Classes of functions with Integer Derivatives at x = 1

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Abstract:

Let f be a function analytic about $x = 1$. If for all natural numbers k the k^{th} derivative of $f(x)$ evaluated at $x = 1$ is an integer, then so is the k^{th} derivative of x to the power of $f(x)$ evaluated at $x = 1$. Furthermore, for the sub-class of functions for which the k^{th} derivative of $f(x)$ evaluated at $x = 1$ is also a multiple of k then, again, so is the k^{th} derivative of x to the power of $f(x)$ evaluated at $x = 1$. For both these classes of functions, this result can then be extended inductively to towers of functions, and both classes are clearly also closed under addition, subtraction and multiplication. A duality, multiplying or dividing by $x - 1$, bijectively mapping an element of one class to the other and vice versa, is also noted. As towers are built, this construct can be repeatedly applied back and forth at various stages along the way, taking the function from one class to the other and back again, etc. We ask whether our results (currently only for positive integer k) can be carried over for fractional derivatives.