ASP2062 Introduction to Astrophysics

Star formation III — Daniel Price

Key revision points

1. Fragmentation stops when the temperature inside the cloud starts to increase.
2. This 'opacity limit for fragmentation' sets the minimum mass for stars.
3. Star formation proceeds via two 'stall' phases where a hydrostatic object is formed, the 'first' and 'second' cores, caused by changes in the thermodynamics of the gas.
4. The *initial mass function* is the number of stars born as a function of mass

1 The physics of star formation, cont...

1.5 When does fragmentation stop?

Fragmentation stops when the gas starts to heat. This occurs when radiation can no longer escape, i.e. the gas becomes optically thick. This occurs due to dust opacity when

$$\rho \gtrsim 10^{-13}\mathrm{g/cm^3}. \tag{1}$$

The Jeans mass at this density defines *the minimum mass for a star* and is known as the *opacity limit for fragmentation*, first proposed by Low and Lynden-Bell (1976) and Rees (1976). We can easily calculate this minimum mass using (1) as

$$\begin{aligned} M_{\rm min} &= \left(\frac{\pi}{G}\right)^{3/2} \left(c_{\rm s} = 2\times 10^4 \mathrm{cm/s}\right)^3 \left(\rho = 10^{-13}\mathrm{g/cm^3}\right)^{-1/2} \\ &\approx 8.6\times 10^{30}\mathrm{g} \\ &\approx \mathit{0.0043}\mathrm{M}_\odot \\ &\approx \mathit{4.5} M_{\rm Jupiter} \end{aligned}$$

Once the opacity limit for fragmentation is reached, a hydrostatic object is formed, known as the 'first hydrostatic core' (Larson, 1969).

How big is the first hydrostatic core?
Just like we calculated the minimum mass for a star, what is the typical size of the 'fragments' that the molecular cloud breaks up into? i.e. what is the Jeans length at this density?) Express your answer in both AU and solar radii.

1.6 First and second core

As matter continues to accrete from the surrounding cloud, the first core continues to contract and heat (radiation is trapped). Pressure increases with density approximately as $P \propto \rho^{7/5}$, and the temperature continues to increase. At $\rho \approx 10^{-8}$ g/cm^3 the temperature reaches 2000K. At this point molecular hydrogen dissociates, which absorbs energy (an *endothermic* reaction) and a second stage of nearly isothermal collapse occurs, with $P \propto \rho^{1.1}$ until a density of $\rho \sim 10^{-3}$ g/cm^3. Above this $P \propto \rho^{5/3}$ and a hydrostatic object is again formed — the 'second' or 'protostellar' core.

Detailed one dimensional computer simulations of the collapse process that account for all of the chemistry show how the central temperature of the collapsing fragment evolves as the density increases (Figure 1)

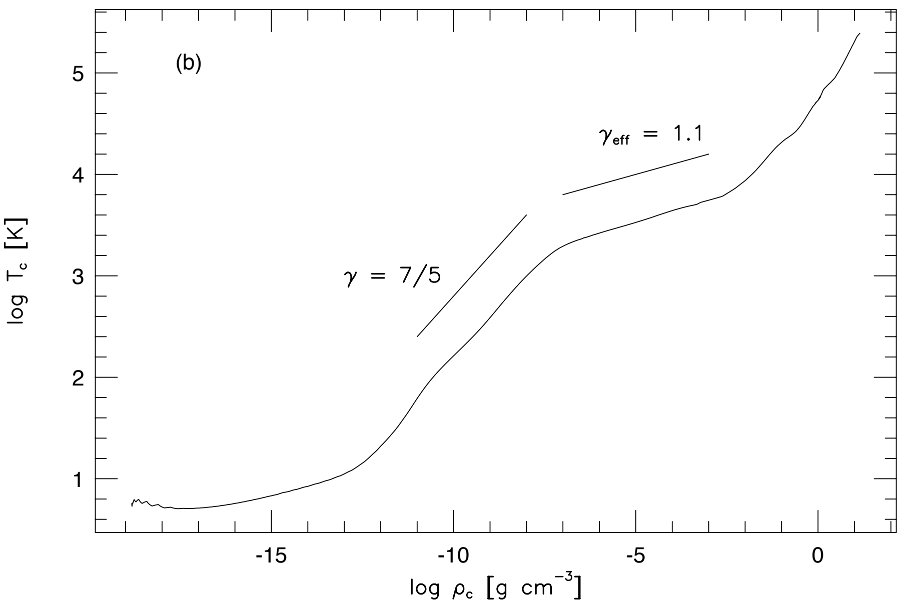

Figure 1: Temperature as a function of density during the collapse of an isolated molecular cloud core, from 1D radiation hydrodynamics calculations including dust and molecule chemistry by Masunaga and Inutsuka (2000).

The temperature evolution can be parameterised into a simple 'barotropic' equation of state $P = K\rho^\gamma$, where

$$\gamma = \begin{cases} 1 & \rho < 10^{-13}\mathrm{g/cm^3} & (T = \mathrm{const}) \\ 7/5 & 10^{-13} \lesssim \rho \lesssim 10^{-8}\mathrm{g/cm^3} & (T \propto \rho^{2/5}) \\ 1.1 & 10^{-8} \lesssim \rho \lesssim 10^{-3}\mathrm{g/cm^3} & (T \propto \rho^{0.1}) \\ 5/3 & \rho > 10^{-3}\mathrm{g/cm^3} & (T \propto \rho^{2/3}) \end{cases} \tag{2}$$

With this it is possible to run 3D simulations of star formation without worrying about details of the radiation and dust chemistry. We can simply prescribe how the temperature changes as a function of density. State-of-the-art 3D simulations now include radiation.

Above 10^{-3} g/cm^3 radiation is trapped and the protostar is powered by gravitational contraction, i.e. $P \propto \rho^{5/3}$ with the density continuing to slowly rise until the temperature is high enough for nuclear fusion to start. At this point a star is born. The contraction-powered, 'protostellar' or 'pre-main sequence' phase lasts for ∼10 Myr (see problem sheet).

Can we observe the first core?

The first core phase of star formation has been predicted from simulations for more than 40 years, but has not yet been definitively observed due to its extremely short lifetime (100 – 1000 yrs) and low luminosity. Seeing the first core in nearby molecular clouds is one of the key targets of the Atacama Large Millimetre/submillimetre Array (ALMA) telescope.

Material continues to accrete onto the protostar via an *accretion disc* (these are the sites of planet formation; see next week's lectures). The final mass of the star depends on how much gas it can accrete from the parent cloud. Objects with masses less than $\sim 80\ \mathrm{M_{Jupiter}}$ are too small to fuse hydrogen and are known as brown dwarfs (these burn Deuterium if $M \gtrsim 13\ \mathrm{M_{Jup}}$ and also Lithium if $M \gtrsim 65\ \mathrm{M_{Jup}}$). Around 80 Jupiter masses is the threshold for nuclear fusion of H→He to be possible, the definition of a star.

1.7 The initial mass function

Salpeter (1955) first inferred the birth rate of stars as a function of their mass, based on the observed luminosity of stars near the Sun (he did much of this work during a sabbatical visit to Mt Stromlo observatory in Canberra). He inferred that the relative number of stars as a function of mass $N(m)$ is a steeply decreasing function of mass,

$$N(m) \propto m^{-\alpha}, \tag{3}$$

with $\alpha \approx 1.35$ for $m > 1\mathrm{M_\odot}$.

How many stars?

The classic 'Salpeter slope' of the IMF implies that star formation produces many more low mass stars than high mass stars. For every $10M_\odot$ star born in a molecular cloud there are twenty-two $1M_\odot$ stars formed. How many solar-mass stars would you produce for every $100M_\odot$ star?

At lower masses the distribution flattens and turns over, with a peak around $\sim 0.5M_\odot$. Determining the Initial Mass Function (IMF) is very important for understanding the universe, since stars of different masses evolve *very* differently. Remarkably, this basic shape of the initial mass function (IMF) seems, within statistics, to be the same everywhere we look in the Galaxy, suggesting a universal physical process at work.

1.8 Origin of the IMF

Predicting the IMF is a key goal of star formation theory, but its origin is not yet fully understood. Two leading explanations are *turbulence* and *competitive accretion*.

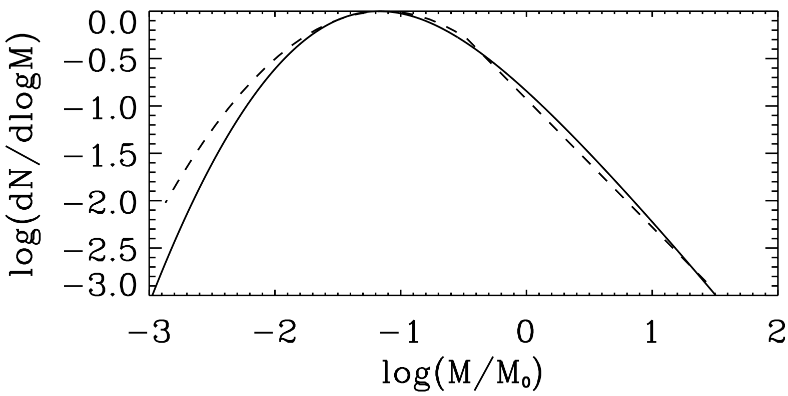

Figure 2: Comparison between the theoretical IMF (solid lines) produced by integrating the density PDF produced in supersonic turbulence and the observed IMF in the Galaxy (dashed lines), from the theory of Hennebelle and Chabrier (2008).

1.8.1 Turbulence

Computer simulations of supersonic turbulence show that it produces a probability distribution function (PDF; the probability of a parcel of gas being at a given density) in gas density with a 'log-normal' shape, similar to the observed shape of the IMF at low masses. The idea, first put forward by Padoan and Nordlund (2002) and developed further by Hennebelle and Chabrier (2008) is that turbulence first shapes the density distribution in the cloud, and that the high density tail of the PDF is then converted into stars under the influence of gravity. However, a definitive link between the PDF (a function of density) and the IMF (a function of mass) is yet to be made, either in simulations or observationally.

Log-normal distributions

A normal distribution, having the shape of a Gaussian,

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\bar{x})^2}{2\sigma^2}\right], \tag{4}$$

arises frequently in Nature because the central limit theorem tells us that the addition of many independent random fluctuations will result in a normal distribution. The *log-normal* distribution is a normal distribution in the logarithm, i.e.

$$f(\log x) = \frac{1}{\sqrt{2\pi\sigma_{\log}^2}} \exp\left[-\frac{(\log x - \overline{\log x})^2}{2\sigma_{\log}^2}\right]. \tag{5}$$

Log-normal distributions arise from the *multiplication* of many random, independent variables. The log-normal PDF in supersonic turbulence is thought to arise from the multiplication effect of many independent random shocks compressing the gas.

1.8.2 Competitive accretion

Bate and Bonnell (2005) suggested, on the basis of their computer simulations of star cluster formation, that the IMF arises from a process of 'competitive accretion' between protostars. That is, all stars are born equal, at the opacity limit for fragmentation, but then *compete* with each other from a finite reservoir of gas. Low mass stars and brown dwarfs are those that are ejected quickly from the cloud, before accreting much gas. Massive stars are those that remain in the potential well and thus accrete the most gas.

All stars are born equal, but some are more equal than others

In the competitive accretion picture, all stars are born with the same mass, but the stars that gain mass quickly can out-compete the other stars, since they create a deeper potential well into which more gas is drawn. These stars 'run away' to high mass, producing the power-law shape of the IMF at the high mass end observed by Salpeter. In this picture nearly all stars form in binary and multiple systems, and low mass stars and brown dwarfs are formed by ejection from unstable triple or multiple systems (three or more bodies in orbit are in general unstable, leading to ejection of the lowest mass object).

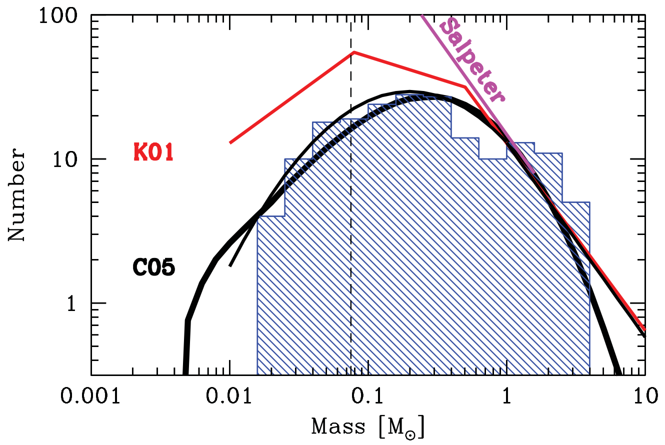

Figure 3: Initial mass function produced in the largest computer simulation of star cluster formation performed to date (Bate, 2012) compared to the observational determinations of the low-mass end of the IMF from Chabrier (2005) (C05) and Kroupa (2001) (K01) together with the classical Salpeter slope at high masses. The black line shows the prediction from the simple accretion/ejection model proposed by Bate and Bonnell (2005).

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