Beta transition rates and EOS for massive stars

(New Nuclear Theory: TOAMD)

Hiroshi Toki (RCNP/Osaka)

with

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Toshio Suzuki (Nihon)
Ken'ichi Nomoto (WPI/Tokyo)
Hong Shen (Nankai)
Jinniu Hu (Nankai)

STELLAR EVOLUTION

A CRASH COURSE

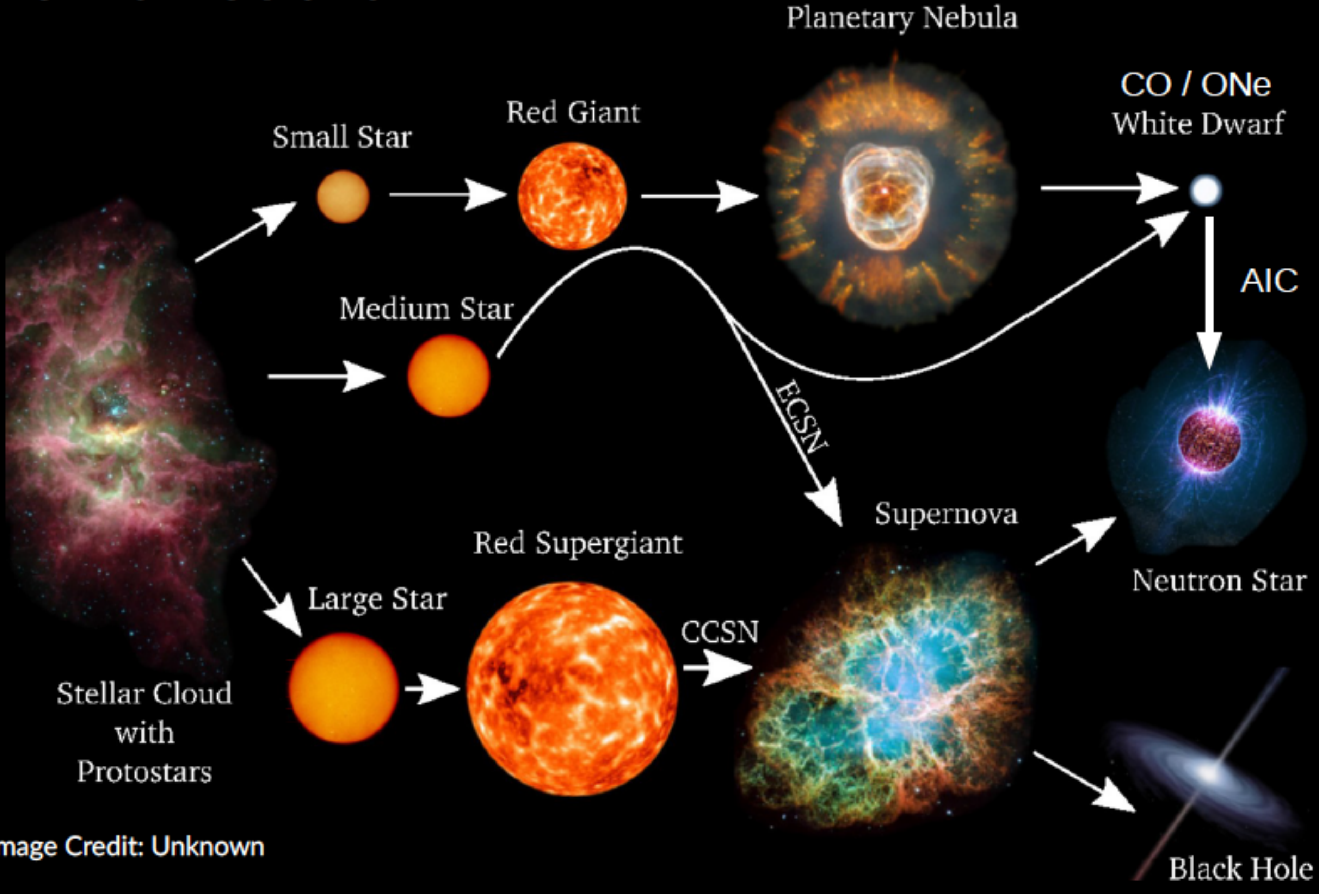


Image Credit: Unknown

My desire:

1. Provide best NP results for astrophysics
2. EOS table: H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi Nucl. Phys. A637 (1998) 435
3. Beta transition rate for nuclear URCA: H. Toki, T. Suzuki, K. Nomoto, S. Jones, R. Hirschi, Phys. Rev. C88 (2013) 015806

Content:

1. Beta transition rates for URCA nuclei
2. EOS table for supernova and neutron star (URCA)
3. New theory using nuclear interaction (TOAMD)

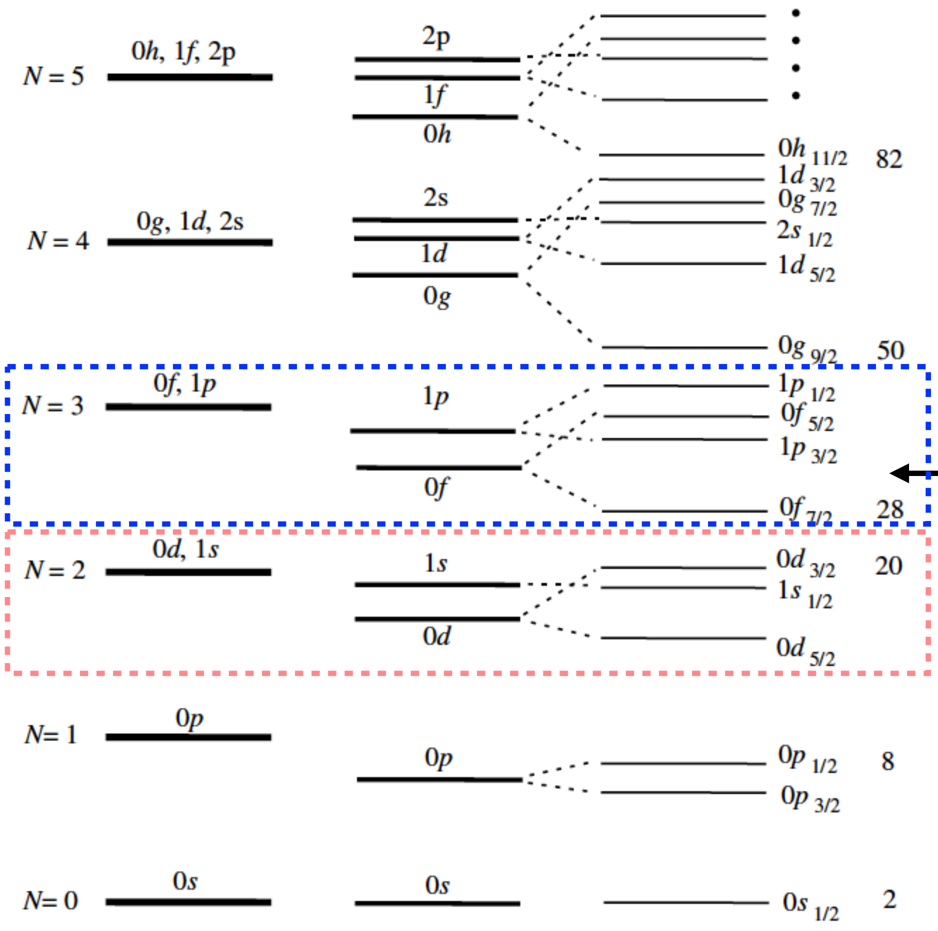
Beta transition rates in sd-shell nuclei

Several works:

1. T.Kajino, E.Shiino, H.Toki, A.Brown, H.Wildenthal, NP (1988)
2. T.Oda et al, Atomic and nuclear data table (1994)
3. H.Toki, T.Suzuki, K.Nomoto, S.Jones, R.Hirschi, PR (2013)
4. T.Suzuki, H.Toki, K.Nomoto, APJ (2016)

These works are all based on the shell model for sd-shell nuclei by A. Brown and H. Wildenthal

Nuclear shell structure



^{54}Fe (Most stable)

$pf-shell$

$sd-shell$

(Suzuki)

Ann. Rev. Nucl. Part. Sci. 1988. 38: 29–66

STATUS OF THE NUCLEAR SHELL MODEL

B. A. Brown *B. H. Wildenthal*

Phenomenological
$sd-shell$

$$H\Psi = E\Psi$$

$$H = \sum_{i=1}^{A} [T+U]_i + \sum_{i\neq j}^{A} V_{ij}$$

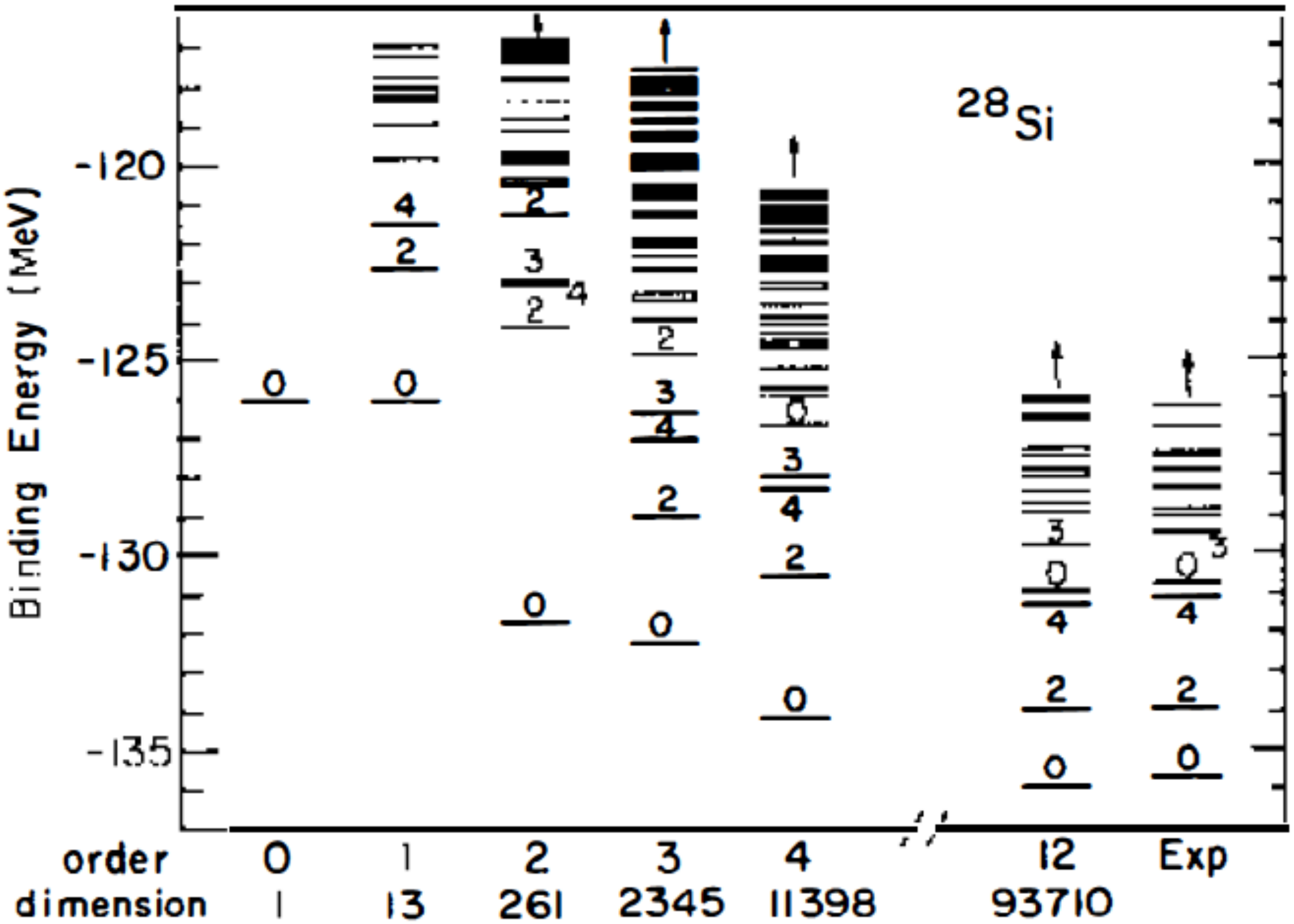
$$\Psi = \sum_C A_C \left| (s)^{n1} (d_{3/2})^{n2} (d_{5/2})^{n3} \right\rangle$$

No. of parameters	
ε_j	3
$\langle sd \| V \| sd \rangle$	63

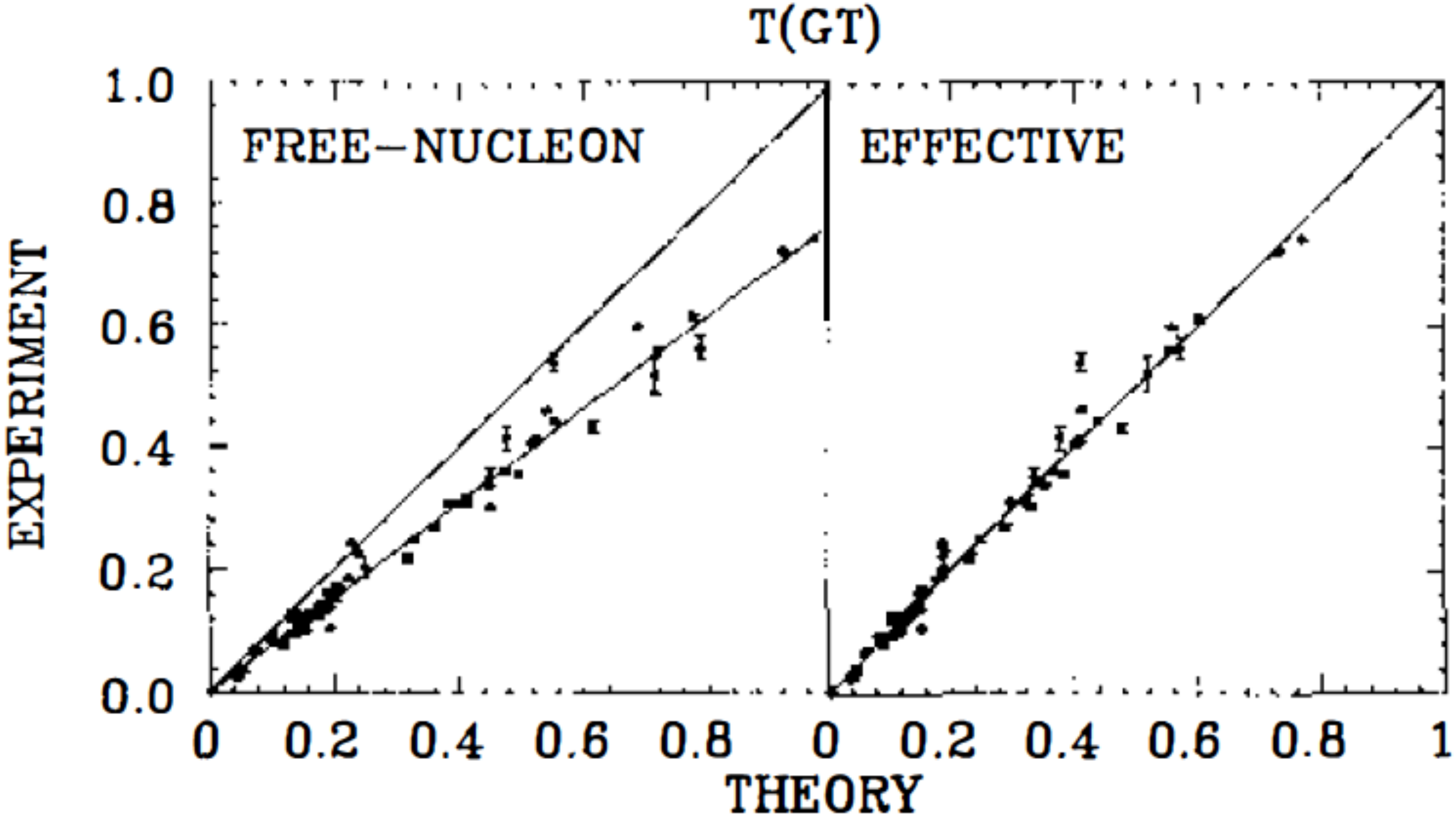
$$[T+U]\psi_j = \varepsilon_j \psi_j$$

$$\left\langle (s)^{n1} (d_{3/2})^{n2} (d_{5/2})^{n3} \middle| V \middle| (s)^{n1} (d_{3/2})^{n2} (d_{5/2})^{n3} \right\rangle = \sum C \langle sd | V | sd \rangle$$

Level scheme



$\langle f|\sigma\tau|i\rangle$ Beta transition rate



$$\tilde{g}_A = 0.77\, g_A$$

PHYSICAL REVIEW C **88**, 015806 (2013)

Detailed β-transition rates for URCA nuclear pairs in 8–10 solar-mass stars

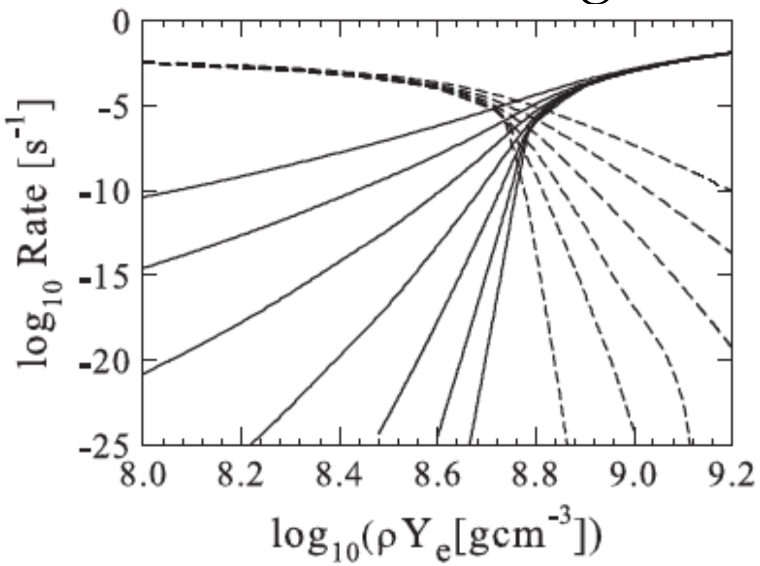
Hiroshi Toki,[1,*] Toshio Suzuki,[2,†] Ken'ichi Nomoto,[3,‡] Samuel Jones,[4] and Raphael Hirschi[4,3]

URCA process for rapid cooling of star

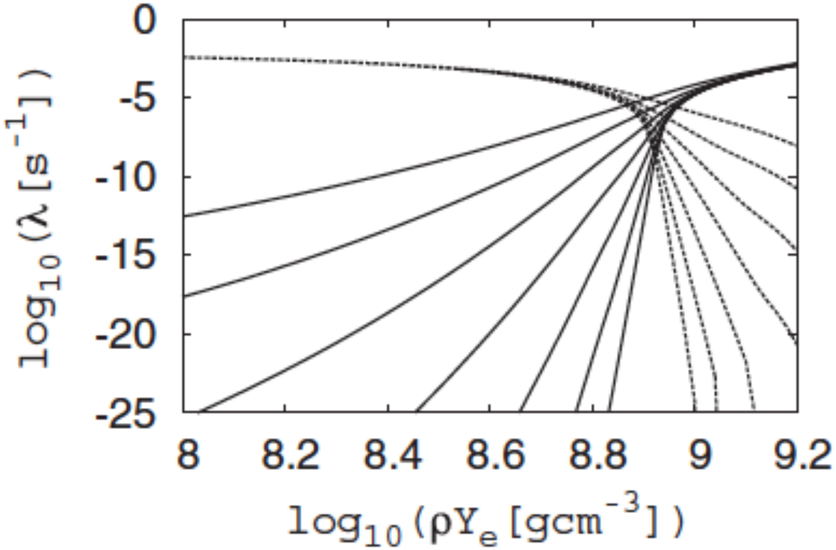
$$(A,Z)+e^- \rightarrow (A,Z-1)+\nu$$

$$(A,Z-1) \rightarrow (A,Z)+e^- +\bar{\nu}$$

$$^{25}Na - {}^{25}Mg$$

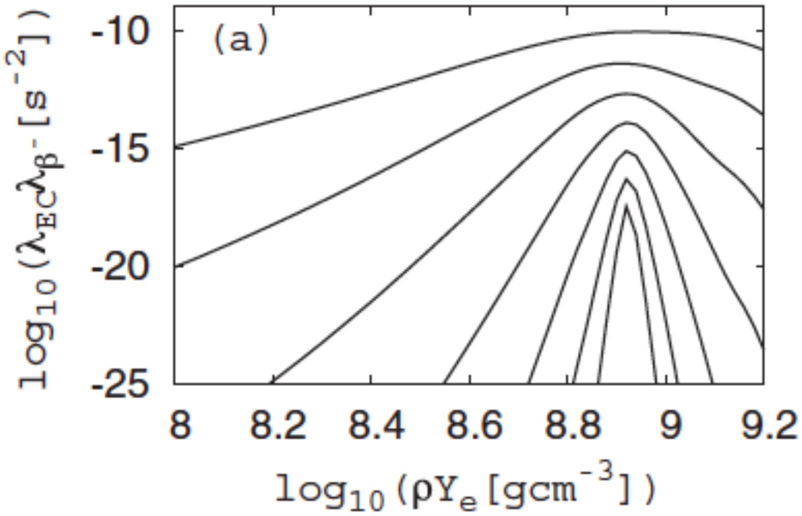


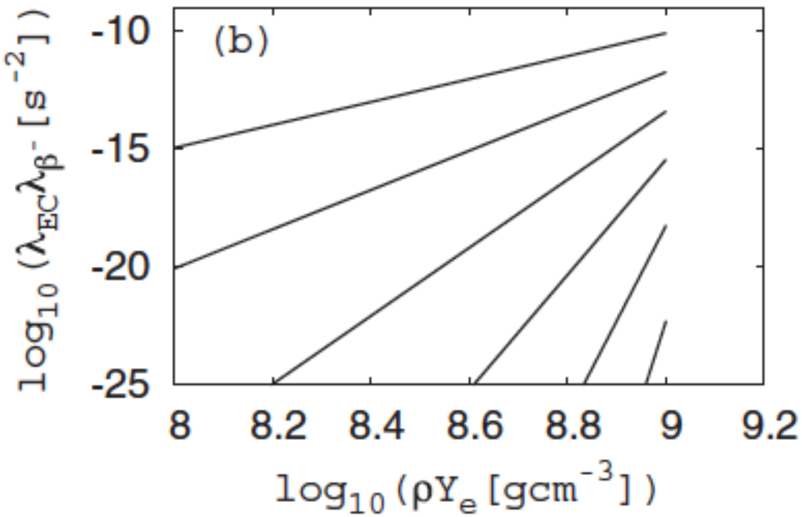
$$^{23}Na - {}^{23}Ne$$



sd-shell model calculation

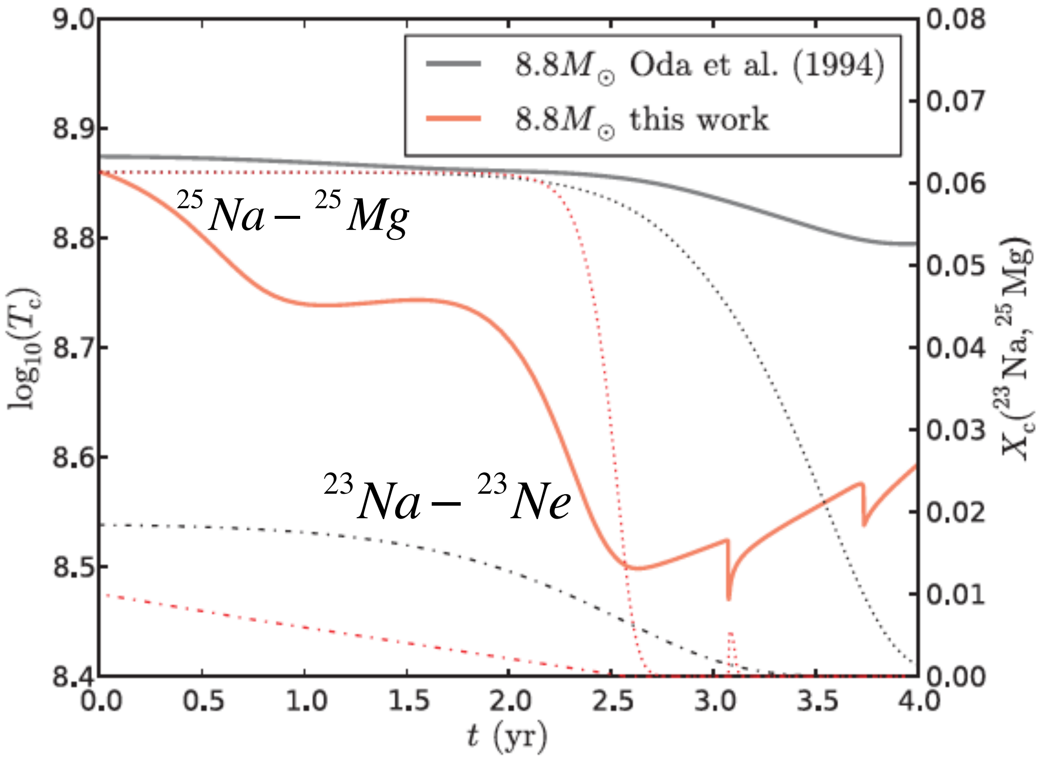
Toki-Suzuki table





Oda table

Jones calculation



other beta rates

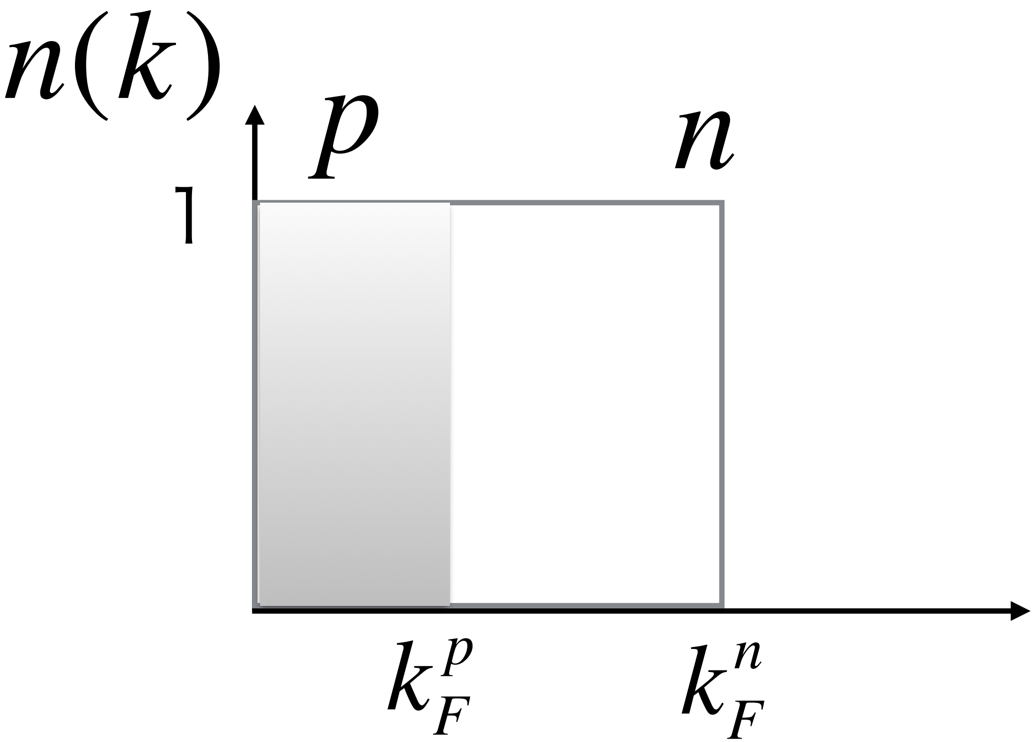
Suzuki, Toki, Nomoto

Astro J. (2016)

URCA process for neutron star cooling

$$p + e^- \rightarrow n + \nu$$

$$n \rightarrow p + e^- + \bar{\nu}$$



$$\mu_n = \mu_p + \mu_e$$

(Beta equilibrium)

$$k_F^p = k_F^e$$

(Charge neutrality)

Momentum conservation

$$k_F^p + k_F^e > k_F^n$$

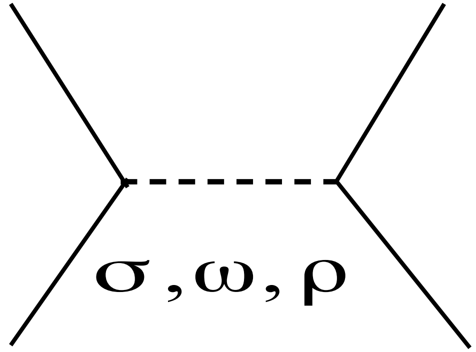
$$(2k_F^p)^3 > (k_F^n)^3$$

$$8\rho_p > \rho_n$$

$$\frac{\rho_p}{\rho} > \frac{1}{9}$$

EOS with the RMF theory (Shen EOS)

Shen Toki Oyamatsu Sumiyoshi
Nucl. Phys. A637 (1998) 435



Relativistic Mean Field Theory (RMF)

mean-field approximation: *meson field operators are replaced by their expectation values*

no-sea approximation: *contributions from the negative-energy Dirac sea are ignored*

Applications

	flavor SU(2)	flavor SU(3)
infinite matter:	*nuclear matter*	*strange hadronic matter*
finite system:	*nuclei*	*hypernuclei*

Relativistic Mean Field Theory
(Phenomenological model)

Lagrangian

$$L = \bar{\psi}[i\gamma_\mu \partial^\mu - M - g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \gamma_\mu \tau_a \rho^{a\mu}]\psi$$
$$+ \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4$$
$$- \frac{1}{4} W_{\mu\upsilon} W^{\mu\upsilon} + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2$$
$$- \frac{1}{4} R^a_{\mu\upsilon} R^{a\mu\upsilon} + \frac{1}{2} m_\rho^2 \rho^a_\mu \rho^{a\mu} + \dots$$

6 parameters

TM1 parameter set

Lagrangian → Equations → Mean-Field Approximation

↓

Calculate everything such as $\varepsilon, p, s \dots$

EOS

Comparison with nuclear physics data

2157 nuclei

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}\left(M^{i}_{\text{theo}} - M^{i}_{\text{expt}}\right)^2}{n}} = 2.1$$

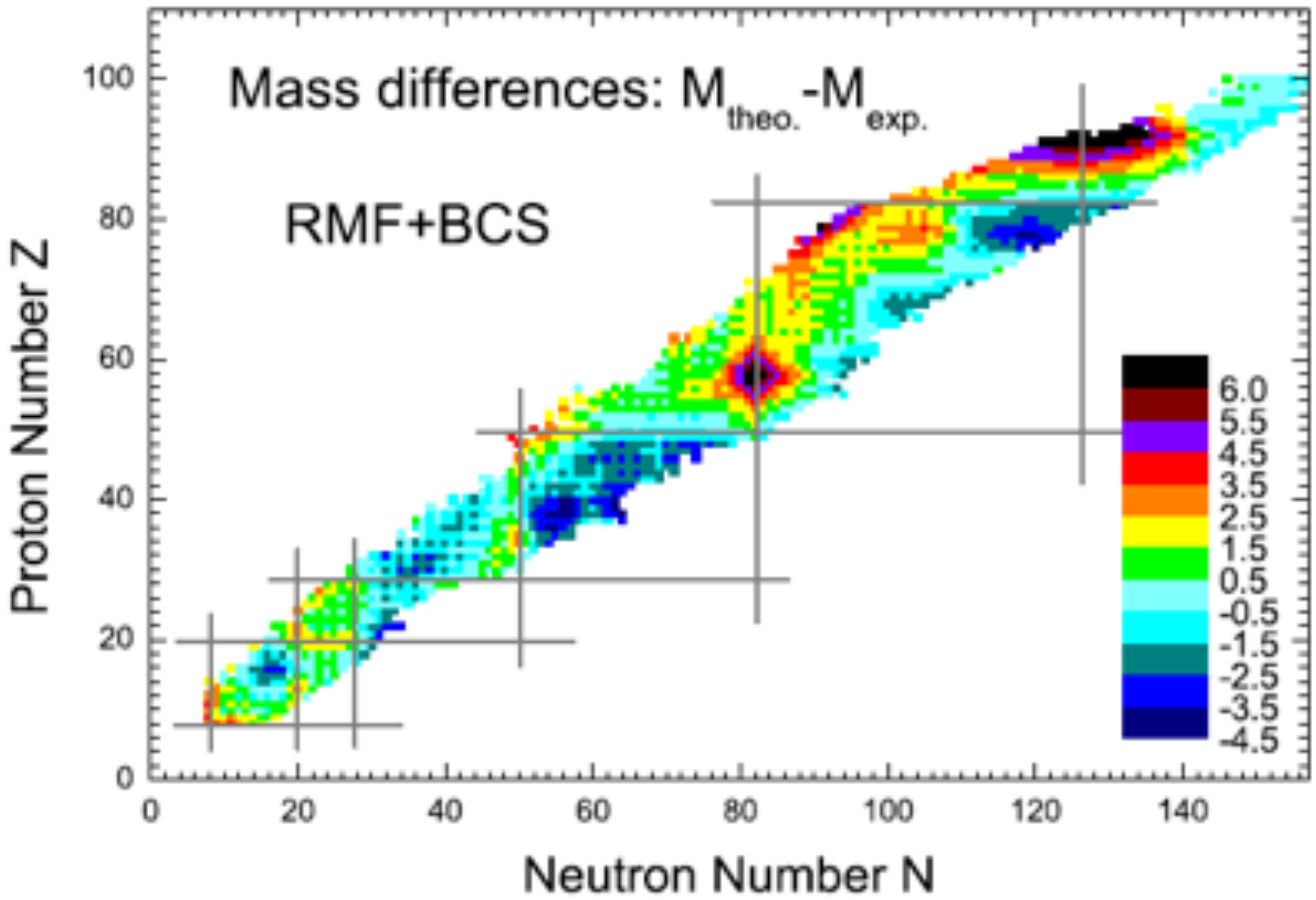


Fig. 2. Mass differences between the predictions of the present work and the experimental data for 2157 nuclei whose measured uncertainties for the masses are less than 0.2 MeV.[34)]

L. S. Geng, H. Toki, J. Meng, Prog. Theor. Phys. 113 (2005) 785

Shen EOS

H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi, Prog. Theor. Phys. 100, 1013 (1998)
H. Shen, H. Toki, K. Oyamatsu, K. Sumiyoshi, Astrophys. J. Suppl. 197, 20 (2011)

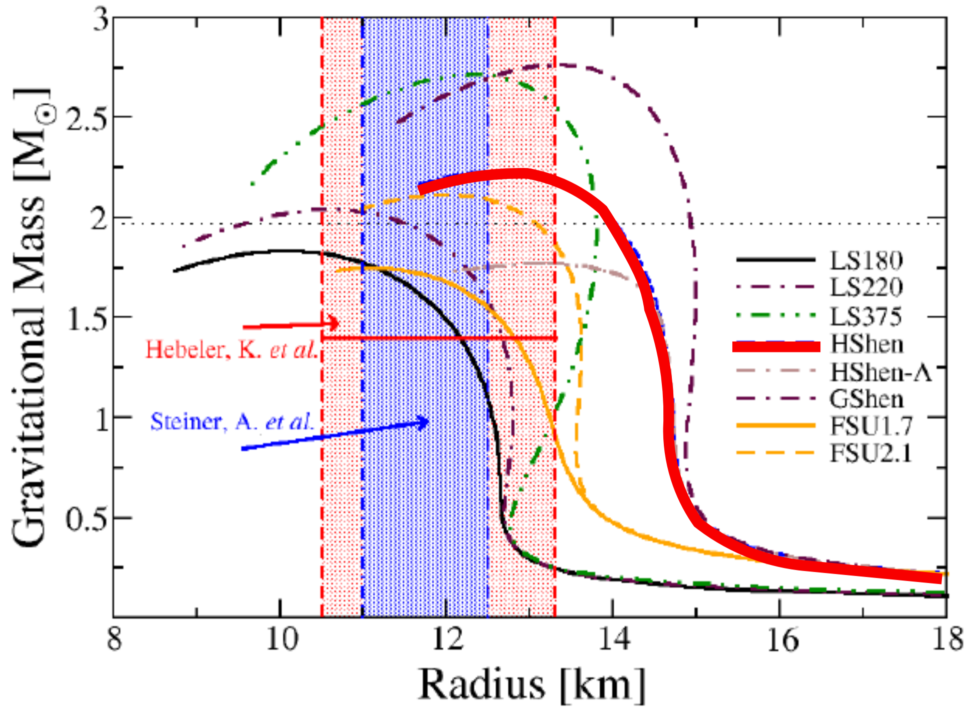


Figure by
Evan O'Connor

$$\rho_c \sim 2.2\rho_\odot$$

$$\rho_p \sim 0.08\rho$$

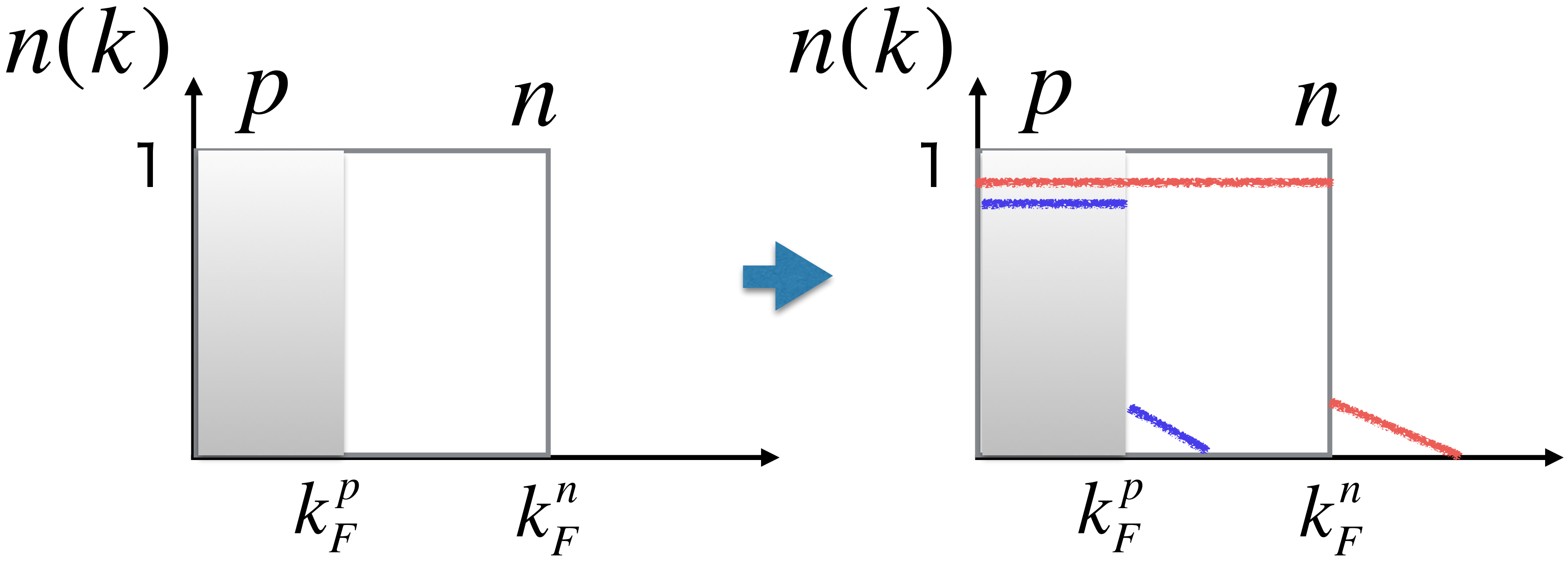
$$\frac{\rho_p}{\rho} > \frac{1}{9}$$ (URCA condition)

LS PSU (non-relativistic) soft EOS

H.Shen G.Shen (relativistic) hard EOS

URCA process does not occur for neutron star cooling

New theory with tensor and short range correlations



Brueckner-Hartree-Fock　　　　TOAMD

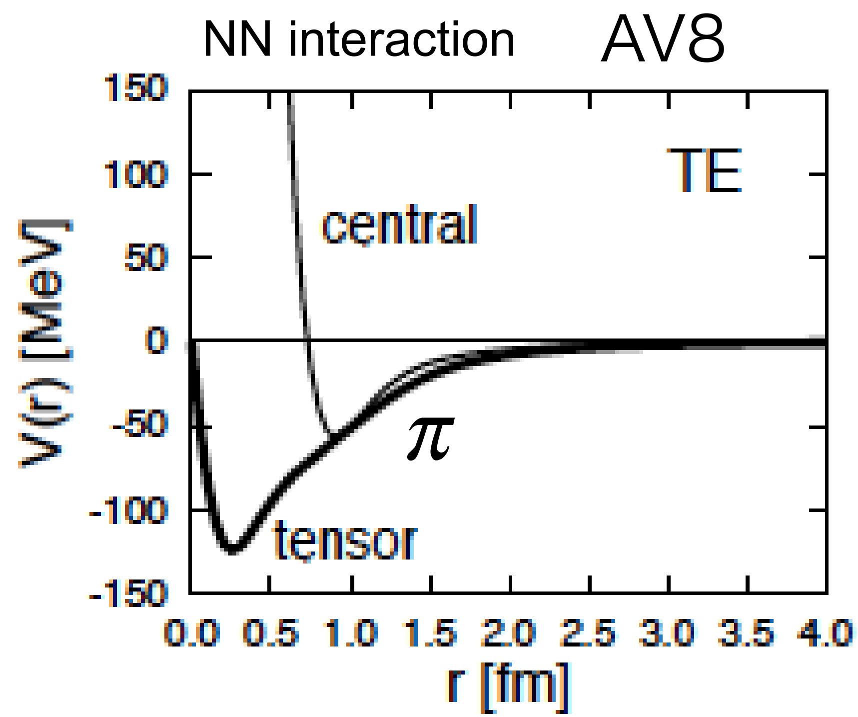
$\vec{k}_p + \vec{k}_e = \vec{k}_n$ becomes possible!!

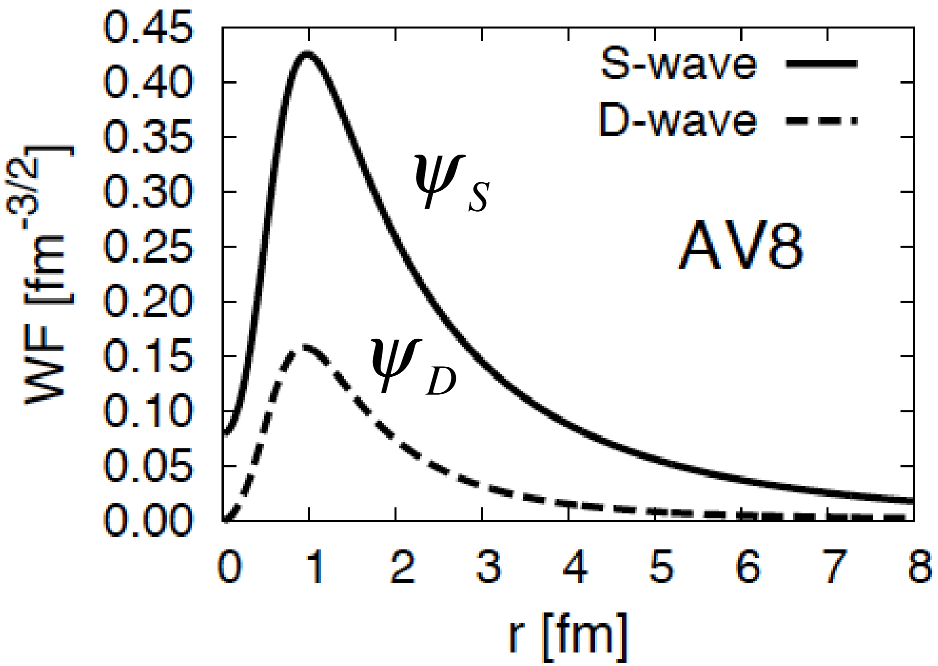
URCA becomes possible in neutron star cooling

Deuteron (1^+)

S=1 and L=0 or 2

$$\Psi_{J=1} = \psi_S Y_0 \chi_{S=1} + \psi_D \left[Y_2 \chi_{S=1} \right]_1$$





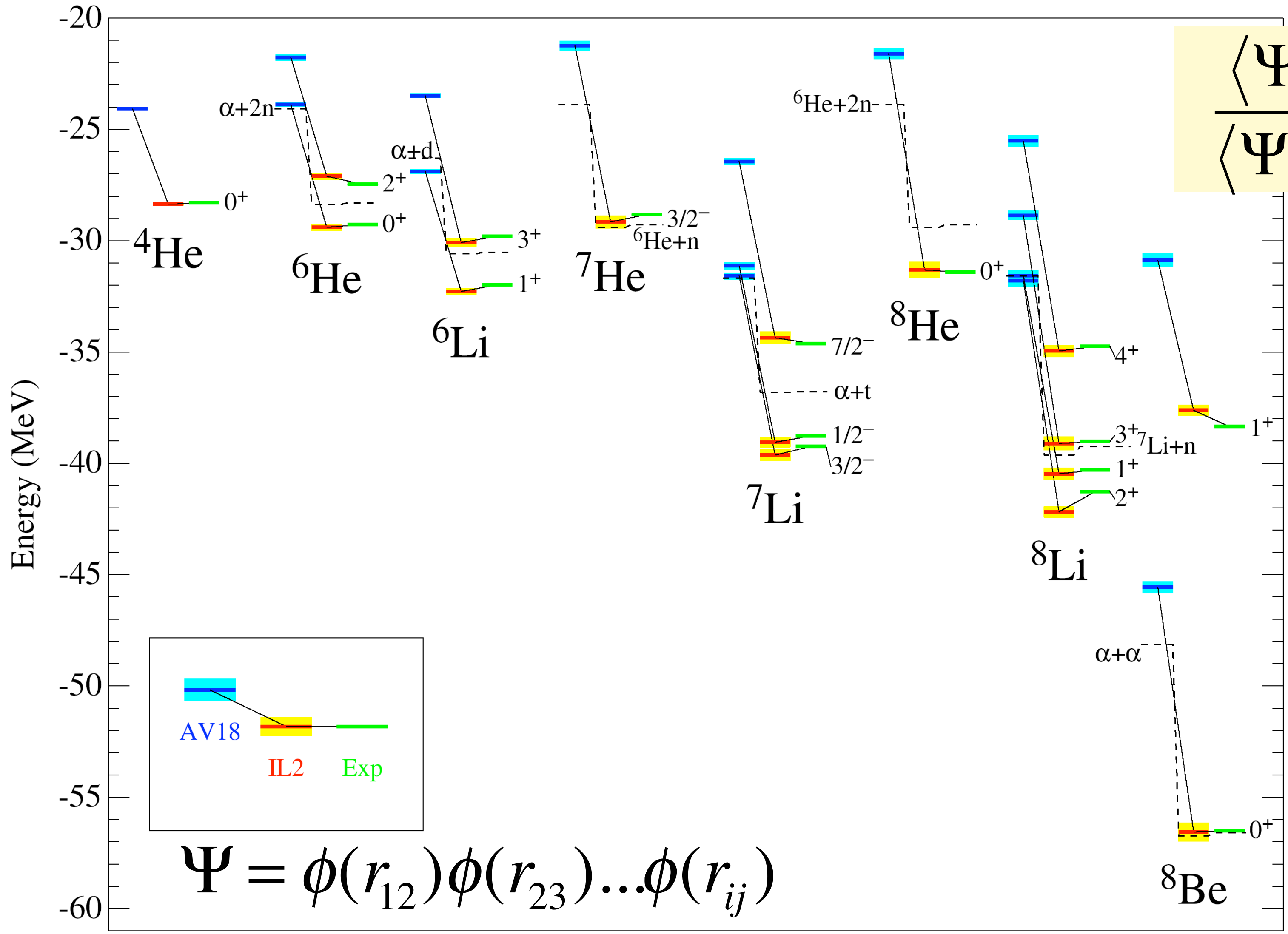
pion

Energy	−2.24 [MeV]
Kinetic	19.88
(SS)	11.31
(DD)	8.57
Central	−4.46
(SS)	−3.96
(DD)	−0.50
Tensorc	−16.64
(SD)	−18.93
(DD)	2.29
LS	−1.02
P(*D*)	5.78 [%]
Radius	1.96 [fm]
(SS)	2.00 [fm]
(DD)	1.22 [fm]

Variational calculation of light nuclei with NN interaction

VMC+GFMC

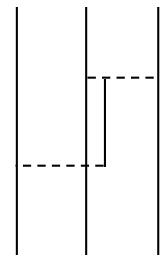
$$\frac{\langle \Psi | V_\pi | \Psi \rangle}{\langle \Psi | V_{NN} | \Psi \rangle} \sim 80\%$$



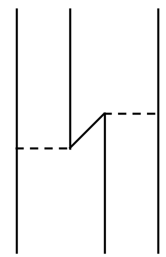
$$\Psi = \phi(r_{12})\phi(r_{23})\ldots\phi(r_{ij})$$

C. Pieper and R. B. Wiringa, Annu. Rev. Nucl. Part. Sci.51(2001)

V_{NNN}



Fujita-Miyazawa



Relativistic

Heavy nuclei (Super model)

Pion is key

Pion is important in nucleus

- 80% of attraction is due to pion
- Tensor interaction is particularly important and difficult to handle (50%)

Pion	Tensor	spin-spin

$$\frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{m_\pi^2 + q^2} = \frac{1}{3}\frac{q^2}{m_\pi^2 + q^2} S_{12}(\hat{q}) + \frac{1}{3}\frac{q^2}{m_\pi^2 + q^2}\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$= \frac{1}{3}\frac{q^2}{m_\pi^2 + q^2} S_{12}(\hat{q}) + \frac{1}{3}\left(1 - \frac{m_\pi^2}{m_\pi^2 + q^2}\right)\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

high momentum　　low momentum

Tensor Optimized Antisymmetrized Molecular Dynamics (TOAMD)

Myo Toki Ikeda

Tensor optimized shell model (TOSM)

1. We include tensor interaction most effectively to shell model
2. Difficult to treat cluster structure

+

Horiuchi Enyo Kimura..

Antisymmetrized molecular dynamics (AMD)

1. Cluster+shell structure is handled on the same footing with effective interaction
2. Difficult to treat bare nucleon-nucleon interaction

↓

Study nuclear structure based on nuclear interaction

PTEP

Prog. Theor. Exp. Phys. **2015**, 073D02 (38 pages)
DOI: 10.1093/ptep/ptv087

Tensor-optimized antisymmetrized molecular dynamics in nuclear physics

Takayuki Myo[1,2,*], Hiroshi Toki[2], Kiyomi Ikeda[3], Hisashi Horiuchi[2], and Tadahiro Suhara[4]

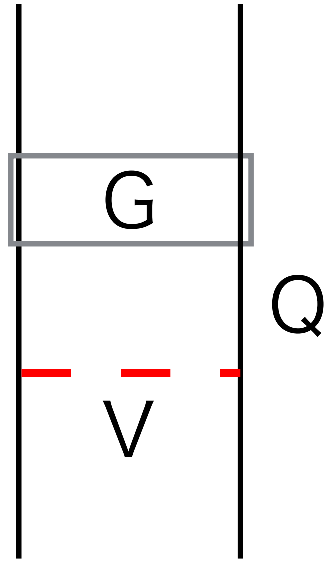
TOAMD project

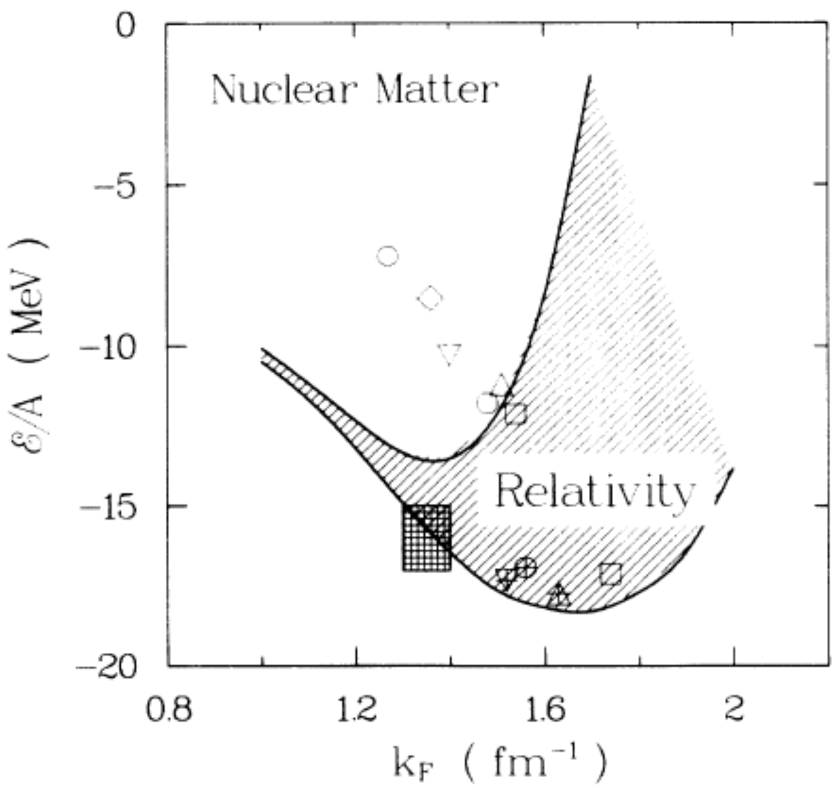
1. T. Myo : S-shell nuclei (He3, He4)
   Make fundamental programs and establish the TOAMD concept → (almost finished)
2. T. Suhara : P-shell nuclei
   Establish the treatment of shell structure
3. H. Toki, T. Yamada : Nuclear matter
   Study infinite matter
4. Many collaborations : China, Korea

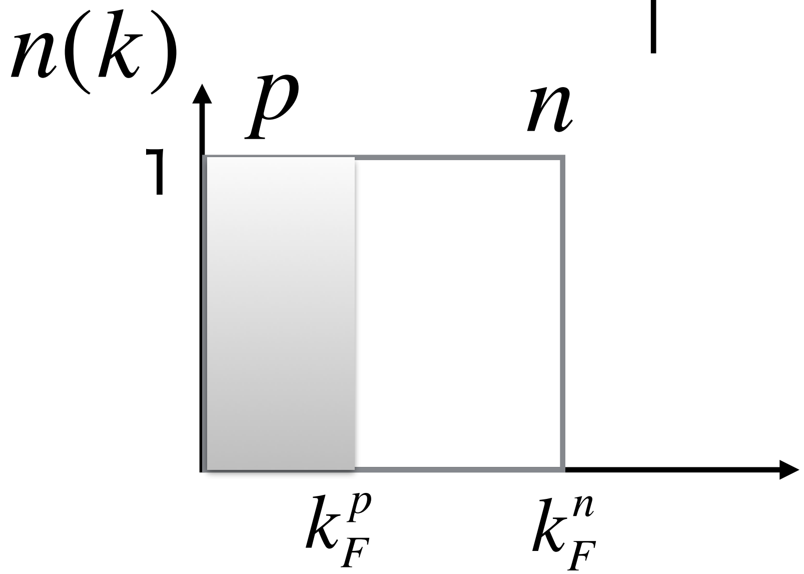
Nuclear Matter（Relativistic effect）

Brockmann Machleidt : PRC42(1990)1965

Relativistic Brueckner-Hartree-Fock with Bonn-potential







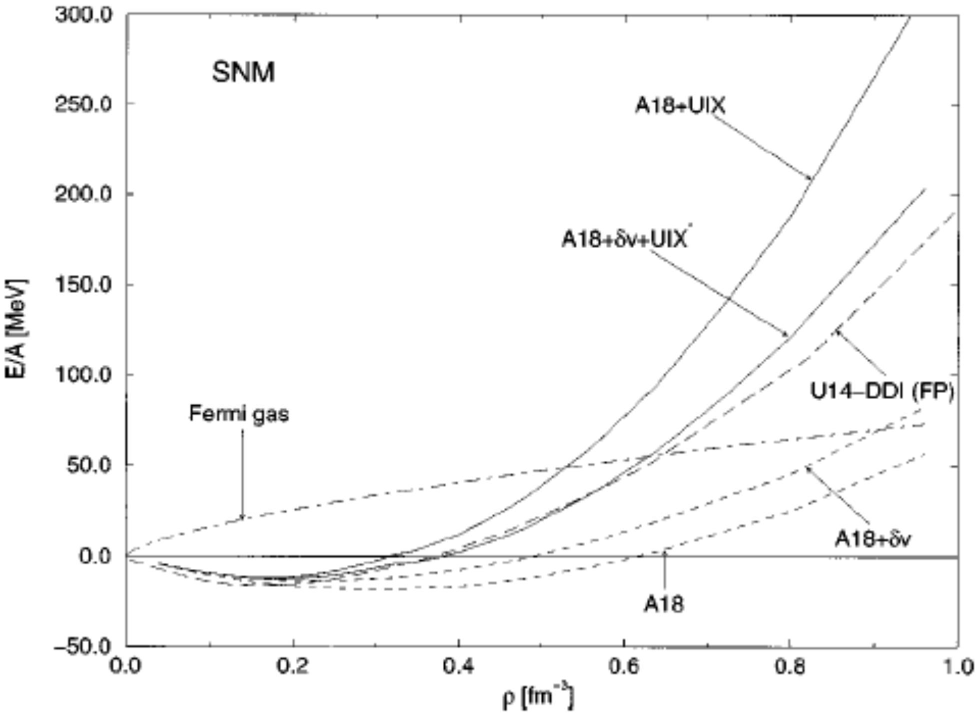
$$(\alpha \cdot p + \beta m + U)\tilde{\psi} = E\tilde{\psi}$$

$$U_k(\tilde{m}) = \sum_p \langle kp | G(\tilde{m}) | kp - pk \rangle$$

$$U = \beta U_S + U_V \qquad \tilde{m} = m + U_S(\tilde{m})$$

Infinite matter（non-relativistic framework：
3 body repulsion +Boost corrections)

Akmal Pandhyaripande Ravenhall : PRC58(1998)1804



Relativistic effect

Effective mass

＝3 body repulsion

C.M. boost effect

＝C.M. boost interaction

＋

3 body attraction（Δ）

Variational chain summation (VCS)

$$\Psi = \prod_{ij} \left(1 + F_{ij}\right) \Phi$$

F_{ij}^{p} correlation function

Extension of Hartree–Fock theory including tensor correlation in nuclear matter Prog. Theor. Exp. Phys. **2013**, 103D02 (17 pages)

Jinniu Hu[1,2,*,†], Hiroshi Toki[1,*,†], and Yoko Ogawa[1,*,†]

TOSM for relativistic matter

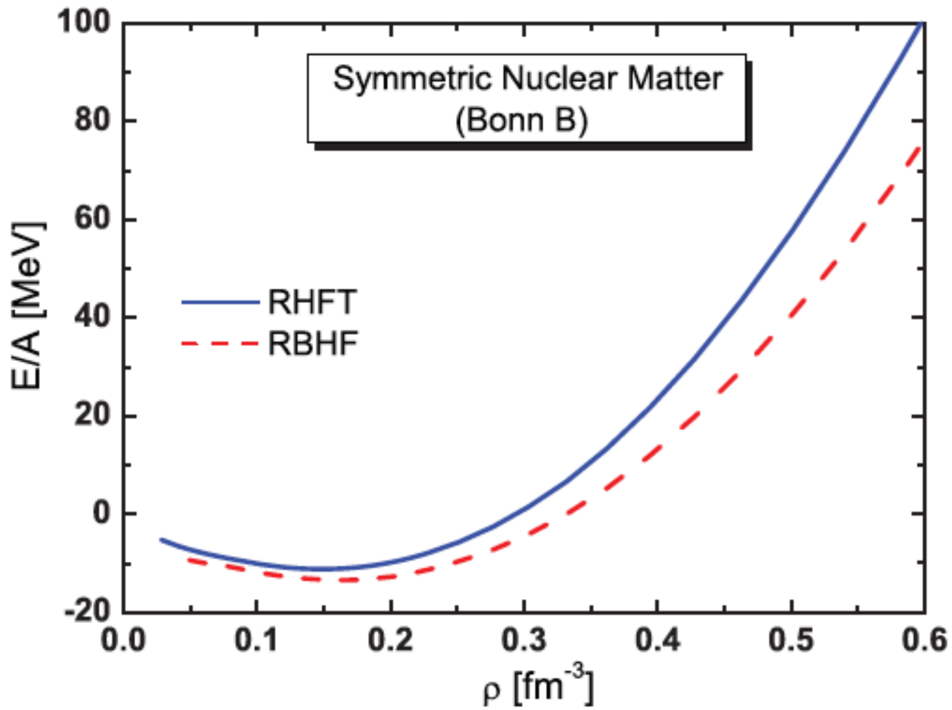
$$\Psi = C_0|0\rangle + \sum_\alpha C_\alpha |2p2h:\alpha\rangle \qquad |C_0|^2 + \sum_\alpha |C_\alpha|^2 = 1$$

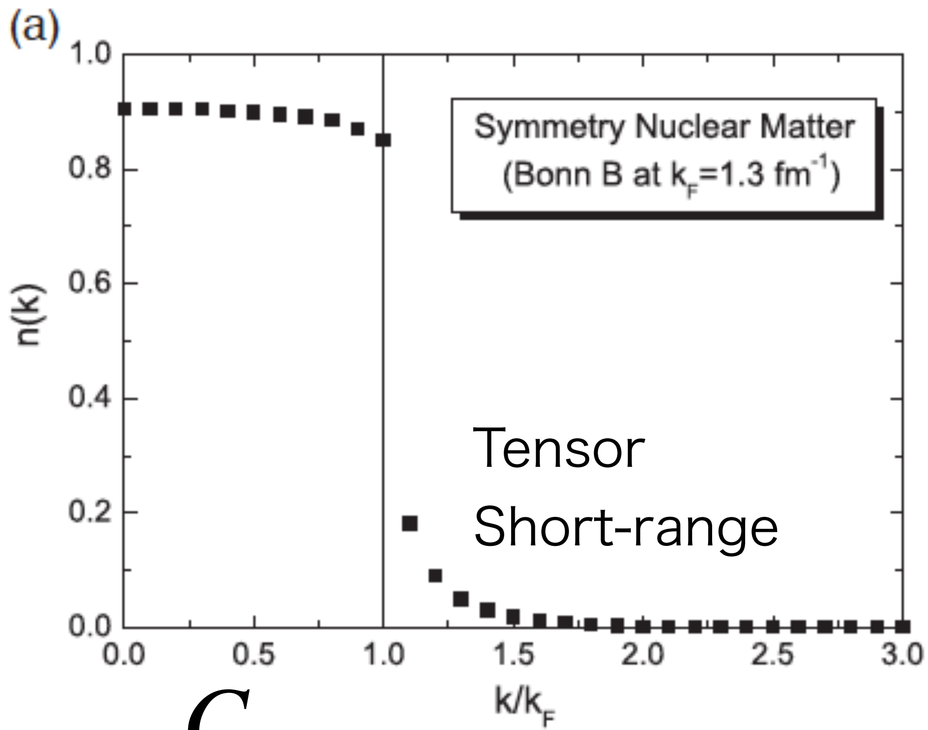
$$\langle\Psi|H|\Psi\rangle = |C_0|^2\langle 0|H|0\rangle + \sum_\alpha C_0^* C_\alpha \langle 0|H|\alpha\rangle + \sum_\beta C_\beta^* C_0 \langle\beta|H|0\rangle + \sum_{\alpha,\beta} C_\beta^* C_\alpha \langle\beta|H|\alpha\rangle$$

$$\langle 0|H_{\text{eff}}|0\rangle = |C_0|^2\langle 0|T+V|0\rangle - |C_0|^2 \sum_{\alpha,\beta}\langle 0|V|\alpha\rangle\langle\alpha|\frac{1}{H-E}|\beta\rangle\langle\beta|V|0\rangle$$

Brueckner-Hartree-Fock type equation

Numerical results of TOSM and comments

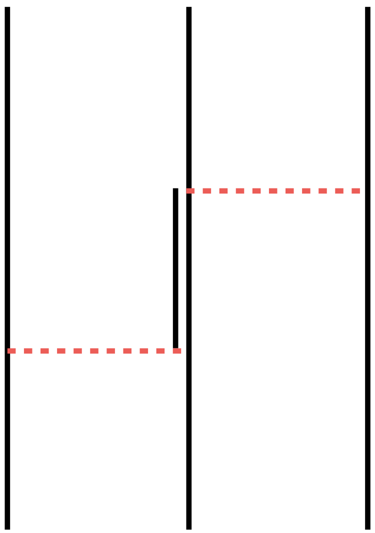




C_0
(low momentum)

C_α
(high momentum)

5MeV/A short



3 body interaction
(Fujita-Miyazawa delta term)

It is difficult to include 3 body interaction in TOSM

TOAMD for nuclear matter

$$\Psi = (1+F_S)(1+F_D)\Phi(RNM)$$

$$\Phi(RNM) = \prod_p^A \left| \psi_p(r,s)\xi_p(t) \right|$$

$$\psi_p(r,s) = \sqrt{\frac{E_p+\tilde{m}}{2\tilde{m}}} \begin{pmatrix} \chi_p(s) \\ \dfrac{\sigma \cdot p}{E_p+\tilde{m}} \chi_p(s) \end{pmatrix} \frac{1}{\sqrt{V}} e^{ipr}$$

$$F_D = f_D(r_{ij})(3(2m)^2 \gamma_{5i}\gamma_{5j} - k^2 \sum_x \gamma_{5i}\gamma_i^x \gamma_{5j}\gamma_j^x)\tau_i \cdot \tau_j \quad \rightarrow 3\sigma_1 \cdot k\sigma_2 \cdot k - k^2 \sigma_1 \cdot \sigma_2$$

$$F_S = f_S(r_{ij})\gamma_i^0 \gamma_j^0 \quad \rightarrow 1$$

$$H = T + V_{Bonn} + U_\Delta \text{(Three body interaction)}$$

Formulation is simple（2 body＋3 body..)

2 body term (we can use Feynmann rule)

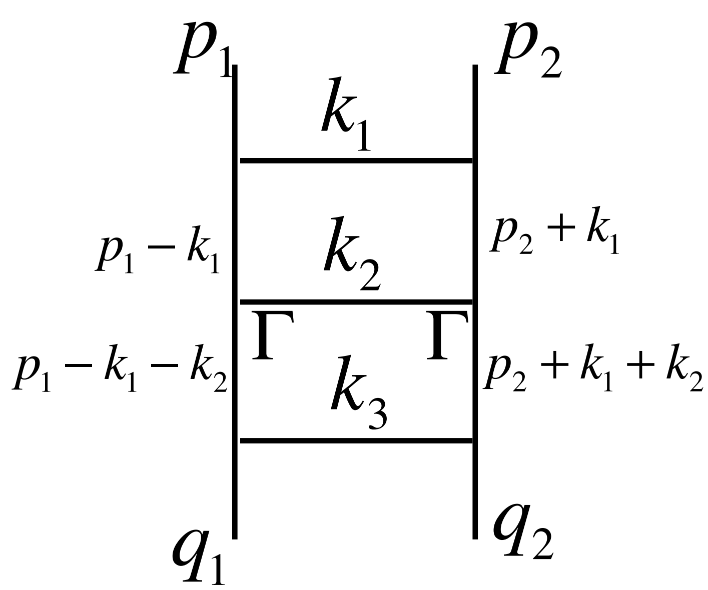
$$\langle RNM | F_S V F_S | RNM \rangle = \frac{1}{2} \sum_{p_1 p_2 : q_1 q_2} C(p_1 p_2 : q_1 q_2) \sum_{\mu_1 \mu_2 \mu_3} C_{\mu_1} C_{\mu_2} C_{\mu_3}$$

$$\sum_{k_1 k_2} e^{-k_1^2/k_{\mu 1}^2} e^{-k_2^2/k_{\mu 2}^2} e^{-(p_1 - q_1 - k_1 - k_2)^2/k_{\mu 3}^2} M(p_1 - k_1 | \Gamma | p_1 - k_1 - k_2) M(p_2 + k_1 | \Gamma | p_2 + k_1 + k_2)$$

$$M(p|1|q) = \sqrt{\frac{E_p + m}{2E_p}} \sqrt{\frac{E_q + m}{2E_q}} \chi_p^\dagger \left(1 - \frac{\sigma \cdot p}{E_p + m} \frac{\sigma \cdot q}{E_q + m} \right) \chi_q$$

$$M(p|\gamma_5|q) = \sqrt{\frac{E_p + m}{2E_p}} \sqrt{\frac{E_q + m}{2E_q}} \chi_p^\dagger \left(-\frac{\sigma \cdot p}{E_p + m} + \frac{\sigma \cdot q}{E_q + m} \right) \chi_q$$

$$C(p_1 p_2 : q_1 q_2) = \delta_{p_1 q_1} \delta_{p_2 q_2} - \delta_{p_1 q_2} \delta_{p_2 q_1}$$



MC(Metropolis) method for integration

$$\langle f \rangle = \int dp_1 dp_2 dk_1 dk_2 f(p_1 p_2 k_1 k_2) \theta(p_1 - k_F) \theta(p_2 - k_F) e^{-k_1^2/k_{\mu 1}^2} e^{-k_2^2/k_{\mu 2}^2}$$

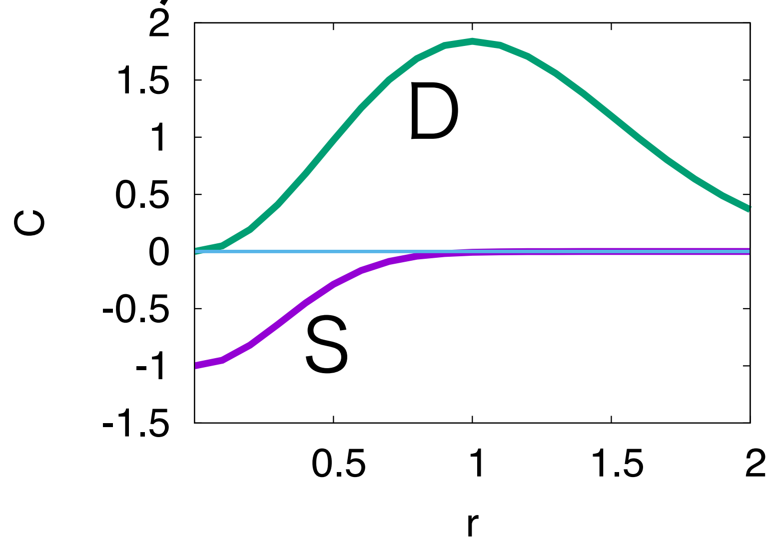
Present status (preliminary)

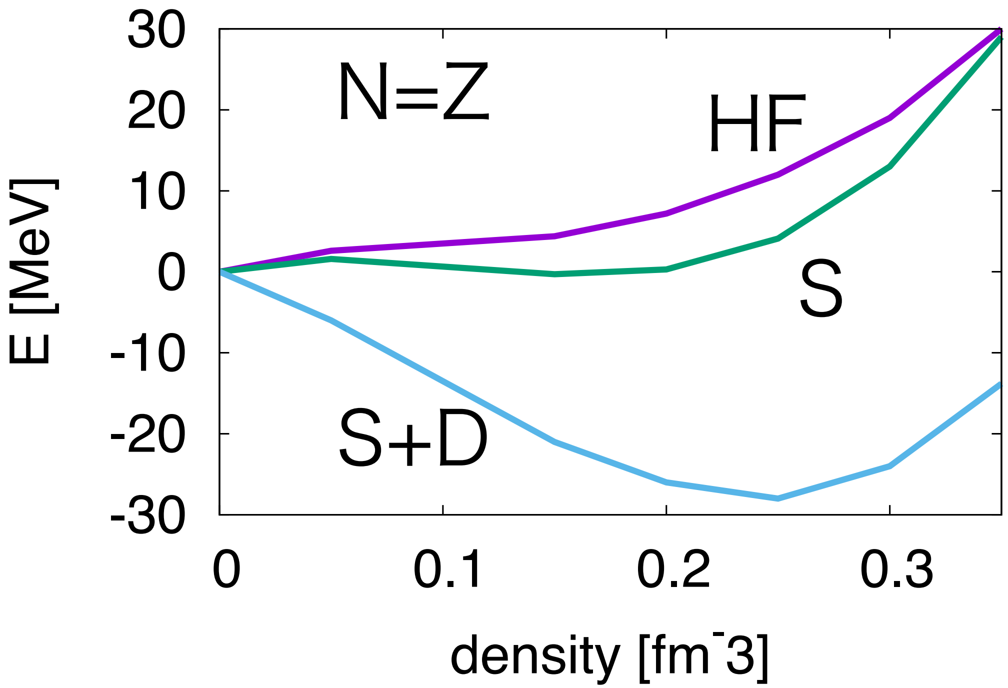
$\sigma + \omega + \pi + \delta + \eta + (\rho)$ (Bonn potential)

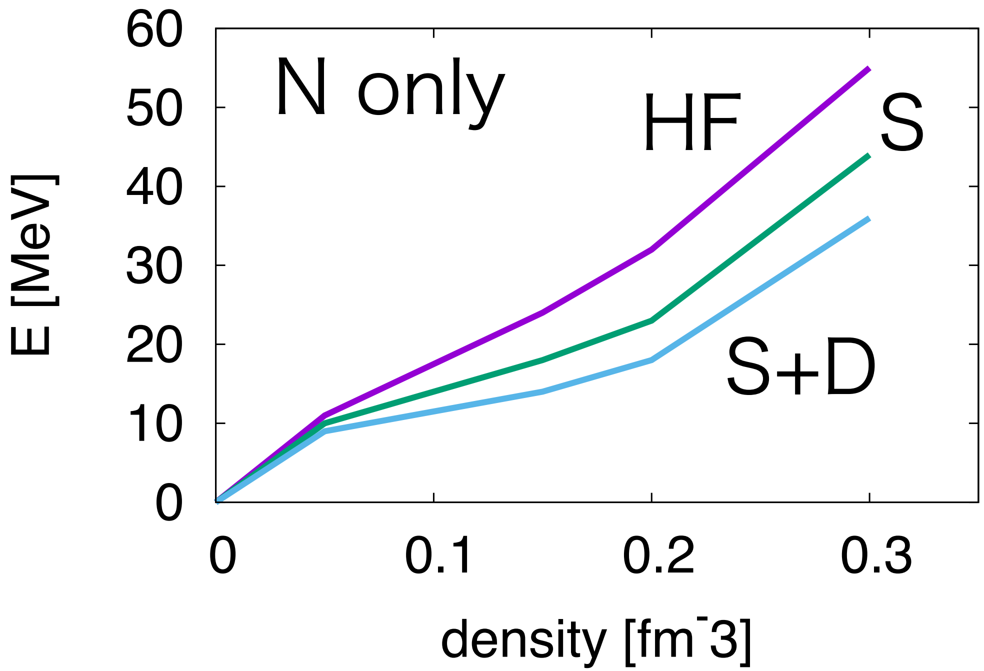
One gaussian -> many gaussians

Two body term -> many body term

Two -> Three body interaction

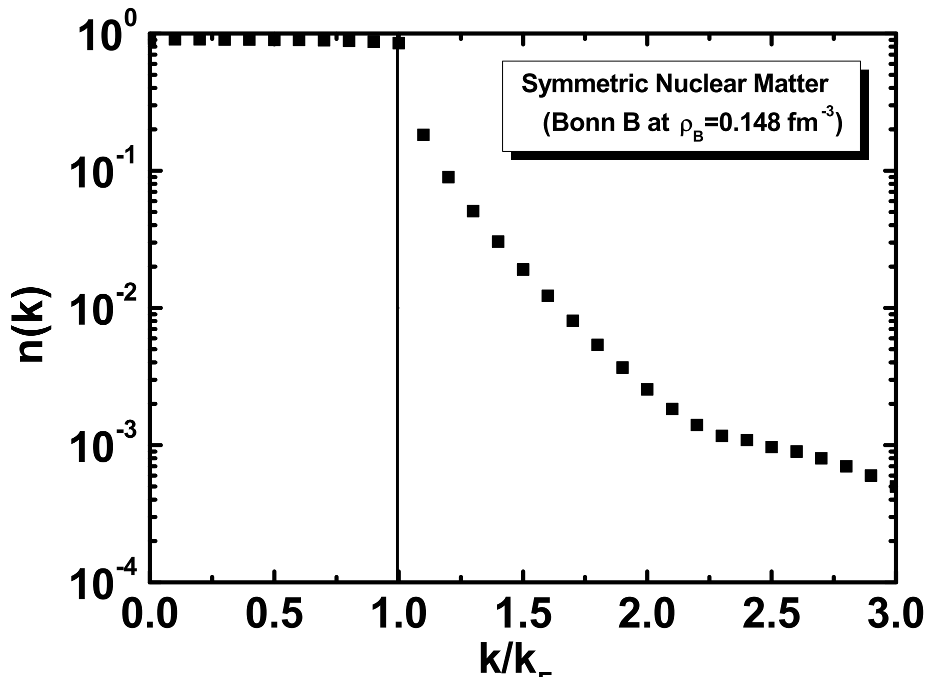




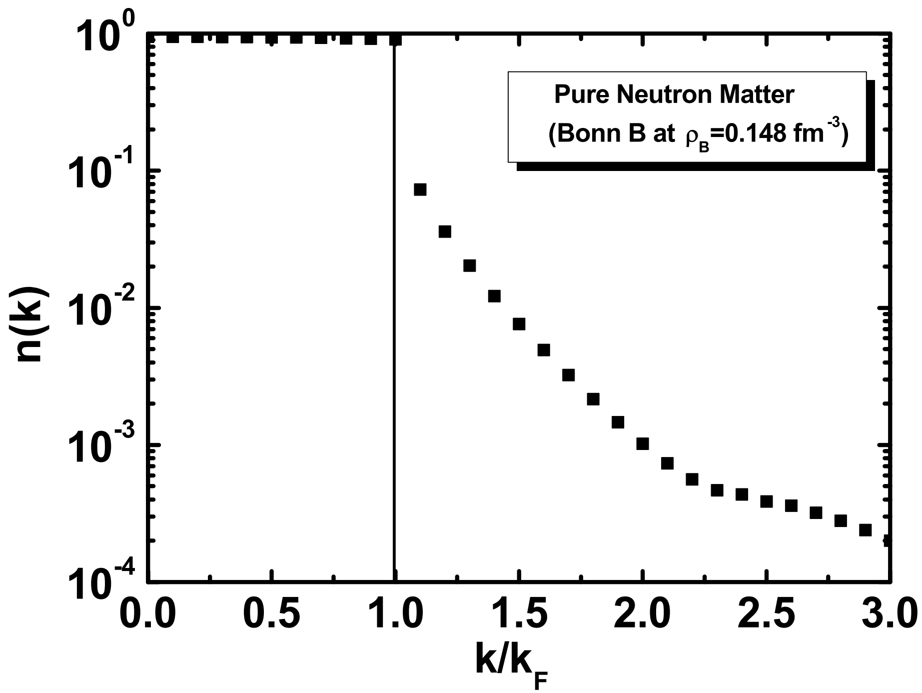


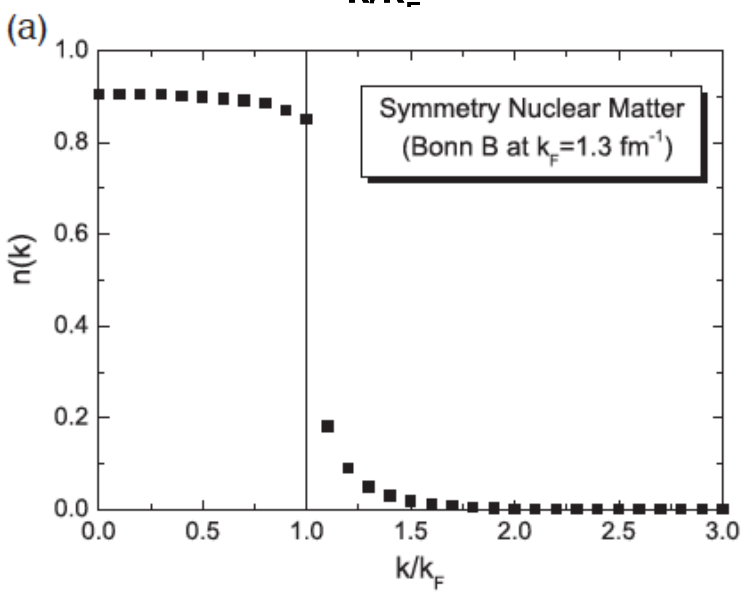
TOSM=Two body version of TOAMD

Nuclear matter



Neutron matter





URCA

$$M(p \rightarrow n) \sim (1 - n(k_n))n(k_p)$$

$$M(n \rightarrow p) \sim (1 - n(k_p))n(k_n)$$

possible!!

Conclusion

1. Nuclear URCA process (sd-shell)
2. Fine mesh table —> T. Suzuki talk
3. URCA in neutron stars: RMF NO!!
4. Many body theory with correlation YES!!
5. BHF should be extended to TOAMD
6. We are close to get numbers with TOAMD
7. URCA in neutron star cooling will be possible
8. EOS using TOAMD will come soon