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Rapid AGN accretion from counter–rotating discs

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ABSTRACT

Accretion in the nuclei of active galaxies may occur chaotically. This can produce accretion discs which are counter–rotating or strongly misaligned with respect to the spin of the central supermassive black hole (SMBH), or the axis of a close SMBH binary. Accordingly we consider the cancellation of angular momentum in accretion discs with a significant change of plane (tilt) between inner and outer parts. We estimate analytically the maximum accretion rate through such discs and compare this with the results of Smoothed Particle Hydrodynamics (SPH) simulations. These suggest that accretion rates on to supermassive black holes may be larger by factors $\gtrsim 100$ if the disc is internally tilted in this way rather than planar. This offers a natural way of driving the rapid growth of supermassive black holes, and the coalescence of SMBH binaries.

Key words: accretion, accretion discs – black hole physics – galaxies: formation – galaxies: active – cosmology: theory

1 INTRODUCTION

Astronomers still do not know how the supermassive black holes (SMBH) in galaxy centres accrete gas and grow. The problem lies in the very small angular momentum this gas must have in order to accrete on a reasonable timescale. For example, the viscous time of a standard accretion disc of ~ 1 pc radius accreting on to a SMBH of $10^8 \mathrm{M}_{\odot}$ approaches a Hubble time. A natural remedy for this is to assume that on small scales near the hole, accretion occurs in episodes whose angular momenta are randomly oriented with respect to each other. This chaotic picture predicts that the spin of the hole remains low, allowing rapid mass growth provided that there is an adequate mass supply (King & Pringle 2006; King & Pringle 2007; King et al. 2008; Fanidakis et al. 2011). A characteristic feature of this type of accretion is that the disc flow on to the SMBH is often retrograde (King et al., 2005) with respect to the hole spin, or to the rotation of an SMBH binary resulting from a galaxy merger.

In a recent paper (Nixon et al., 2011) we showed that a retrograde coplanar external gas disc can be markedly more efficient in shrinking an SMBH binary than a prograde one. A retrograde disc can cancel the binary orbital angular momentum directly, rather than being slowed by the resonances which always arise in a prograde circumbinary disc. It is natural to ask if this angular momentum cancellation can work if we replace the SMBH binary with a pre–existing gas disc surrounding a single SMBH. If so, this may be a mechanism promoting much faster delivery of gas on to the black hole. This situation may arise naturally from misaligned accretion events, for example where an accretion event forms a disc around the black hole, and a second event forms an outer disc which is misaligned with respect to the first one. Similar cases occur as a misaligned accretion disc attempts to reach an axisymmetric (co– *or* counter–aligned) configuration around a spinning black hole (King et al. 2005; Lodato & Pringle 2006), and also during the closely related co– or counter–alignment of an accretion disc around a central binary (Nixon, King & Pringle, 2011).

In all these cases integrating the full dynamics of the accretion disc is complex and time–consuming because the component discs can warp before interacting. To study the mixing of distinct or opposing disc angular momenta we investigate a simple form of this kind of accretion. We assume an accretion disc with initially distinct planes in its inner and outer parts. We let these two parts interact viscously and determine how much the accretion is enhanced compared with a disc lying in a single plane. In Section 2 we give simple analytical estimates of this. In Section 3 we perform global 3D hydrodynamical simulations to examine the disc's dynamical behaviour in the light of these estimates. We interpret our results in Section 4.

2 COUNTER–ROTATING DISCS

We consider an inner and outer disc counter–rotating with respect to each other but not coplanar, that is, the angle θ between the disc angular momentum vectors obeys $\pi/2 < \theta < \pi$.

If the discs were in perfect contact the gas would share opposed angular momenta from almost the same radii. This may be unphysical (or imply a very rapid evolution away from the initial state) since the discs may attempt to open a gap of some size, as we

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discuss later. For generality we choose an arbitrary gap between the discs of size a.

Now we assume that the gas efficiently shares angular momentum across this gap and that an equal mass of gas from each disc takes part in the interaction. The resultant angular momentum of the interacting gas is

$$L_{\rm new} = \frac{1}{2}\left[L\left(R_{\rm gap}+a\right)+L\left(R_{\rm gap}\right)\right] \approx \frac{1}{2}\sqrt{GMR_{\rm gap}}\left(1+\frac{1}{2}\frac{a}{R_{\rm gap}}+\cos\theta\right) \quad (1)$$

to first order in $a/R_{\rm gap}$, where $R_{\rm gap}$ is the outer edge of the inner disc (so that $R_{\rm gap}+a$ is the inner edge of the outer disc), G is the gravitational constant, M is the mass of the central object, and we have assumed a Keplerian potential for the gas.

The circularisation radius for this interacting gas is

$$R_{\rm circ} = \frac{1}{4}R_{\rm gap}\left(1+\cos\theta+\frac{1}{2}\frac{a}{R_{\rm gap}}\right)^2 \quad (2)$$

We expect the size of the gap between the two discs to be roughly the scale of the epicyclic motions in the gas disc as this is where fluid orbits cross and interact. This implies a gap size $\sim H(R_{\rm gap})$, where H is the semi–thickness of the disc, so that

$$R_{\rm circ} = \frac{1}{4}\left(1+\cos\theta+\frac{1}{2}\frac{H}{R}\right)^2 R_{\rm gap}. \quad (3)$$

For typical AGN disc parameters, $H/R \sim 10^{-3}$, and there are two distinct cases. For θ close to π, we can have $1+\cos\theta \ll H/R$, so that

$$R_{\rm circ} \approx \frac{1}{16}\left(H/R\right)^2 R_{\rm gap}. \quad (4)$$

This suggests that the gas orbits would decrease by ~ 7 orders of magnitude in radius which implies direct accretion of the gas as long as its path to the hole is clear.

If however θ is not close to π we have $1+\cos\theta \gg H/R$, and

$$R_{\rm circ} \approx \frac{1}{4}\left(1+\cos\theta\right)^2 R_{\rm gap}. \quad (5)$$

Then the gas orbit decreases least when $\theta \sim \pi/2$, where $R_{\rm circ} \sim \frac{1}{4}R_{\rm gap}$. The orbit scale (3) varies smoothly between these two extreme values.

The accretion rate through a disc is approximately

$$\dot{M} \sim \frac{M_{\rm disc}}{\tau_{\rm visc}} \quad (6)$$

where $M_{\rm disc}$ is the disc mass and $\tau_{\rm visc}$ is the viscous timescale

$$\tau_{\rm visc} \sim \frac{R^2}{\nu} \quad (7)$$

where R is a characteristic radius for the disc and ν is a measure of the disc viscosity. Thus for constant viscosity[1] the accretion rate is $\propto R^{-2}$. This suggests that for $\theta > \pi/2$, where the gas orbits decrease by a factor > 4, there is potential for the accretion rate to increase by a factor > 16. For large $\theta \approx \pi$ the cancellation can lead to direct accretion of gas on a dynamical timescale. This allows accretion on a local disc filling timescale at the radius of the gap. This timescale is much shorter than the viscous timescale as we only need move gas to $R_{\rm gap}$ and not right on to the hole. Note that the rate at which mass can be supplied to $R_{\rm gap}$ is clearly an upper bound on the possible accretion rate from this process.

[1] This is only approximate as one expects ν to increase with radius.

If the gas does not accrete directly then the viscous timescale is shortened to

$$\tau_{\rm visc,\ circ} \sim \left(R_{\rm circ}/R_{\rm gap}\right)^2 \tau_{\rm visc,\ gap}. \quad (8)$$

Although these estimates are suggestive, we caution that we have assumed that the gas shares angular momentum efficiently, and that an equal mass of gas interacts from each disc. In reality neither of these simplifications may be valid. Accordingly we use Smoothed Particle Hydrodynamics (SPH) to simulate the interactions between the discs.

3 SIMULATIONS

3.1 Code Setup

We use PHANTOM, a low-memory, highly efficient SPH code optimised for the study of non-self-gravitating problems. This code has performed well in related simulations. For example Lodato & Price (2010) performed simulations of warped accretion discs and found excellent agreement with the analytical work of Ogilvie (1999) on the nature of the internal accretion disc torques in response to warping.

The implementation of accretion disc α–viscosity (Shakura & Sunyaev, 1973) in PHANTOM is described in Lodato & Price (2010). Specifically, we use the 'artificial viscosity for a disc' described in Sec. 3.2.3 of Lodato & Price (2010), similar to earlier SPH accretion disc calculations (e.g. Murray, 1996). The main differences compared to standard SPH artificial viscosity are that the disc viscosity is applied to both approaching and receding particles and that no switches are used. Our implementation also differs slightly from Lodato & Price (2010) in that we retain the $\beta^{\rm AV}$ term in the signal velocity in order to prevent particle interpenetration from occurring in high-velocity stream-disc collisions. The disc viscosity in PHANTOM was extensively calibrated against a 1D thin α–disc evolution in Lodato & Price (2010) (c.f. Fig. 4 in that paper) and the disc scale heights employed here are similar. As the exact value of α is unimportant given the dynamical nature of the simulations, we simply use $\alpha^{\rm AV} = 1$ and $\beta^{\rm AV} = 2$ which at the employed resolution (see below) corresponds to a physical viscosity $\alpha_{\rm SS} \approx 0.02$–$0.06$.

For these simulations we model the gravity of the accretor with a Newtonian point mass potential, adopting an accretion radius of 0.1 in code units. We start with a flat disc of gas, composed of 10 million SPH particles, in hydrostatic equilibrium between 1.0 to 2.0 in radius, and surface density distribution $\Sigma \propto R^{-1}$, setup using the usual Monte-Carlo technique. We also choose a vertical density profile corresponding to $H/R = 0.02$ at $R = 1$ and employ an isothermal equation of state. To produce the two distinct planes for the disc we rotate the outer half (in radius) of the disc by an angle θ. We then perform simulations with different values of θ to find how the accretion rate changes with inclination angle.

3.2 Tilted disc evolution

We perform seven simulations corresponding to inclination angles $\theta = 30n^\circ$ where $n = 0, 1, ..., 6$. The $\theta = 0^\circ$ simulation gives a flat–disc accretion rate to which we compare the other simulations. We note that the 180° case is a rather unrealistic setup as it would require perfectly anti-parallel accretion events. Even in the co-and counter-aligned discs predicted by the numerical simulations of Lodato & Pringle (2006) the discs never achieve a configuration where θ is precisely 180°.

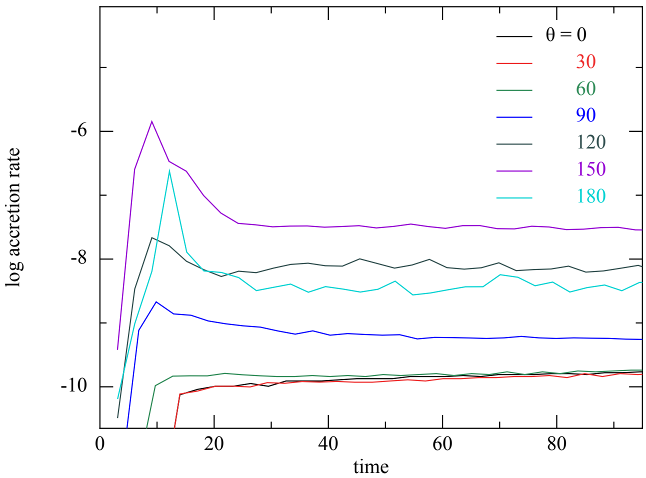

Figure 1. The accretion rates for the isothermal simulations with different inclination angles θ. Time is in units of the dynamical time at $R = 1$. The discs with a tilt $\geq 90^\circ$ are much more efficient at driving accretion, as these involve a degree of counter–rotation and so direct cancellation of angular momentum.

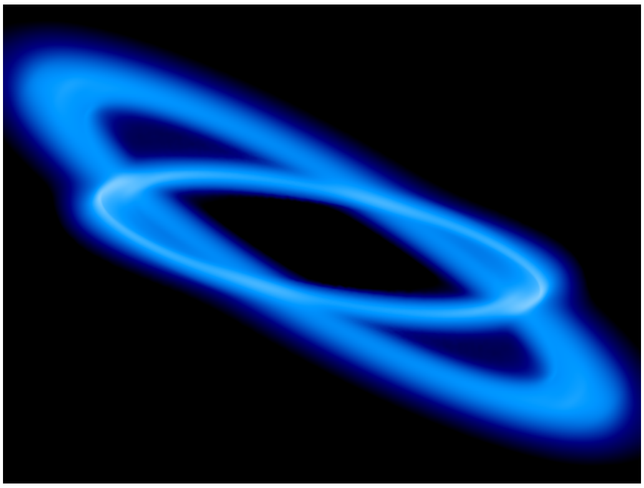

Figure 2. Column density projection showing the disc structure for the $\theta = 30^\circ$ simulation. The disc has evolved for $\sim$ 50 dynamical times ($\sim$ 10 orbits).

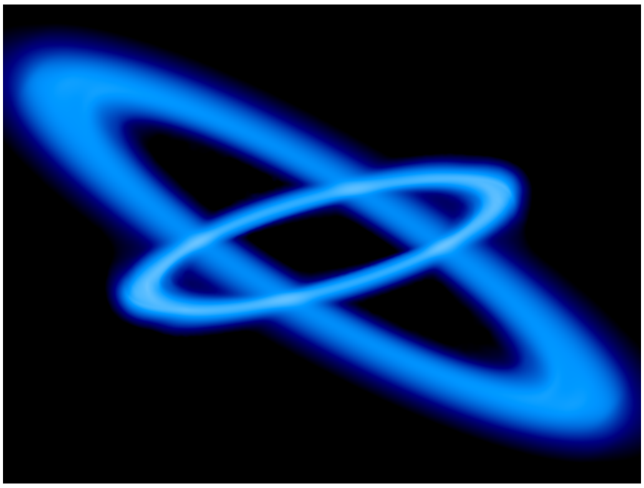

Figure 3. Column density projection showing the disc structure for the $\theta = 60^\circ$ simulation. The disc has evolved for $\sim$ 50 dynamical times ($\sim$ 10 orbits).

The simulations were run with an isothermal equation of state, i.e. the temperature of each particle was held constant for each simulation (see Section 3.2.2 below for a discussion of the effects of different equations of state). This implies that the pressure, determined by $P = c_s^2 \rho$, is simply proportional to ρ, and the sound speed c_s is constant in time and the same for all particles.

In Fig. 1 we report the accretion rates achieved for the different values of θ. This shows that for discs inclined by less than 90° the accretion rates are similar to that of a flat disc for the duration of our simulations. In this case the interaction between inclined gas orbits is relatively weak with the gas staying on near circular orbits, with planes slowly varying from the inner to outer disc. In contrast the simulations with $\theta \geq 90^\circ$ show a dramatic increase in the accretion rate. This is caused by the direct cancellation of orbital angular momentum and energy where the orbits of the disc gas cross.

In Fig. 2 we show the disc structure for the simulation where $\theta = 30^\circ$. Here the two discs initially join to form a coherent disc with a warped region joining the still misaligned inner and outer discs. Gas is quickly depleted from the warp region, leaving a clear drop (of 2-3 orders of magnitude) in projected density in the warp region. This break between the discs persists throughout the simulations as the two discs slowly align with each other, reaching an angular separation of $\sim$ 10° by the time the simulation ends. The large gradient in specific angular momentum in the warp region causes gas to quickly move on past the warp. This causes the low density in the warp region which is the disc break (Nixon & King, 2012). The disc therefore maintains two distinct planes. We still resolve the rings of gas between the two discs but this is clearly of much lower density than the inner and outer discs (cf Lodato & Price 2010). In Fig. 3 we show the similar disc structure for the simulation where $\theta = 60^\circ$.

The $\theta = 90^\circ$ simulation (not illustrated) has an initial period of chaotic flow between the two discs, until the inner disc moves towards an inclination of $\sim$ 85° after mixing with some of the gas from the outer disc. At this point a strong warp is set up and the flow continues in a similar way to $\theta = 60^\circ$.

The evolution of the $\theta = 120^\circ$ and $\theta = 150^\circ$ simulations proceed in a dramatically distinct way to those with smaller θ. This is simply because the sharing of angular momentum now causes significant changes to the gas orbits. In the $\theta = 120^\circ$ simulation gas falls from the region between the two discs and circularises at a radius inside the inner accretion disc. In this process some of gas is also accreted, as it carries a spread of angular momentum. Clearly this gas does not have precisely the angular momentum of the inner disc; it is a mix of gas from both discs. Therefore the steady state for this simulation is an outer disc causing cancellation with the outer edge of the original inner disc. This gas falls and feeds both direct accretion and the new innermost disc (white region of high density in Fig. 4). This innermost disc is strongly warped with respect to the original inner disc and so there is again extra dissipation between these two discs as they try to straighten. The disc structure can be seen in Fig. 4.

In the $\theta = 150^\circ$ simulation the gas falls to a much smaller radius from the region between the two discs. Initially a large amount of gas is directly accreted on to the sink particle representing the central accretor. The strong dissipation occurring early in the sim-

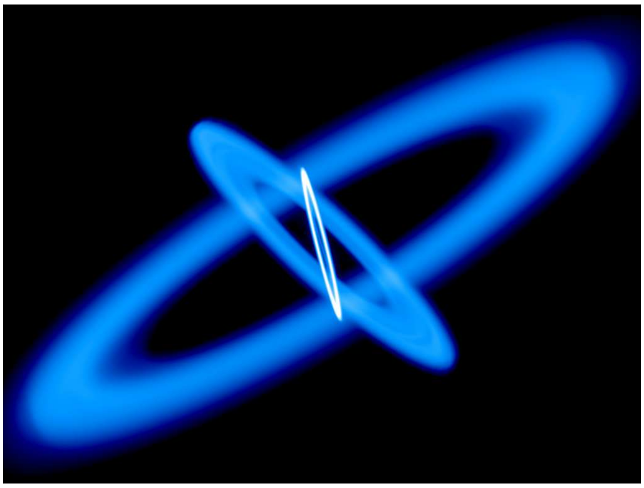

Figure 4. Column density projection showing the disc structure for the $\theta = 120°$ simulation. The disc has evolved for $\sim$ 50 dynamical times ($\sim$ 10 orbits). Note the new innermost disc (of high density and therefore white) which has formed from the dynamical infall of gas from the warp region.

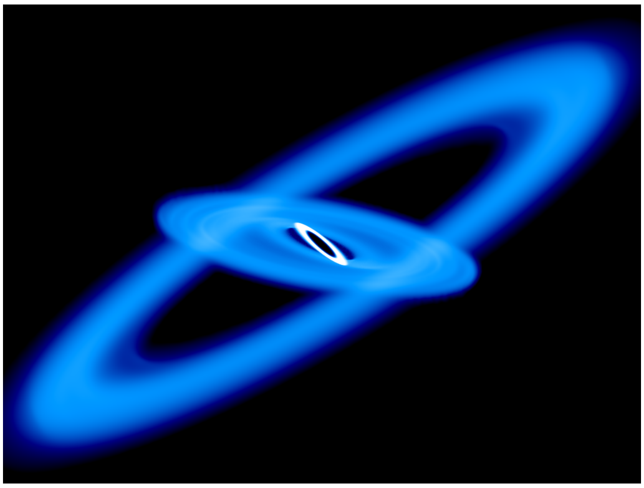

Figure 5. Column density projection showing the disc structure for the $\theta = 150°$ simulation. The disc has evolved for $\sim$ 50 dynamical times ($\sim$ 10 orbits).

ulation also causes the inner disc to spread all the way in to the accretion radius. Some of the falling gas tries to circularise at a radius where it is forced to impact upon the inner disc. As it still has the sense of angular momentum of the outer disc this causes more cancellation of angular momentum, driving more accretion. For these reasons this simulation generates the highest accretion rates – this is simply because it causes the greatest mixing of angular momentum in the gas. The disc structure for this simulation can be seen in Fig. 5.

3.2.1 $\theta = 180°$

The $\theta = 180°$ case does not initially appear a very realistic initial setup. However something close to this configuration may appear in the evolution of accretion discs around spinning black holes (cf King et al. 2005; Lodato & Pringle 2006) and also during the evolution of an accretion disc around a binary system (cf Nixon et al. 2011).

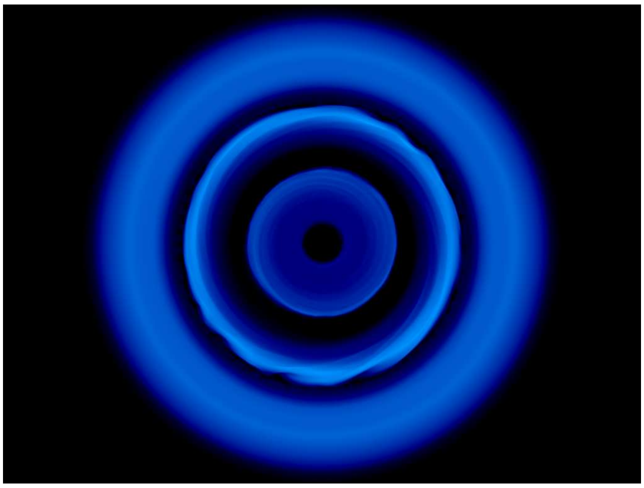

Figure 6. Column density projection showing the disc structure for the $\theta = 180°$ simulation. The disc has evolved for $\sim$ 50 dynamical times ($\sim$ 10 orbits). The innermost disc is formed of gas falling from the gap between the original inner disc (now the middle disc) and the outer disc. In this picture the innermost and outermost discs are rotating clockwise and the middle disc rotates counter–clockwise.

Initially while the two discs (inner and outer) are in contact there is great cancellation of angular momentum for the gas. This gas then attempts to fall towards the central sink particle. It therefore impacts upon the inner disc. The pressure of the inner disc gas is enough to force the falling gas out of the plane of the disc where it can continue on a low angular momentum orbit towards the centre. It then circularises inside the inner disc (with some accretion) after colliding with gas on similar orbits and losing orbital energy in shocks. This new innermost disc clearly has the same sense of angular momentum as the outer disc since this has the larger specific angular momentum. Once the original inner disc has spread inwards (and this new innermost disc has spread outwards), exactly the same process starts again between these two discs. See Fig. 6 for the disc structure.

Angular momentum cancellation thus proceeds as follows. Imagine the two discs are not in contact. They spread viscously until gas from the discs can interact, sharing and so destroying angular momentum. Now consider two test particles (one from the inner disc and one from the outer disc) which interact. The particle from the inner disc adopts a slightly eccentric orbit, which passes through its parent disc. It rejoins its own disc, which shrinks slightly to accommodate the loss of angular momentum. The particle from the outer disc also adopts a slightly eccentric orbit (inside its original orbit). But this forces it to interact more with the 'hostile' inner disc. This causes runaway angular momentum loss for the outer disc particle, almost to the point of free–fall to the centre. Now the particle's free–fall energy is enough to move it out of the disc plane (helped by the pressure force from the inner disc). It thus moves over and then inside the inner disc. The overall effect is to shrink the inner disc and move the outer disc particles on to orbits that pass inside the inner disc.

At this point one of three things can happen to the outer disc particle:
1) it may have cancelled enough angular momentum to accrete directly.

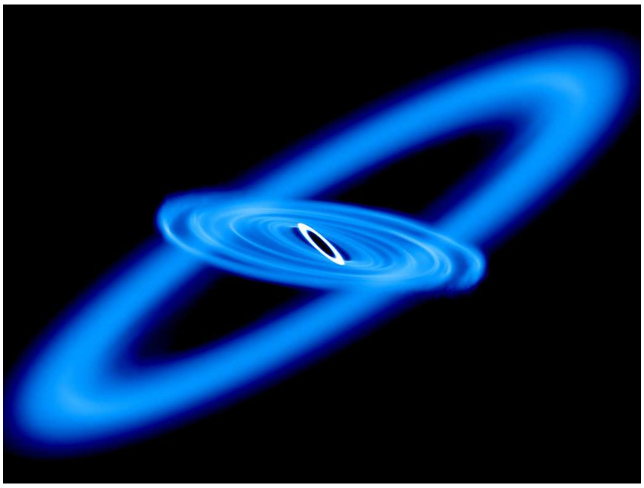

Figure 7. Column density projection showing the disc structure for the $\theta = 150°$ simulation with an isothermal equation of state. The disc has evolved for ~ 20 dynamical times.

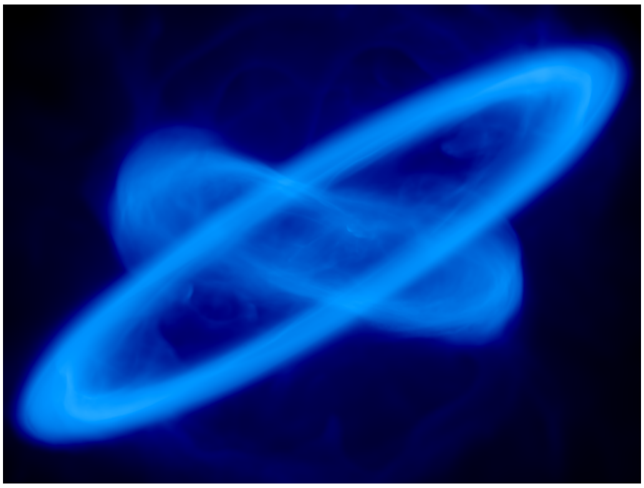

Figure 8. Column density projection showing the disc structure for the $\theta = 150°$ simulation with an "adiabatic" equation of state (see the text for a full definition). The disc has evolved for ~ 20 dynamical times. Note that the gas no longer can cool to form a smaller disc (cf fig 7), but instead forms outflows which return on chaotic orbits generating a lot of cancellation of angular momentum.

2) it may have cancelled enough to fall inside the inner edge of the discs where it crosses the plane of the inner disc and meets other particles on similar orbits: the particles shock and dissipate energy, forming a new disc at much smaller radii. This disc should be planar, but any small perturbations in the initial discs will be amplified here at smaller radii and could produce a disc with a substantial warp.
3) The particle's new orbit may take it through the inner (initial) disc. If this is the case it again interacts with the (hostile) disc and falls further. There are again two possibilities here. Either the particle bounces all the way in past the hostile disc, or it has so many interactions that it adopts the sense of rotation of the hostile disc. So the particle either accretes or drives the inner disc to shrink, and hence drives accretion through the inner disc.

All sequences here greatly enhance the central accretion rate while the discs are trying to mix their angular momenta.

3.2.2 Effect of the gas equation of state

We have also investigated the effect of changing the thermal treatment of the gas. In the simulations detailed above we used an isothermal equation of state. The gas was assumed to radiate away any dissipative heating instantly. The other extreme is to assume that any dissipation feeds the internal energy of the particles, which are not allowed to cool by radiative losses. Therefore we also simulate the $\theta = 150°$ case using an adiabatic equation of state (with $\gamma = 5/3$), transferring the kinetic energy dissipated by the viscous terms into internal energy. In all other ways this simulation is precisely the same as the corresponding isothermal simulation. In Figs. 7 & 8 we show the disc evolution after ~ 20 dynamical times. In the isothermal case the gas cancels angular momentum and falls to circularise at a new radius. In comparison Fig. 8 shows that the equation of state can play a significant role in the hydrodynamics. In this case the gas is significantly heated when it shocks and thus initially expands violently. This process drives more mixing of the gas and thus the accretion is now almost a factor of two higher here than in the isothermal case. The gas is still bound to the black hole and so returns on chaotic orbits to eventually accrete.

4 DISCUSSION

Retrograde circumbinary accretion discs are natural in the chaotic accretion picture (King & Pringle 2006; King & Pringle 2007), and offer the possibility of cancelling the binary orbital angular momentum (Nixon et al., 2011). We have performed numerical simulations of accretion discs with two initially distinct planes. In the region between the two discs angular momentum is shared, driving accretion, particularly if $\theta > \pi/2$.

The highest accretion rates come from discs with inclination $\theta \approx 150°$. Here the gas cancels angular momentum when the discs meet, allowing this gas to fall directly to the centre without the need to interact with the inner disc. Clearly this gas would in reality have some residual angular momentum and so would circularise at a new smaller radius. In the simulations however the gas falls inside our accretion radius for the central sink particle and so is directly accreted. Gas falling from between the discs always has angular momentum with the same sense of direction as the outer disc. Therefore if the gas tries to circularise outside the accretion radius it impacts the inner disc, cancelling more angular momentum and falling further until it is accreted.

The accretion rates found here are > 100 times those of a flat planar disc. This increase comes from the mixing of angular momenta and consequent dynamical infall of gas to smaller radii. As gas is depleted from the gap the disc tries to spread and fill the gap, promoting more cancellation of angular momentum and so more accretion. The timescale for gas accretion is shortened (cf. equation 8). If enough angular momentum is cancelled then the gas accretes directly on a dynamical timescale.

Our simulations show that discs with an initial inclination less that $90°$ evolve into coherent discs joined by a warp. Over time, dissipation from this warp brings the two discs into a common plane. This extra dissipation enhances accretion, the effect increasing with inclination angle (although negligible in comparison to the enhanced accretion when $\theta > \pi/2$). The discs appear to break, in the sense that the surface density of the disc is greatly reduced in the warp. However in these high–resolution simulations we still resolve the orbits of the disc particles in the warp. This disc breaking is predicted by the numerical simulations of warp propagation in accretion discs by Nixon & King (2012) using the constrained viscosities of Ogilvie (1999), and has been also been seen in the SPH simulations of Larwood & Papaloizou (1997) and Lodato & Price (2010).

Inclinations greater than 90° cancel rather than sharing angular momentum between particles. This leads to particles falling on to smaller orbits, and so the accretion rates in these simulations are significantly higher. The accretion rate for $\theta = 180°$ is lower than that of $\theta = 150°$. This is probably because gas must be pushed out of the plane of the discs before it can leap over the inner disc.

The numerical simulations do not predict accretion rates as high as the analytical approach (3). This was based on two assumptions: that the angular momentum cancellation is perfect, and an equal mass from each disc interacts. The simulations suggest instead that the gas falling from the gap has a spread of angular momentum, where some accretes and some falls to a smaller radius.

The main result of this paper is that internally tilted accretion discs can generate enhanced accretion rates of up to $\sim 10^4$ times that of a plane disc (cf Fig. 1). In a quasi–steady state (such discs never reach a full steady state) we see accretion rates that are $\gtrsim 100$ times those of a planar disc. As remarked above, discs tilted in this way arise naturally during the alignment of a disc around a spinning black hole or a central binary (cf. Lodato & Pringle 2006 & Nixon et al. 2011). We also expect such structures to occur during chaotic gas infall into galactic centres. This is a very efficient way of driving gas right down to the vicinity of an SMBH or an SMBH binary and thus causing accretion. Nested discs suggest a way of transferring gas from large radii (approaching galaxy scales) to small radii.

We note finally that our result, that tilted and nested discs imply enhanced accretion, has implications for simulations where subgrid models are used to mimic feedback from the SMBH. The dynamics of misaligned accretion events may imply much larger accretion rates than predicted by the simple assumption of a planar disc with a fixed direction of angular momentum.

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