, to preserve distance 0. ...
[That is, we will assume that for cusps at 0 and 1, translation is along a horocycle, so we don't have to worry about rescaling these under the holonomy elements.]
- Proof of Theorem 3.19: In the second paragraph, we state that we will use the fact that local homeomorphism, plus path lifting, implies covering map. This result is not trivial, but a proof appears, for example, in Theorem 4.19 of [Forster, Lectures on Riemann surfaces, Graduate Texts in Mathematics, Volume 81, Springer-Verlag, New York, 1991.]

Chapter 5

- Definition 5.1: The *trace* of A is the trace of its matrix. [Remove the word normalized.]
- Discussion under statement of Theorem 5.22 (Universal Elementary Neighborhoods):
We will give a proof of theorem 5.22 in section 5.5. Before that, a few remarks are in order. First, the Universal Elementary Neighborhoods theorem holds when we allow elliptics; ....
[That is, replace "Margulis lemma" with "Universal Elementary Neighorhoods theorem"]
- Theorem 5.24: The theorem needs to exclude elementary examples, namely the solid torus and the product of a torus and an interval. Reword the theorem as follows:
A hyperbolic 3-manifold M has finite volume if and only if M is closed (compact without boundary), or M is homeomorphic to the interior of a compact manifold $\overline{M}$ with torus boundary components that is not elementary, i.e. $\overline{M}$ is not a solid torus or the product of a torus and an interval.
- Beginning of proof of theorem 5.24: Need to add a few words to the proof to deal with the two elementary cases of solid torus and product of torus and interval:
If M is closed then a fundamental domain for M in its univeral cover $\mathbb{H}^3$ is a compact set, hence has finite volume. If M is the interior of a manifold with torus boundary that is not a solid torus nor the product of a torus and an interval, then we claim that each torus boundary component will be realized as a unique cusp in the complete hyperbolic structure on M. This follows from the fact that M is not the interior of a solid torus nor $T^2 \times I$. For if M is not one of these two elementary manifolds, then the fundamental group of each torus boundary component of M injects into $\pi_1(M)$ and no two fundamental groups of distinct torus boundary components give the same subgroup of $\pi_1(M)$. We will assume this statement for now; it follows from tools that will be developed in chapters 8 and 12.

Thus each torus boundary component gives a unique $\mathbb{Z} \times \mathbb{Z}$ subgroup of $\pi_1(M)$. Because M is hyperbolic, it follows from exercise 5.11 (see corollary 5.32) that this subgroup is generated by parabolics with the same fixed point, giving a cusp.

[Then continue the proof as written.]

- Very end of proof of theorem 5.24: Add a sentence about the elementary case, again:

  Finally, observe that N cannot be a solid torus or $T^2 \times I$, because a hyperbolic structure on the interior of these manifolds is given by the quotient of $\mathbb{H}^3$ by an elementary group, and has infinite volume.

Chapter 6

- (Minor) Definition 6.10: Remove repeated statement of "suppose the interior of M has a hyperbolic structure" on lines 3-4.

Chapter 7

- After definition 7.10, there is a note that the complement of an augmented link is homeomorphic to the complement of the link with all even crossings removed, then the definition of a *half-twist*. At the end of that paragraph, add the following:

  Because of this homeomorphism, from now on, we require the diagram of a fully augmented link to consist of one or zero crossings adjacent to each crossing circle.
- Definition 7.12: Part (2), the definition of *prime* for a fully augmented link, needs to be expanded to rule out the diagram consisting of a crossing circle encircling an unknot diagram with no crossings. For example add the following:

  Additionally, we do not allow a diagram consisting of a single crossing circle encircling the diagram of an unknot with no crossings.
- Proof of Lemma 7.15: We need to indicate where we are using the hypothesis that there are at least two crossing circles. This is done by adding the following to the very end of the proof, on the very last line:

  Observe, however, that we will obtain a degenerate ideal polyhedron if we remove from $\mathbb{H}^3$ regions on both sides of one of the gray planes. This can occur, but only if two of the gray circles agree for two different regions complementary to the white circles. If this happens, these must be the only complementary regions, and the circle packing consists only of three circles whose nerve is a triangle, and there are exactly three white regions on the plane of projection. The regions must be separated by two link components on the plane of projection, and thus there is at most one crossing circle in the diagram. Because we assumed there are at least two crossing circles, this will not occur, and the polyhedron will be non-degenerate. This proves the lemma.

Chapter 8

- Proof of theorem 8.15:

  ... Under the hyperbolic structure on the interior of M, torus boundary components become cusps, using the fact that M has finite volume (hence is not a solid torus or $T^2 \times I$). ...
- Theorem 8.17 (Thurston, Hyperbolization): The statement needs to change to deal with elementary groups again, as follows:

  A knot complement admits a complete hyperbolic structure if and only if it is not a satellite knot or a torus knot.

  More generally, let M be a compact 3-manifold with nonempty boundary consisting only of tori, and suppose M is neither a solid torus nor $T^2 \times I$. Then M has interior admitting a compete hyperbolic structure if and only if it is irreducible, boundary irreducible, atoroidal, and anannular.

- Definition 8.31: Add to item (2), so it reads:
  (2) Every normal disk has non-negative combinatorial area, and each normal disk that is parallel to an ideal vertex of a polyhedron has combinatorial area zero.
- Proof of theorem 8.36, second paragraph of the proof:
  ... By the definition of an angled polyhedral structure, definition 8.31, and the definition of combinatorial area, definition 8.30, it follows that each such disk has combinatorial area zero.
  [Note this line is not true without adding the extra condition to definition 8.31 above.]
- Proof of theorem 8.42: In the very last paragraph, it could be the case that at most $m-2$ boundary components of the punctured sphere come from intersections with cusps. The contradiction is the same, but the paragraph needs to be changed a bit for accuracy, to the following.
  On the other hand, each $\ell(s_j) > 6$, and there are m of these, where m is the number of boundary components of S coming from intersections with the R_j. If S is a punctured sphere, then it arose from an essential sphere, disk, or annulus in $M(s_1, \ldots, s_n)$, and all but at most two of its punctures come from intersections with the R_i. Then its Euler characteristic is either $m-2$, $m-1$, or m. In any case, the right hand side of equation (8.3) is at most $6m$. Then equation (8.3) implies $6m < 6m$, which is a contradiction. If S is a punctured torus, equation (8.3) implies $6m < 6m$, again a contradiction.

Chapter 9

- (Minor) Two lines before the statement of lemma 9.8, Schläfli is misspelled.
- In the statement of lemma 9.18, the notation $\Re$ denotes the real part of a complex number (and $\Im$ will denote its imaginary part).
- In the proof of lemma 9.18, an instance of t_i should be ζ_i, namely:
  ... so that α_i is the angle cut off by ζ_i.
- Proof of proposition 9.19, second paragraph of the proof: Theorem 9.9 is cited. While this is ok, it is more direct to cite instead lemma 9.15.
- In the discussion below conjecture 9.22 and theorem 9.23, remove the extra copy of the word "equation".

Chapter 10

- In figure 10.6, each instance of S should be S_i.

Chapter 11

- In the discussion at the beginning of section 11.1, in the second and third paragraphs (just before definition 11.1), there are two instances of chapter 1 that should both be chapter 0.
- Proof of proposition 11.7: After fixing the definition of an angled polyhedral structure, definition 8.31, we need to ensure that the added condition holds. This is straightforward. The following lines should be added to the second paragraph of that proof:
  The second condition takes the most work. We need to show that every normal disk has non-negative combinatorial area, and that normal disks parallel to ideal vertices have zero combinatorial area. Recall that ... [continue with the paragraph as written, past two displayed equations]
  [Then just before the paragraph "Notice..." add the following:]
  Notice that if D is a disk parallel to an ideal vertex, then the fact that ideal vertices are 4-valent implies that D is a quad meeting exactly four ideal edges, hence its combinatorial area is zero as required. More generally, notice that if ∂D meets at least four ideal edges, or at least two boundary faces, then the combinatorial area of D is non-negative. ... [continue the proof as written from here to the end]

- Definition 11.11: Let γ be a simple closed curve meeting the diagram of K transversely exactly four times in knot strands, with two intersections adjacent to a crossing on the outside of γ. A *flype* is a move on the diagram that rotates the region inside γ by 180° about an axis in the plane, rotating in a third dimension. Equivalently, it reflects in an axis in the plane. In any case, it moves the crossing outside γ to lie between the opposite two strands. See figure 11.4, right.
- Definition 11.27: Rewrite item (2):

  (2) Each normal disk in a polyhedron has nonnegative combinatorial area, with the combinatorial area of any disk parallel to an ideal vertex equal to zero.
- Proof of proposition 11.28: ... Then the proof of proposition 11.7 carries through to show that every normal disk has nonnegative combinatorial area, and those parallel to ideal vertices have zero combinatorial area. ...