“The Americans, they always depend on a method what I call . . . stupid, silly. All I ask is check yourself. Do not in fact repeat their lies”

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1 Introduction

Magnetic fields play an important, in some cases crucial, role in many areas of astrophysics: in the production of jets and outflows from a wide range of sources; in Star and Planet formation; in Accretion Discs; in compact star mergers, Supernovae and Gamma Ray Bursts (GRBs); in the Sun and other stars; the Interstellar Medium; in galaxy collisions and in galaxy clusters, to name just a few. In fact it is difficult to name an area of astrophysics in which magnetic fields are *not* important at some level. However, despite the relative simplicity and well-studied nature of the equations which describe them, their effects are complicated and both analytic and numerical studies present severe technical challenges. It is for this reason that despite a large theoretical effort over the past few decades profound questions remain over the role, configuration, effects and origin of magnetic fields in many astrophysical contexts.

In no field is this more relevant than that of star formation. For many years magnetic fields were thought to play the decisive role in the star formation process, leading to the so-called ‘standard model’ of star formation as a quasi-static process mediated by slow diffusion of the gas through a supporting magnetic field (Shu et al., 1987). In recent years, however, with a substantial increase in spatial resolution in the observations of star forming regions and the development of sophisticated codes for magnetohydrodynamics (MHD), the standard model of star formation has become extremely difficult to reconcile with both observational fact and numerical investigation. In its place a new ‘standard model’ has taken hold, where the crucial ingredient in the star formation recipe is not the magnetic field, but rather the details of the supersonic turbulence in the parental molecular cloud (Mac Low and Klessen, 2004).

This new picture of the star formation process has come through the dramatic increase in computational power over the last decade or so and with it the ability to study the properties of compressible MHD turbulence in detail. A series of grid-based MHD simulations (e.g. Stone et al. 1998; Klessen et al. 2000) have demonstrated that magnetic fields in realistic turbulent configurations cannot provide the support required to prevent molecular clouds from collapsing. However, the role of magnetic fields in other parts of the star formation process remains unknown. Importantly, magnetic fields may control the overall star formation efficiency in molecular clouds either by preventing material from collapsing or by inducing feedback via jets and outflows. Magnetic fields are also thought to play a crucial role in angular momentum transport both in the infalling gas and by the magnetic braking of cores. The effect of magnetic fields on fragmentation is also unclear and represents an issue of key importance. Magnetic fields are also thought to play a crucial role in the accretion discs which are observed to form around newborn stars.

More generally, the formation of an accretion disc is thought to be the primary means by which gas is accreted onto astrophysical objects, since it is in a disc that the substantial angular momentum of the infalling material can be dissipated. The means by which this might be achieved remained for many years a subject of much speculation since the seminal work of Shakura and Sunyaev (1973) in which the source of the angular momentum dissipation was parametrised into a viscosity-like term, although the physical source of such a viscosity remained unknown. This so-called 'α-disc model' provided a standard model for describing the accretion process in thin discs (Pringle, 1981) which could be used to explain many astrophysical phenomena (such as the outbursts observed in Dwarf Novae[1]). However a physical source of the viscosity term remained elusive until Balbus and Hawley (1991) rediscovered[2] a powerful instability present in shear flows with a weak magnetic field, dubbed the Magneto-Rotational Instability (MRI) (see the review by Balbus and Hawley, 1998). The effect of the MRI is to drive magnetic turbulence in the disc, leading to significant dissipation of energy and hence angular momentum transport. These theoretical expectations have been confirmed by direct numerical simulations (e.g. Hawley et al., 1995). Since the requirements for the MRI to operate are quite general, it is the leading candidate for driving angular momentum transport in most (but not all) classes of accretion discs, such as those found in Active Galactic Nuclei (AGN), in stellar and compact binary systems and around young stars.

An alternative, though still magnetic, mechanism for removing angular momentum in accretion discs is via the outflows and powerful, collimated jets which are observed in nearly all of the classes of object in which accretion discs are found (Livio, 1999). Although jets were first observed in the centres of active galaxies, they are now routinely observed in Young Stellar Objects (YSOs), stellar and compact binary systems, and even in planetary nebulae. Despite the wide variety of jets observed the ultimate source of their acceleration and high degree of collimation over substantial length scales remains uncertain. What is known is that the acceleration and most likely also the collimation mechanism are almost certainly magnetic in origin, from both observational constraints and theoretical ideas. Various such mechanisms have been proposed invoking either large or small scale magnetic fields present in the accretion disc (e.g. Blandford and Payne, 1982; Heinz and Begelman, 2000), however the quest to understand the origin of jet production remains one of the most longstanding problems in theoretical astrophysics.

Magnetic fields are also thought to be the main driving mechanism behind the most powerful and luminous objects ever observed in the universe, the mysterious Gamma Ray Bursts (GRBs). Much progress has been made in this area recently, with GRBs now observed in two general classes – those of long ($\sim 10s$) and short ($\sim 0.1s$) duration. In the former case rapid-response observations have been able to capture the fading afterglows of such bursts through longer wavelengths, in many cases clearly

[1] for this and other examples see Frank et al. (2002).

[2] although the instability was known previously (e.g. Chandrasekhar, 1961), Balbus and Hawley were the first to recognise the importance of this instability in accretion discs.

identifying their origin in the host galaxy. The standard 'fireball' model for such bursts (e.g. Mészáros, 2002) suggests that the long duration bursts are caused by extremely powerful supernovae explosions in which an ultrarelativistic (Lorentz factor $\gtrsim 100$) jet penetrates the surrounding material blown off in the explosion. Such events would be viewed as GRBs when the jet is directed towards the observer. The mechanism by which such a jet is produced is widely believed to be magnetic in origin. The origin of the short duration bursts remains somewhat less certain, primarily because of the difficulty of making follow-up observations on such short timescales. However a leading suggestion is that the burst is produced following the merger of two neutron stars. The mechanism by which it might do so is highly speculative, although a prime candidate is that the energy extraction is due to a magnetic field which is wound up by differential rotation in the merger remnant (Rosswog et al., 2003).

Thus there are a wide range of astrophysical problems which require a detailed understanding of the role of the magnetic field in many different physical regimes and in highly dynamical environments. In order to tackle these problems two general approaches may be taken. The first is to attempt to simplify the physics to the point where simple solutions are possible which focus on a few narrow aspects of the problem. The second approach is to undertake full numerical simulations, which in general involves solving the equations of magnetohydrodynamics (MHD) numerically.

In the first part of this thesis (Chapter 2) we take the first approach, that of simplification, to one specific astrophysical problem in which the magnetic field is thought to play a dominant role, namely in the production of jets from accretion discs. Numerical simulations incorporating both the accretion disc and the jet formation process are extremely difficult in this case due to the large range in length scales and the extreme timestepping constraints caused by the disc corona. We focus on the jet acceleration process, making simple physical assumptions in order to compare the acceleration of jets in both relativistic and non-relativistic environments in a fairly generic manner. The aim of the investigation is to examine the hypothesis of a common acceleration mechanism (ascribed to the magnetic field) for jets born in such different environments by reconciling the observed jet velocities in each class of object to a common energy input rate. The results of this investigation are presented at the end of Chapter 2 and summarised in Chapter 6.

In the second part of the thesis (Chapters 3, 4 and 5) we focus on developing new methods for solving the MHD equations numerically in an astrophysical context. The MHD equations represent a one-fluid approximation to the equations of plasma physics (a combination of gas dynamics with Maxwell's equations for the electromagnetic field). Traditionally MHD studies have been the domain of solar physicists, due to the clear and unmistakable presence of magnetic phenomena in the Sun. However in the last decade or so the importance of magnetic phenomena in many other areas of astrophysics has become clear (mostly in connection with accretion phenomena) and therefore a substantial research effort has been devoted to the development of accurate numerical methods for solving the compressible MHD equations, albeit on fixed spatial grids. This has also been fuelled by a dramatic increase in computational power which has made previously inaccessible problems open to study. The development of such algorithms has enabled significant new insights to be made into a wide range of problems, many of which have been described above. However, the primary disadvantage of such methods is that adaptivity is a crucial requirement for astrophysical problems, since problems are frequently highly asymmetric with important dynamical effects occurring over length and time scales of many orders of magnitude. This has been redressed somewhat in recent years with the development of (somewhat complicated) procedures for adaptive mesh refinement (AMR), although there remains significant scope for other adaptive methodologies since there are also many disadvantages (such as high numerical transport of angular momentum) in performing simulations involving highly asymmetric flow geometries (ie. non-Cartesian) on (fixed or adaptive) Cartesian grids. Furthermore the complexity of such algorithms means that introducing even moderate amounts of new physics is a lengthy and time-consuming process.

Smoothed Particle Hydrodynamics (SPH) is a unique numerical method widely used for astrophysical problems since it involves no spatial grid. Rather, fluid quantities are carried by a set of Lagrangian 'particles' which move with the flow, meaning that complicated dynamics and asymmetric phenomena are treated with ease. Since adaptivity is a built-in feature of the method there is no need to resort to complicated additional mesh refinement procedures. The implementation of MHD into SPH has been studied in detail by several authors. However a substantial number of issues remain to be addressed, particularly with respect to the recent rapid progress in MHD algorithms developed for grid-based codes. The remainder of this thesis (Chapters 3, 4 and 5) is dedicated to addressing many of these issues in order to provide a sufficiently robust and accurate numerical method for the simulation of magnetic phenomena in many of the problems considered above. Doing so involves a comprehensive review of the SPH method itself (Chapter 3) before discussing the implementation of MHD (Chapter 4) and the many further issues involved in multidimensional MHD related to the divergence-free (no monopoles) constraint for the magnetic field (Chapter 5). A discussion of the main results is presented at the end of each chapter and summarised in Chapter 6, along with a brief discussion of problems to which the algorithm can be applied.