Weak rates for ECSN progenitor evolution and nucleosynthesis

Gabriel Martínez Pinedo

Electron Capture Supernova & Super-AGB Star Workshop,
Melbourne, February 1-6, 2016
Stellar Evolution Intermediate mass stars

- Stellar Cloud with Protostars
- Low-mass Star
- Intermediate-mass Star
- Massive Star
- Red Giant
- Red Supergiant
- Supernova
- Planetary Nebula
- White Dwarf
- Neutron Star
- Black Hole

Mass vs. Age Diagram:

- 8 – 12 M⊙
- CCSN
- ECSN
- Stellar Cloud with Protostars
- Massive Star
- Red Supergiant
- Intermediate-mass Star
- Low-mass Star
- Planetary Nebula
- White Dwarf
- Neutron Star
- Black Hole
Core evolution (intermediate stars)

Threshold densities for electron capture

<table>
<thead>
<tr>
<th>Nuc.</th>
<th>$\epsilon_0$ (MeV)</th>
<th>$2Y_e\rho$ (g/cm$^3$)</th>
<th>Nuc.</th>
<th>$\epsilon_0$ (MeV)</th>
<th>$2Y_e\rho$ (g/cm$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1$H</td>
<td>0.782</td>
<td>$2.44\times10^7$</td>
<td>$^{28}$Si</td>
<td>4.643</td>
<td>$1.97\times10^9$</td>
</tr>
<tr>
<td>$^3$He</td>
<td>0.0186</td>
<td>$3.94\times10^4$</td>
<td>$^{29}$Si</td>
<td>3.681</td>
<td>$1.05\times10^9$</td>
</tr>
<tr>
<td>$^4$He</td>
<td>20.6</td>
<td>$1.37\times10^{11}$</td>
<td>$^{30}$Si</td>
<td>8.539</td>
<td>$1.08\times10^{10}$</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>13.37</td>
<td>$3.89\times10^{10}$</td>
<td>$^{31}$P</td>
<td>1.491</td>
<td>$1.06\times10^8$</td>
</tr>
<tr>
<td>$^{13}$C</td>
<td>13.44</td>
<td>$3.95\times10^{10}$</td>
<td>$^{32}$S</td>
<td>1.710</td>
<td>$1.47\times10^8$</td>
</tr>
<tr>
<td>$^{14}$N</td>
<td>0.156</td>
<td>$1.15\times10^6$</td>
<td>$^{33}$S</td>
<td>0.249</td>
<td>$2.60\times10^6$</td>
</tr>
<tr>
<td>$^{15}$N</td>
<td>9.772</td>
<td>$1.58\times10^{10}$</td>
<td>$^{34}$S</td>
<td>5.38</td>
<td>$2.95\times10^9$</td>
</tr>
<tr>
<td>$^{16}$O</td>
<td>10.42</td>
<td>$1.90\times10^{10}$</td>
<td>$^{35}$Cl</td>
<td>4.854</td>
<td>$2.22\times10^9$</td>
</tr>
<tr>
<td>$^{17}$O</td>
<td>8.480</td>
<td>$1.06\times10^{10}$</td>
<td>$^{36}$A</td>
<td>0.7096</td>
<td>$1.99\times10^7$</td>
</tr>
<tr>
<td>$^{18}$O</td>
<td>14.06</td>
<td>$4.51\times10^{10}$</td>
<td>$^{37}$Cl</td>
<td>0.1675</td>
<td>$1.30\times10^6$</td>
</tr>
<tr>
<td>$^{19}$F</td>
<td>4.819</td>
<td>$2.18\times10^{9}$</td>
<td>$^{38}$A</td>
<td>4.917</td>
<td>$2.30\times10^9$</td>
</tr>
<tr>
<td>$^{20}$Ne</td>
<td>7.026</td>
<td>$6.20\times10^9$</td>
<td>$^{39}$K</td>
<td>0.565</td>
<td>$1.24\times10^7$</td>
</tr>
<tr>
<td>$^{21}$Ne</td>
<td>5.686</td>
<td>$3.44\times10^9$</td>
<td>$^{40}$Ca</td>
<td>1.312</td>
<td>$7.85\times10^7$</td>
</tr>
<tr>
<td>$^{22}$Ne</td>
<td>10.85</td>
<td>$2.13\times10^{10}$</td>
<td>$^{41}$K</td>
<td>2.492</td>
<td>$3.78\times10^8$</td>
</tr>
<tr>
<td>$^{23}$Na</td>
<td>4.374</td>
<td>$1.67\times10^9$</td>
<td>$^{42}$Ca</td>
<td>3.521</td>
<td>$9.34\times10^8$</td>
</tr>
<tr>
<td>$^{24}$Mg</td>
<td>5.513</td>
<td>$3.16\times10^9$</td>
<td>$^{44}$Ca</td>
<td>5.659</td>
<td>$3.39\times10^9$</td>
</tr>
<tr>
<td>$^{25}$Mg</td>
<td>3.833</td>
<td>$1.17\times10^9$</td>
<td>$^{48}$Ti</td>
<td>3.990</td>
<td>$1.30\times10^9$</td>
</tr>
<tr>
<td>$^{26}$Mg</td>
<td>9.325</td>
<td>$1.38\times10^{10}$</td>
<td>$^{52}$Cr</td>
<td>3.976</td>
<td>$1.29\times10^9$</td>
</tr>
<tr>
<td>$^{27}$Al</td>
<td>2.609</td>
<td>$4.25\times10^8$</td>
<td>$^{56}$Fe</td>
<td>3.695</td>
<td>$1.06\times10^9$</td>
</tr>
</tbody>
</table>

David Arnett, *Supernovae and Nucleosynthesis*
Urca pairs: cooling vs heating

mass parabola for isobaric chain

$E_f \propto \rho^{1/3}$

Urca cooling

$(A,Z) + e^- = (A,Z-1) + \nu_e$

$(A,Z-1) = (A,Z) + e^- + \bar{\nu}_e$

heating

$(A,Z) + e^- = (A,Z-1) + \nu_e$

$(A,Z-1) + e^- = (A,Z-2)^* + \nu_e + \gamma$
Description of electron capture and beta decay rates

Both rates are given by a thermal average over states in the initial nucleus:

$$\lambda = \frac{\sum_{if} (2J_i + 1)\lambda_{if} e^{-E_i/(kT)}}{\sum_i (2J_i + 1) e^{-E_i/(kT)}}$$

Allowed approximation (Gamow-Teller transitions)

$$\lambda_{if} = \frac{\ln 2}{K} B_{if} \Phi(q_{if}, \mu_e, T), \quad K = 6144 \text{ s}$$

- $B_{if}$: transition matrix element. Most of the relevant transitions are experimentally known. Shell-model calculations are possible.
- $\Phi(q_{if}, \mu_e, T)$: “trivial” phase space integral that accounts for the strong sensitivity of rates to temperature and density. Implementation in stellar evolution codes requires special care.
What to include in a weak interaction rate table?

- Directly the rates: Requires very fine grids in density and temperature to achieve accurate interpolations. Particularly relevant at the low temperatures relevant for ONeMg core evolution.

- Instead of the rate tabulate an effective matrix element (Fuller, Fowler and Newmann 1985). For electron capture

\[
\lambda^{ec} = \frac{\ln 2}{K} B_{eff} \Phi^{ec}(q_{gs}, \mu_e, T), \quad q_{gs} = Q_{gs}/(m_e c^2)
\]

Phase space can be expressed via Fermi integrals:

\[
\Phi^{ec}(Q, \mu_e, T) = \left( \frac{kT}{m_e c^2} \right)^5 \left\{ F_4 \left( \frac{\mu_e - Q}{kT} \right) + 2 \frac{Q}{kT} F_3 \left( \frac{\mu_e - Q}{kT} \right) + \left( \frac{Q}{kT} \right)^2 F_2 \left( \frac{\mu_e - Q}{kT} \right) \right\}
\]

Allows to use approximate expressions for Fermi integrals: fast and accurate up to 10-20%.

- An extension to $\beta^-$ decay is necessary.
Example: Electron capture on $^{23}\text{Na}$


- Direct interpolation in sparse density grid results in 1-2 orders of magnitude uncertainty.
- Interpolation matrix element results in a maximum error of a factor 2.
How to do better?

In general, all rates relevant for ONeMg core evolution are determined by a few transitions. It is possible to provide analytical expressions for each individual rate [GMP+, PRC 89, 045806 (2014)]

\[
\begin{array}{c|c}
\hline
\text{State} & \text{Rate} \\
\hline
\text{1}^+ & 1.057 \\
\text{2}^+ & 0.0 \\
\text{20}^9 \text{p} & 11.163 \text{ s} \\
\end{array}
\]

- Low densities (all temperatures): Rate determined by \( 2^+ \rightarrow 2^+ \) (\( Q = 5.902 \text{ MeV} \)) transition (experimentally known from beta decay).

- Intermediate densities (\( T < 0.9 \text{ GK} \)): determined second forbidden transition \( 0^+ \rightarrow 2^+ \) (\( Q = 7.536 \text{ MeV} \)) (only an experimental limit)

- Higher densities: transition \( 0^+ \rightarrow 1^+ \) (\( Q = 8.592 \text{ MeV} \) determines rate (experimentally known from \((p, n)\) charge exchange).
Electron capture on $^{20}\text{Ne}$

$T = 0.4 \text{ GK}$

$T = 1 \text{ GK}$

Mayor uncertainty is due to second forbidden transition.
Second forbidden calculation

\[ \lambda = \frac{\ln 2}{K} \Phi^{2nd}(q, \mu_e, T) \]

\[ \Phi^{2nd}(q, \mu_e, T) = \int_{q}^{\infty} wp(q + w)^2 C(w)F(Z, w) f_e(w, \mu_e, T) dw \]

- \( C(w) \) is the shape factor: Linear combination of matrix elements and energy factors.
- Relevant matrix elements (Behrens & Bühring 1971)
  \[ V F_{211} \sim \left[ r \otimes p_{if} \right]^2 t_+, \quad p_{if} = (p_i + p_f)/2 \]
  \[ V F_{220} \sim r^2 Y_2 t_+ \]
  \[ A F_{221} \sim r^2 \left[ Y_2 \otimes \sigma \right]^2 t_+ \]
Shell-model calculations

sd-shell shell-model calculation using USDB interaction (Idini, Brown, Langanke, GMP, in preparation)

<table>
<thead>
<tr>
<th></th>
<th>Harmonic Oscillator</th>
<th>Wood–Saxon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V F_{211}$</td>
<td>0.</td>
<td>0.0048</td>
</tr>
<tr>
<td>$V F_{220}$</td>
<td>0.8035</td>
<td>1.3353</td>
</tr>
<tr>
<td>$A F_{221}$</td>
<td>0.2423</td>
<td>0.3257</td>
</tr>
</tbody>
</table>

The beta-decay theoretical matrix element is $B = \langle C(w) \rangle = 1.36 \times 10^{-7}$ using $g_A = 1.27 \ (1.11 \times 10^{-7} \ for \ g_A = 1.0)$.

The experimental upper limit is $1.94 \times 10^{-7}$. 
Impact on electron capture and beta decay

Blue dashed: Experimental limit
Red: Wood-Saxon wave functions
Black: Harmonic oscillator wave functions
Screening of weak interaction rates

The presence of a degenerate electron background can affect both beta-decays and electron capture rates:

- Correction to nuclear binding energy (DeWitt, Graboske, and Cooper 1973; Hix and Thielemann 1996, Bravo and García-Senz 1999, Juodagalvis et al. 2010). Q-value increases by 0.1–0.3 MeV.

- Correction to electron energy (Itoh et al. 2002). Chemical potential reduced by 0.02–0.05 MeV.

- Net effect is a reduction of electron capture rate and an increase of the beta-decay rate.

Having an analytical scheme allows to consider screening corrections consistent with the underlying EoS.
Impact evolution core

Based on ONeMg cores from Schwab, Quataert, and Bildsten 2015. Convection does not develop in the core.

How sensitive is this result to the set of nuclear reactions included?
Larger network

Möller, Jones, GMP, in preparation

Increased to account for possible role of $^{20}\text{O}(\alpha, n)^{23}\text{Ne}$. This rate dominates over $^{20}\text{Ne}(\alpha, \gamma)^{24}\text{Mg}$ during Neon burning.
Evolution larger network

Convection does in fact develops in some of the models.
Evolution very sensitive to variations of $\text{^{20}O(\alpha, n)^{23}Ne}$ rate. It may affect the density at which oxygen deflagration initiates.
O DEFLAGRATION
MULTI-DIMENSIONAL SIMULATIONS


Isothermal ONe core/WD in HSE with a **range of central (ignition) densities**

**Centrally-confined ignition**: 300 'bubbles' within 50 km sphere, \(< 5 \times 10^{-4} \, M_\odot\)** inside initial flame surface

In **laminar regime**, flame speeds from Timmes+ (1992);
in **turbulent regime**, flame speeds from subgrid scale model of turbulence (Schmidt+ 2006)
O DEFLAGRATION

3D $4\pi: 512^3$

THERMONUCLEAR EXPLOSION?
O DEFLAGRATION

3D $4\pi \times 512^3$

THERMONUCLEAR EXPLOSION?

Scale: 2500 km
Time: 1.3 s

56 Ni
Scale: 400,000 km
Time: 60 s

O DEFLAGRATION

3D $4\pi \cdot 512^3$

THERMONUCLEAR EXPLOSION?

$^{56}\text{Ni}$
\( \rho_{\text{ign}} = 10^{9.9} \text{ g cm}^{-3} \)

THERMONUCLEAR EXPLOSION?
\[ \rho_{\text{ign}} = 10^{10.2} \text{ g cm}^{-3} \]

CORE COLLAPSE
Heavy elements and metal-poor stars


- Stars rich in heavy r-process elements ($Z > 50$) and poor in iron (r-II stars, [Eu/Fe] > 1.0).
- Robust abundance pattern for $Z > 50$, consistent with solar r-process abundance.
- These abundances seem the result of events that do not produce iron. [Qian & Wasserburg, Phys. Rept. 442, 237 (2007)]
- Possible Astrophysical Scenario: Neutron star mergers.

- Stars poor in heavy r-process elements but with large abundances of light r-process elements (Sr, Y, Zr)
- Production of light and heavy r-process elements is decoupled.
- Astrophysical scenario: neutrino-driven winds from core-collapse supernova

Nucleosynthesis in neutrino-driven winds

Main processes:

\[
\nu_e + n \leftrightarrow p + e^- \\
\bar{\nu}_e + p \leftrightarrow n + e^+ 
\]

Neutrino interactions determine the proton to neutron ratio.

Neutron-rich ejecta:

\[
\langle E_{\bar{\nu}_e} \rangle - \langle E_{\nu_e} \rangle > 4\Delta np - \left[ \frac{L_{\bar{\nu}_e}}{L_{\nu_e}} - 1 \right] \left[ \langle E_{\bar{\nu}_e} \rangle - 2\Delta np \right]
\]

- neutron-rich ejecta: r-process
- proton-rich ejecta: \( \nu p \)-process

We need accurate knowledge of \( \nu_e \) and \( \bar{\nu}_e \) spectra
Weak rates in the decoupling region

Neutrino mean-free paths at high densities:

- $\nu_e$ emission: mainly determined by charged-current $\nu_e + n \leftrightarrow p + e^-$. Depends on equation of state properties.

- $\bar{\nu}_e$ emission: strong sensitivity to the processes considered and equation of state properties.

Neutrino interactions at high densities

Most of Equations of State treat neutrons and protons as “non-interacting” (quasi)particles that move in a mean-field potential $U_{n,p}(\rho, T, Y_e)$.

\[ E_n = \frac{p_n^2}{2m_n^*} + m_n^* + U_n \]

\[ E_p = \frac{p_p^2}{2m_p^*} + m_p^* + U_p \]

\[ Q = m_n^* - m_p^* + U_n - U_p \]

- Energy difference between neutrons and protons is directly related to nuclear symmetry energy.
- Symmetry energy enhances $\nu_e$ absorption and suppresses $\bar{\nu}_e$ absorption.
- Symmetry energy determines the spectral differences between $\nu_e$ and $\bar{\nu}_e$ and consequently the nucleosynthesis.

GMP, Fischer, Lohs, Huther, PRL 109, 251104 (2012)
Constrains in the symmetry energy

- Combination nuclear physics experiments and astronomical observations (Lattimer & Lim 2013)
- Isobaric Analog States (Danielewicz & Lee 2013)
- Chiral Effective Field Theory calculations (Drischler+ 2014)

Figure data from Matthias Hempel (Basel)
Impact on neutrino luminosities and $Y_e$ evolution

1D Boltzmann transport radiation simulations (artificially induced explosion) for a $11.2 \, M_\odot$ progenitor based on the DD2 EoS (Stefan Typel and Matthias Hempel).

$Y_e$ is moderately neutron-rich at early times and later becomes proton-rich.
Nucleosynthesis

Elements between Zn and Mo ($A \sim 90$) are produced

Mainly neutron-deficient isotopes are produced

Uncertainties: Equation of State, neutrino reactions (mainly $\bar{\nu}_e$), Neutrino oscillations(?).
Neutron decay

The neutron-proton energy difference in the medium could be of the order of several 10s MeV. Neutron decay is important for low energy neutrinos.

\[
\bar{\nu}_e + p \leftrightarrow n + e^+
\]

\[
\bar{\nu}_e + e^- + p \leftrightarrow n
\]

This is part of the direct URCA process in neutron stars [Lattimer et al, (1991)]

Fischer, Lohs, GMP, Qian, in preparation
Additional opacity channels for $\bar{\nu}_e$

![Graph showing additional opacity channels for $\bar{\nu}_e$]
Summary

- Most of the weak interaction rates relevant for ONeMg cores evolution are well constrained by experimental data.
- Challenge: accurate and fast implementation of rates in stellar evolutionary codes.
- Core evolution sensitive to weak rates and thermonuclear rates.
- Final outcome sensitive to density of oxygen ignition. 3D simulations by Jones et al
- Electron capture supernova constitute an ideal test ground to explore the impact of neutrino opacities on heavy element nucleosynthesis.
- It is important to improve the description of $\bar{\nu}_e$ opacities in transport codes.