"I hope we get to the bottom of the answer. It's what I'm interested to know"

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6 Conclusions

In the introductory chapter (Chapter 1) the importance of magnetic fields in many astrophysical problems was highlighted. In this final chapter we summarise the results contained in this thesis and discuss ways in which the work can be applied and extended in order to provide answers to some of these problems.

6.1 Summary

In Chapter 2 we have used simple physical models in order to compare the mechanisms for jet acceleration in both relativistic (pertaining to AGN jets) and non-relativistic (pertaining to jets produced in Young Stellar Objects) environments. Time-dependent, spherically symmetric wind models in Newtonian and relativistic gravitational fields were used to examine whether or not the observed jet velocities in both classes of object could be reconciled to a common (appropriately scaled) energy input rate. It was found that the energy input rate required to produce observed outflow velocities of $v_{\rm jet} \simeq 2v_{\rm esc}$ in the Newtonian (YSO) case could give rise to outflows with a Lorentz factor $\gamma_{\rm jet} \simeq 11$ in the strongly relativistic case (as observed in AGN jets). Thus it was concluded that it is not unreasonable to suggest, on the basis of the simple physical models employed, that the relativistic jets observed in AGN are simply scaled-up versions of their non-relativistic (YSO) counterparts and that the intrinsic acceleration process is the same in both classes of object. For this to be the case, two further conditions were required. The first was that jet acceleration must occur close to the central gravitating object, in order to make use of the speed of light as a limiting velocity in the black hole case. The second was that, since the dimensionless heating rates required are much larger than unity, the energy released in the outflow must be imparted to only a small fraction of the available accreting material.

The remainder of the thesis was dedicated to the accurate numerical simulation of magnetic fields in an astrophysical context using the Smoothed Particle Hydrodynamics (SPH) method. A thorough review of the SPH algorithm was presented in Chapter 3. Various aspects of the algorithm were considered in detail, including the choice of smoothing kernel, the evaluation of first and second derivatives, the self-consistent formulation of the discrete equations from a variational principle and the more accurate formulation which can be derived by incorporating terms relating to the spatial variation of the smoothing length (the ‘variable smoothing length terms’). Artificial dissipation terms were used in order to capture shocks and in particular the potential advantages of introducing a small artificial thermal conductivity were discussed. Switches were proposed to reduce the spurious effects of the dissipative terms away from shocks. The hydrodynamic algorithm was tested against a variety of problems, including linear waves, shocks, Cartesian shear flows and on a class of exact non-linear solutions known as ‘Toy Stars’, in which various effects were highlighted.

In Chapter 4 the SPH algorithm was extended to the MHD case. Particular attention was paid to the self-consistent formulation of the discrete equations (achieved using a variational principle) which is important in the MHD case due to the presence of terms proportional to the divergence of the magnetic field which are in general non-zero in a numerical context. Consistent alternative formulations of the SPMHD equations were also derived as well as formulations incorporating the variable smoothing length terms. Stability considerations were extensively discussed, with a variety of solutions to the known instability associated with an exactly momentum-conserving form of the SPMHD force in the presence of tension forces examined. An approach suggested by Monaghan (2000) for solid mechanics problems was extensively investigated, although not found to be universally effective for astrophysical problems due to the spatial variations in the smoothing length. The best approaches to eliminate the tensile instability were found to be either to subtract any constant field components from the gradient terms in the magnetic force or to use a simple modification of the anisotropic force term due to Morris (1996) which retains the conservation of momentum in a continuum sense although not discretely. Dissipative terms for shock capturing analogous to those used in the SPH case were derived which ensure a positive definite contribution to the entropy and thermal energy. The shock capturing abilities of the resulting algorithm were extensively tested against a variety of one dimensional shock tube problems used to test recent grid-based MHD codes. Linear wave tests were also presented which highlighted the increased accuracy resulting from inclusion of the variable smoothing length terms.

Finally, multidimensional aspects of the SPMHD algorithm were examined in Chapter 5. Various methods for maintaining the divergence-free condition in an SPH context were discussed, including the consistent formulation of the MHD equations in the presence of magnetic monopoles (the ‘source term’ approach), projection methods and a hyperbolic divergence cleaning recently proposed by Dedner et al. (2002). Using an approximate projection method based on the Green’s function solution to Poisson’s equation was found to give good results, although the method is computationally expensive and difficult to implement in the case of periodic boundary conditions. The hyperbolic approach was found to be particularly simple and efficient to implement but limited in some problems in which divergence errors are generated very quickly by the flow. Various multidimensional numerical tests used to test recent grid-based MHD algorithms were also presented, including a divergence advection problem, non-linear circularly polarized Alfvén waves, two dimensional shock tubes, spherically symmetric blast waves and the Orszag-Tang vortex problem. Particular attention was paid to the divergence errors resulting in these problems. The single biggest factor in determining the magnitude of the divergence errors in a given simulation was found to be the size of the smoothing region (ie. the number of contributing neighbours). It was therefore concluded that a slightly larger number of neighbours should be used for MHD problems (typically $h \gtrsim 1.5(m/\rho)^{1/\nu}$ where ν is the number of spatial dimensions).

6.2 Future work: Applications

6.2.1 Star formation

Understanding the role of magnetic fields in star formation involves two distinct but not inseparable issues. The first is the role that magnetic fields, in the form of compressible MHD turbulence, play in the support of molecular clouds against collapse. Related to this issue is to determine the timescale on which the initial turbulent spectrum of the molecular cloud dissipates in order to allow collapse to occur. The second issue is the role of magnetic fields in the collapse phase, ie. during formation of cores (via fragmentation) and particularly their role in angular momentum transport and feedback (by generating outflows). The first problem has been the subject of a substantial research effort over the past decade, primarily enabled by the development of accurate algorithms for MHD simulations within grid-based codes. However, the latter problem has received surprisingly little attention, mainly due to the difficulty of implementing adaptive mesh refinement procedures and incorporating new physics (such as changes in the equation of state) into grid-based MHD codes which rely on complicated shock-capturing procedures. Furthermore even with adaptive meshes, using Cartesian grids on problems which are highly asymmetric presents some difficulty due to the substantial numerical transport of angular momentum.

Although the turbulence simulations seem to indicate that magnetic fields do not play the dominant role in core formation and support of clouds, their role in other parts of the star formation process remains unknown. An issue of key importance is whether magnetic fields control the overall star formation efficiency in molecular clouds, or whether this is due to other processes such as radiative or mechanical feedback from massive protostars. Most of the gas in hydrodynamic collapse simulations (e.g. Bate et al., 2003) is accreted on a free-fall timescale, leading to a discrepancy with observed lifetimes of molecular clouds which may be resolved by the support provided by MHD turbulence to low density regions of the cloud (so that not all the gas would fall onto the protostars). Magnetic fields are often invoked to solve the angular momentum problem via magnetic braking of cores. Some calculations indicate that such angular momentum transport may make it difficult to form binaries from collapsing magnetic cores (Hosking, 2002), although only a few different cases were considered and the calculations did not involve turbulent initial conditions. Other calculations (e.g. Boss 2000, 2002) suggest that magnetic fields may enhance fragmentation, however these calculations use only an approximate treatment of MHD forces. Magnetic fields are the most likely mechanism for the production of jets and outflows commonly observed in star forming regions.

The algorithm developed for SPMHD within this thesis is ideally suited to star formation problems, since the adaptivity is a built-in feature of the numerics and resolution is automatically concentrated in regions of high density which is where the stars form. The use of sink particles in SPH (Bate et al., 1995) has enabled simulations to be followed beyond the point where stars form to study the subsequent accretion and dynamics which turn out to be crucial in determining the final properties of the newborn stars (such as their mass). Ultimately the aim would be to answer both questions self-consistently by following the collapse from the initial turbulent decay all the way to the formation of stars and beyond. Purely hydrodynamic simulations of this type have been performed recently by Bate et al. (2003) and succeed remarkably well in predicting the statistical properties of the stars which form. An MHD version of these simulations would be highly desirable. However, some caution is required in simulating the initial turbulent decay using the SPMHD algorithm described here because the physical dissipation time of the MHD turbulence is quite important and this may be difficult to disconnect from the effects of the artificial dissipation terms employed for shock capturing (at the very least it must be shown that the numerical results are converged). Thus SPMHD may, in the short term at least, be best suited to answering the second question, that is, what effects do magnetic fields have on fragmentation and in providing angular momentum transport and feedback in star forming cores? Preliminary calculations exploring these questions are currently being performed using a version of the SPMHD algorithm incorporated into a 3D SPH code which has been used for many of the hydrodynamic star formation calculations (Bate, 1995).

6.2.2 Neutron star mergers

Compact binary systems consisting of two neutron stars will eventually spiral towards each other and merge due to the energy and angular momentum loss caused by the emission of gravitational waves. The coalesced central object resulting from the merger is probably too massive to form a single neutron star, whilst the substantial angular momentum prevents the merger remnant being swallowed immediately by the black hole. Thus the most likely scenario is the formation of a single black hole surrounded by a disc-like merger remnant from which matter is accreted. The dynamics of this problem present a severe challenge for numerical simulation, not least because the gravitational dynamics are strongly relativistic and ultimately require the full solution of Einstein's equations. Whilst a significant research effort is directed towards the gravitational side of the problem (with recent promising results by Shibata and Uryū 2000), the astrophysical aspects are equally challenging, drawing on almost every field of astrophysics. The problem is important firstly because such events are known to occur regularly in sufficient numbers to present a substantial background of gravitational wave sources which may be detected with the next (or perhaps even current) generation of gravitational wave detectors.

From an astrophysical perspective Rosswog and Davies (2002) have presented detailed numerical simulations of this problem using SPH incorporating many aspects of the microphysics, including a detailed nuclear equation of state and neutrino emission of all flavours. SPH has significant advantages over grid-based methods for this problem, in particular the spurious numerical transport of angular momentum is much lower and the stars do not have to be embedded in an artificial background medium which can cause artificial shock waves at the stellar surfaces (e.g. the simulations of Ruffert and Janka, 2001, using a nested-grid code based on the Piecewise Parabolic Method). However a major piece of physics missing from the simulations is the magnetic field. Magnetic fields may play a decisive role in determining, via the transport of angular momentum, whether or not the central coalesced object collapses into a black hole (and if so the timescale on which this occurs). If the central object can remain stabilised against collapse for a substantial length of time, estimates by Rosswog et al. (2003) suggest that magnetic fields could wind up by differential rotation in the merger remnant to strengths of up to $\sim 10^{17}$G (depending on the rotation period). Such field strengths would provide the conditions required for magnetically powered Gamma-Ray Bursts. The implementation of the SPMHD algorithm described in this thesis enables such possibilities to be explored.

6.2.3 Accretion discs

Another area to which we intend to apply the SPMHD algorithm is in the simulation of accretion discs. SPH is widely used to simulate accretion disc phenomena, particularly in mass-transferring binary systems where the dynamics of the disc can be extremely complicated due to the tidal influence of the secondary (Murray, 1996). SPH has also been used to study gravitational instabilities in discs (Lodato and Rice, 2004), to study planet-disc interactions (e.g. Schäfer et al., 2004) and to study accretion discs around black holes (e.g. Molteni et al., 1994). However in all of these simulations the transport of angular momentum is induced by introducing a viscosity term similar to the original Shakura and Sunyaev (1973) parametrization. Whilst this is a useful approach, primarily because of its simplicity, it would be very interesting to study the dynamics of the magnetic field in such accretion discs, particularly with respect to the Magneto-Rotational Instability (MRI) which is believed to provide the main source of angular momentum transport.

6.3 Future work: Algorithms

In addition to applying SPMHD to interesting physical problems there are many aspects of the algorithm which can be improved and extended. In particular we intend to investigate the following:

- Extension of the algorithm to non-ideal MHD. There are many astrophysical problems in which non-ideal effects become important, such as the Hall effect and the effects of ion-neutral diffusion. The latter has been implemented using a two-fluid SPMHD code by Hosking and Whitworth (2004).

- Combining the algorithm with other physics. In particular it is our intention to merge the SPMHD code with the algorithms of Whitehouse and Bate (2004) for radiative transfer in SPH in order to study star formation related problems.

- A General Relativistic implementation. Algorithms for SPH on a fixed background metric have been presented by Monaghan and Price (2001). However many of the interesting fixed-metric problems also involve magnetic fields (for example in studying accretion flows onto black holes). A General Relativistic version would also be useful for the implementation of the algorithm in different co-ordinate systems.

- Better ways of maintaining the divergence-free constraint. Several approaches to maintaining the divergence-free condition relevant in an SPH context were discussed in Chapter 5, however many other approaches are also possible and it would be interesting to investigate and compare such possibilities.

- Improvements to the shock capturing scheme. In particular it would be highly desirable to eliminate the use of artificial dissipation terms in order to capture shocks. Simple methods for incorporating Godunov-type schemes into SPH have been presented recently by Cha and Whitworth (2003). Whilst the Riemann problem is much more complicated in the MHD case an implementation of a Godunov-type scheme for SPMHD would be extremely useful.