Discrete Mathematics Seminar
Monash University
20140519

# New directions in matroidal coding theory


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UNSW

## linear code

# linear code

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

# linear code

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |

linear code

weights

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 |

linear code

weights

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | |
| 0 | 1 | 1 | 0 | 0 | |
| 0 | 0 | 0 | 1 | 1 | |
| 1 | 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 1 | 1 | |
| 0 | 1 | 1 | 1 | 1 | |
| 1 | 1 | 0 | 1 | 1 | |

## linear code

## weights

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | | weights |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 2 |
| 0 | 1 | 1 | 0 | 0 | |
| 0 | 0 | 0 | 1 | 1 | |
| 1 | 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 1 | 1 | |
| 0 | 1 | 1 | 1 | 1 | |
| 1 | 1 | 0 | 1 | 1 | |

## linear code

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## weights

Codewords

| | | | | | weights |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 2 |
| 0 | 1 | 1 | 0 | 0 | 2 |
| 0 | 0 | 0 | 1 | 1 | |
| 1 | 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 1 | 1 | |
| 0 | 1 | 1 | 1 | 1 | |
| 1 | 1 | 0 | 1 | 1 | |

linear code | weights

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 2 |
| 0 | 1 | 1 | 0 | 0 | 2 |
| 0 | 0 | 0 | 1 | 1 | 2 |
| 1 | 1 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 1 | 1 | |
| 0 | 1 | 1 | 1 | 1 | |
| 1 | 1 | 0 | 1 | 1 | |

linear code

weights

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | | weights |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 2 |
| 0 | 1 | 1 | 0 | 0 | 2 |
| 0 | 0 | 0 | 1 | 1 | 2 |
| 1 | 1 | 0 | 0 | 0 | 2 |
| 1 | 0 | 1 | 1 | 1 | |
| 0 | 1 | 1 | 1 | 1 | |
| 1 | 1 | 0 | 1 | 1 | |

## linear code

## weights

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | | |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 2 |
| 0 | 1 | 1 | 0 | 0 | 2 |
| 0 | 0 | 0 | 1 | 1 | 2 |
| 1 | 1 | 0 | 0 | 0 | 2 |
| **1** | 0 | **1** | **1** | **1** | 4 |
| 0 | 1 | 1 | 1 | 1 | |
| 1 | 1 | 0 | 1 | 1 | |

linear code — weights

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | | weights |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 2 |
| 0 | 1 | 1 | 0 | 0 | 2 |
| 0 | 0 | 0 | 1 | 1 | 2 |
| 1 | 1 | 0 | 0 | 0 | 2 |
| 1 | 0 | 1 | 1 | 1 | 4 |
| 0 | **1** | **1** | **1** | **1** | **4** |
| 1 | 1 | 0 | 1 | 1 | |

linear code — weights

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| Codewords | weights |
|---|---|
| 0 0 0 0 0 | 0 |
| 1 0 1 0 0 | 2 |
| 0 1 1 0 0 | 2 |
| 0 0 0 1 1 | 2 |
| 1 1 0 0 0 | 2 |
| 1 0 1 1 1 | 4 |
| 0 1 1 1 1 | 4 |
| 1 1 0 1 1 | 4 |

| linear code | weights |
|---|---|
| 1 0 1 0 0<br>0 1 1 0 0<br>0 0 0 1 1 | |

Codewords

| Codewords | weights |
|---|---|
| 0 0 0 0 0 | 0 |
| 1 0 1 0 0 | 2 |
| 0 1 1 0 0 | 2 |
| 0 0 0 1 1 | 2 |
| 1 1 0 0 0 | 2 |
| 1 0 1 1 1 | 4 |
| 0 1 1 1 1 | 4 |
| 1 1 0 1 1 | 4 |

## linear code

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## weights

Codewords

| | | | | | weights |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 0 | 2 |
| 0 | 1 | 1 | 0 | 0 | 2 |
| 0 | 0 | 0 | 1 | 1 | 2 |
| 1 | 1 | 0 | 0 | 0 | 2 |
| 1 | 0 | 1 | 1 | 1 | 4 |
| 0 | 1 | 1 | 1 | 1 | 4 |
| 1 | 1 | 0 | 1 | 1 | 4 |

Weight enumerator

$$A(z) = 1 + 4z^2 + 3z^4$$

| linear code | weights | supports |
|---|---|---|

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | | weights | supports |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | |
| 1 | 0 | 1 | 0 | 0 | 2 | |
| 0 | 1 | 1 | 0 | 0 | 2 | |
| 0 | 0 | 0 | 1 | 1 | 2 | |
| 1 | 1 | 0 | 0 | 0 | 2 | |
| 1 | 0 | 1 | 1 | 1 | 4 | |
| 0 | 1 | 1 | 1 | 1 | 4 | |
| 1 | 1 | 0 | 1 | 1 | 4 | |

linear code | weights | supports

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| linear code | weights | supports |
|---|---|---|
| 0 0 0 0 0 | 0 | $\varnothing$ |
| 1 0 1 0 0 | 2 | |
| 0 1 1 0 0 | 2 | |
| 0 0 0 1 1 | 2 | |
| 1 1 0 0 0 | 2 | |
| 1 0 1 1 1 | 4 | |
| 0 1 1 1 1 | 4 | |
| 1 1 0 1 1 | 4 | |

linear code | weights | supports

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | | weights | supports |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | $\varnothing$ |
| 1 | 0 | 1 | 0 | 0 | 2 | 13 |
| 0 | 1 | 1 | 0 | 0 | 2 | |
| 0 | 0 | 0 | 1 | 1 | 2 | |
| 1 | 1 | 0 | 0 | 0 | 2 | |
| 1 | 0 | 1 | 1 | 1 | 4 | |
| 0 | 1 | 1 | 1 | 1 | 4 | |
| 1 | 1 | 0 | 1 | 1 | 4 | |

linear code — weights — supports

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| linear code | weights | supports |
|---|---|---|
| 0 0 0 0 0 | 0 | $\varnothing$ |
| 1 0 1 0 0 | 2 | 13 |
| 0 **1 1** 0 0 | 2 | 23 |
| 0 0 0 1 1 | 2 | |
| 1 1 0 0 0 | 2 | |
| 1 0 1 1 1 | 4 | |
| 0 1 1 1 1 | 4 | |
| 1 1 0 1 1 | 4 | |

| linear code | | | | | weights | supports |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | | |
| 1 | 0 | 1 | 0 | 0 | | |
| 0 | 1 | 1 | 0 | 0 | | |
| 0 | 0 | 0 | 1 | 1 | | |

Codewords

| | | | | | weights | supports |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | $\varnothing$ |
| 1 | 0 | 1 | 0 | 0 | 2 | 13 |
| 0 | 1 | 1 | 0 | 0 | 2 | 23 |
| 0 | 0 | 0 | **1** | **1** | 2 | 45 |
| 1 | 1 | 0 | 0 | 0 | 2 | |
| 1 | 0 | 1 | 1 | 1 | 4 | |
| 0 | 1 | 1 | 1 | 1 | 4 | |
| 1 | 1 | 0 | 1 | 1 | 4 | |

| linear code | | | | | weights | supports |
|---|---|---|---|---|---|---|

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | | weights | supports |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | $\varnothing$ |
| 1 | 0 | 1 | 0 | 0 | 2 | 13 |
| 0 | 1 | 1 | 0 | 0 | 2 | 23 |
| 0 | 0 | 0 | 1 | 1 | 2 | 45 |
| 1 | 1 | 0 | 0 | 0 | 2 | 12 |
| 1 | 0 | 1 | 1 | 1 | 4 | |
| 0 | 1 | 1 | 1 | 1 | 4 | |
| 1 | 1 | 0 | 1 | 1 | 4 | |

linear code | weights | supports

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| Codeword | weights | supports |
|---|---|---|
| 0 0 0 0 0 | 0 | $\varnothing$ |
| 1 0 1 0 0 | 2 | 13 |
| 0 1 1 0 0 | 2 | 23 |
| 0 0 0 1 1 | 2 | 45 |
| 1 1 0 0 0 | 2 | 12 |
| 1 0 1 1 1 | 4 | 1345 |
| 0 1 1 1 1 | 4 | |
| 1 1 0 1 1 | 4 | |

| linear code | | | | | weights | supports |
|---|---|---|---|---|---|---|

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| | | | | | weights | supports |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | $\varnothing$ |
| 1 | 0 | 1 | 0 | 0 | 2 | 13 |
| 0 | 1 | 1 | 0 | 0 | 2 | 23 |
| 0 | 0 | 0 | 1 | 1 | 2 | 45 |
| 1 | 1 | 0 | 0 | 0 | 2 | 12 |
| 1 | 0 | 1 | 1 | 1 | 4 | 1345 |
| 0 | **1** | **1** | **1** | **1** | 4 | 2345 |
| 1 | 1 | 0 | 1 | 1 | 4 | |

## linear code

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| Codeword | weights | supports |
|---|---|---|
| 0 0 0 0 0 | 0 | $\varnothing$ |
| 1 0 1 0 0 | 2 | 13 |
| 0 1 1 0 0 | 2 | 23 |
| 0 0 0 1 1 | 2 | 45 |
| 1 1 0 0 0 | 2 | 12 |
| 1 0 1 1 1 | 4 | 1345 |
| 0 1 1 1 1 | 4 | 2345 |
| 1 1 0 1 1 | 4 | 1245 |

linear code | weights | supports

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Codewords

| Codeword | weights | supports |
|---|---|---|
| 0 0 0 0 0 | 0 | $\varnothing$ |
| 1 0 1 0 0 | 2 | 13 |
| 0 1 1 0 0 | 2 | 23 |
| 0 0 0 1 1 | 2 | 45 |
| 1 1 0 0 0 | 2 | 12 |
| 1 0 1 1 1 | 4 | 1345 |
| 0 1 1 1 1 | 4 | 2345 |
| 1 1 0 1 1 | 4 | 1245 |

linear code

vector matroid

linear code

vector matroid

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

linear code

vector matroid

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## linear code

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## vector matroid

Independent sets $\mathcal{I}$

$\varnothing$ 1 2 3 4 5

12 13 14 15 23 24 25 34 35

124 125 134 135 234 235

linear code

vector matroid

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Independent sets $\mathcal{I}$

$\varnothing$ 1 2 3 4 5

12 13 14 15 23 24 25 34 35

124 125 134 135 234 235

linear code

vector matroid

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Independent sets $\mathcal{I}$

$\varnothing$ 1 2 3 4 5

12 13 14 15 23 24 25 34 35

124 125 134 135 234 235

## linear code

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## vector matroid

Independent sets $\mathcal{I}$

$\varnothing$ 1 2 3 4 5

12 13 14 15 23 24 25 34 35

124 125 134 135 234 235

## linear code

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## vector matroid

Independent sets $\mathcal{I}$

$\varnothing$ 1 2 3 4 5

12 13 14 15 23 24 25 34 35

124 125 134 135 234 235

## linear code

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## vector matroid

Maximal independent sets $\mathcal{B}$ases

$\varnothing$ 1 2 3 4 5

12 13 14 15 23 24 25 34 35

124 125 134 135 234 235

linear code

vector matroid

Minimally *dependent* sets $\mathcal{C}$ircuits

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

$123 \quad 45$

linear code

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

vector matroid

Minimally *dependent* sets $\mathcal{C}$ircuits

$123 \quad 45$

linear code

vector matroid

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Minimally *dependent* sets $\mathcal{C}$ircuits

$123$ $45$

linear code

vector matroid

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Rank function $\rho$

$$\rho(45) = 1$$

## linear code

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## vector matroid

Rank function $\rho$

$$\rho(12) = 2$$

linear code

vector matroid

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Rank function $\rho$

$$\rho(123) = 2$$

linear code

vector matroid

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Rank function $\rho$

$$\rho(124) = 3$$

linear code

vector matroid

|  |  |  |  |  |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Rank function $\rho$

$$\rho(124) = 3$$

Tutte polynomial

$$T(x+1, y+1) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)}$$

linear code

vector matroid

| 1 | 2 | 3 | 4 | 5 |
| --- | --- | --- | --- | --- |
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Rank function $\rho$

$$\rho(124) \ = \ 3$$

Tutte polynomial

$$T(x+1, y+1) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)}$$

$$= 6 + 9x + 5x^2 + x^3 + 5y + y^2 + 4xy + x^2y$$

$C \quad =$ a linear code over a finite field $\mathbb{F}_q$

$M_C =$ the associated vector matroid

Crapo Rota 1970 $\qquad M_C$ determines the codeword supports of $C$

$C$ = a linear code over a finite field $\mathbb{F}_q$
$M_C$ = the associated vector matroid

Crapo Rota 1970 $M_C$ determines the codeword supports of $C$
"The Critical Theorem"

$C \quad =$ a linear code over a finite field $\mathbb{F}_q$
$M_C =$ the associated vector matroid

Crapo Rota 1970 — $M_C$ determines the codeword supports of $C$
Britz 2005 — An infinite class of such results, eg. subcode supports

$C \;=\;$ a linear code over a finite field $\mathbb{F}_q$

$M_C =$ the associated vector matroid

| | |
|---|---|
| Crapo Rota 1970 | $M_C$ determines the codeword supports of $C$ |
| Britz 2005 | An infinite class of such results, eg. subcode supports |
| Greene 1976 | A small part of $M_C$ determines $C$'s codeword weights |

$C$ = a linear code over a finite field $\mathbb{F}_q$
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| | "Greene's Theorem" |

$C \;=\;$ a linear code over a finite field $\mathbb{F}_q$

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| Britz 2010 | Tutte polynomial and subcode weights are equivalent |

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|---|---|
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| Skorabogatov 1992 | $M_C$ does *not* determine the covering radius of $C$ |

$C$ = a linear code over a finite field $\mathbb{F}_q$

$M_C$ = the associated vector matroid

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|---|---|
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| BR 2005 | Other properties of $C$ not determined by $M_C$ |

$C$ = a linear code over a finite field $\mathbb{F}_q$

$M_C$ = the associated vector matroid

| | |
|---|---|
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| Britz 2010 | Tutte polynomial and subcode weights are equivalent |
| Skorabogatov 1992 | $M_C$ does *not* determine the covering radius of $C$ |
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| MacWilliams 1963 | The codeword weights of $C$ determine those of $C^{\perp}$ |

$C$ = a linear code over a finite field $\mathbb{F}_q$
$M_C$ = the associated vector matroid

| | |
|---|---|
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$C$ = a linear code over a finite field $\mathbb{F}_q$
$M_C$ = the associated vector matroid

| | |
|---|---|
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| BR 2005 | Other properties of $C$ not determined by $M_C$ |
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| Britz 2005 | An infinite class of MacWilliams-type results |

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# New directions

# New directions

## Quasi-uniform codes

# New directions

Quasi-uniform codes

Symplicial complexes

# New directions

Quasi-uniform codes

Symplicial complexes

Poset codes

# New directions

Quasi-uniform codes

Symplicial complexes

Poset codes

Linear codes over rings

# New directions

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Symplicial complexes

Poset codes

Linear codes over rings

Chains of codes

more

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more

# New directions

Quasi-uniform codes BCG 2013 The Critical Theorem ++

Symplicial complexes Greene's Theorem ++

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Linear codes over rings

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more

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Symplicial complexes BSW Wei's Duality Theorem ++

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BJMS 2012 Wei's Duality Theorem ++

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Symplicial complexes

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# New directions

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Symplicial complexes

Poset codes

Linear codes over rings — BSW 2013 — The Critical Theorem ++

Chains of codes — Greene's Theorem ++

more — The MacWilliams' Identity ++

# New directions

Quasi-uniform codes

Symplicial complexes

Poset codes

Linear codes over rings

Chains of codes

more

# New directions

Quasi-uniform codes

Symplicial complexes

Poset codes

Linear codes over rings

Chains of codes BJM 2014 vector matroids ++

more vector matroid duality ++

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composition series

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We cannot associate matroids to linear codes over rings or to chains of codes

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We cannot associate matroids to linear codes over rings or to chains of codes

— but we can associate demi-matroids to these! (BJM13)

# New directions

Quasi-uniform codes

Symplicial complexes

Poset codes

Linear codes over rings

Chains of codes

more

} Demi-matroids are appropriate

composition series

We cannot associate matroids to linear codes over rings or to chains of codes

— but we can associate demi-matroids to these! (BJM13)

linear code

vector matroid

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

Rank function $\rho$

$$\rho(45) = 1$$

linear code

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

vector matroid

Rank function $\rho$

$$\rho(12) = 2$$

## linear code

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## vector matroid

Rank function $\rho$

$$\rho(123) = 2$$

## linear code

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## vector matroid

Rank function $\rho$

$$\rho(124) = 3$$

$E$ = a finite set

$s, t$ = integer functions on $2^E$

$E\ = $ a finite set

$s, t =$ integer functions on $2^E$

$D = (E, s, t)$ is a demi-matroid

if and only if, for all $X \subseteq Y \subseteq E$,

- $0 \leq\ s(X) \leq\ s(Y) \leq\ |Y|$
- $0 \leq\ t(X) \leq\ t(Y) \leq\ |Y|$
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Example

$E = \{a, b\}$

$s(X) = t(X) = 0$ for $X = \varnothing, \{a\}, \{b\}$

$s(E) = t(E) = 1$

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Example

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$D = (E, s, t)$ is a demi-matroid (but not a matroid)

$E$ = a finite set

$\mathbb{F}$ = a finite field

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$F$ = a chain of linear codes $C_m \subseteq \cdots \subseteq C_2 \subseteq C_1 \subseteq \mathbb{F}^E$

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$$
\begin{aligned}
s_F(X) &= \sum_{i=1}^{m} (-1)^{i-1} \rho_i(X) \\
t_F(X) &= |X| - s_F(E) + s_F(E - X)
\end{aligned}
$$

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BJM13 $D_F = (E, s_F, t_F)$ is a demi-matroid

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Dual

$$D^* = (E, t, s)$$

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**$D = (E, s, t)$ is a demi-matroid**

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**Dual** $D^* = (E, t, s)$

**Supplement** $\overline{D} = (E, \overline{s}, \overline{t})$ where

$$\overline{s}(X) = s(E) - s(E - X)$$
$$\overline{t}(X) = t(E) - t(E - X)$$

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BJM13 $\quad D_{F^\perp} = \begin{cases} \overline{D_F} & m \text{ even} \\ (D_F)^* & m \text{ odd} \end{cases}$

# linear code

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## linear code

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

1 0 1 0 0   0 1 1 0 0   0 0 0 1 1   1 1 0 0 0   1 0 1 1 1   0 1 1 1 1   1 1 0 1 1

subcodes

# linear code

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

| 1 0 1 0 0 | 0 1 1 0 0 | 0 0 0 1 1 | 1 1 0 0 0 | 1 0 1 1 1 | 0 1 1 1 1 | 1 1 0 1 1 |
|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 4 | 4 | 4 |

subcodes

# linear code

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

| $1\ 0\ 1\ 0\ 0$ | $0\ 1\ 1\ 0\ 0$ | $0\ 0\ 0\ 1\ 1$ | $1\ 1\ 0\ 0\ 0$ | $1\ 0\ 1\ 1\ 1$ | $0\ 1\ 1\ 1\ 1$ | $1\ 1\ 0\ 1\ 1$ |
|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 4 | 4 | 4 |

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{matrix} \quad \begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix} \quad \begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{matrix} \quad \begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix} \quad \begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{matrix} \quad \begin{matrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{matrix} \quad \begin{matrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{matrix}$$

subcodes

# linear code

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

subcodes

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 0 1 0 0 | 0 1 1 0 0 | 0 0 0 1 1 | 1 1 0 0 0 | 1 0 1 1 1 | 0 1 1 1 1 | 1 1 0 1 1 |
| 2 | 2 | 2 | 2 | 4 | 4 | 4 |
| 1 0 1 0 0<br>0 1 1 0 0 | 1 0 1 0 0<br>0 0 0 1 1 | 1 0 1 0 0<br>0 1 1 1 1 | 1 1 0 0 0<br>0 0 0 1 1 | 1 1 0 0 0<br>1 0 1 1 1 | 0 0 0 1 1<br>0 1 1 0 0 | 0 1 1 0 0<br>1 0 1 1 1 |
| 3 | 4 | 5 | 4 | 5 | 4 | 5 |

# linear code

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

subcodes

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix}$ 2

$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 2

$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 2

$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}$ 2

$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 4

$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ 4

$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \end{bmatrix}$ 4

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 3

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 4

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$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 5

## linear code

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

## higher weights

$d_1 =$

$d_2 =$

$d_3 =$

subcodes

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \end{matrix}$ — 2

$\begin{matrix} 0 & 1 & 1 & 0 & 0 \end{matrix}$ — 2

$\begin{matrix} 0 & 0 & 0 & 1 & 1 \end{matrix}$ — 2

$\begin{matrix} 1 & 1 & 0 & 0 & 0 \end{matrix}$ — 2

$\begin{matrix} 1 & 0 & 1 & 1 & 1 \end{matrix}$ — 4

$\begin{matrix} 0 & 1 & 1 & 1 & 1 \end{matrix}$ — 4

$\begin{matrix} 1 & 1 & 0 & 1 & 1 \end{matrix}$ — 4

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{matrix}$ — 3

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$ — 4

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{matrix}$ — 5

$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$ — 4

$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{matrix}$ — 5

$\begin{matrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{matrix}$ — 4

$\begin{matrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{matrix}$ — 5

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$ — 5

## linear code

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## higher weights

$d_1 = 2$
$d_2 =$
$d_3 =$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 0 1 0 0 | 0 1 1 0 0 | 0 0 0 1 1 | 1 1 0 0 0 | 1 0 1 1 1 | 0 1 1 1 1 | 1 1 0 1 1 |
| 2 | 2 | 2 | 2 | 4 | 4 | 4 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 0 1 0 0<br>0 1 1 0 0 | 1 0 1 0 0<br>0 0 0 1 1 | 1 0 1 0 0<br>0 1 1 1 1 | 1 1 0 0 0<br>0 0 0 1 1 | 1 1 0 0 0<br>1 0 1 1 1 | 0 0 0 1 1<br>0 1 1 0 0 | 0 1 1 0 0<br>1 0 1 1 1 |
| 3 | 4 | 5 | 4 | 5 | 4 | 5 |

| |
|---|
| 1 0 1 0 0<br>0 1 1 0 0<br>0 0 0 1 1 |
| 5 |

subcodes

## linear code

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

## higher weights

$d_1 = 2$
$d_2 = 3$
$d_3 =$

subcodes

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \end{matrix}$ — 2

$\begin{matrix} 0 & 1 & 1 & 0 & 0 \end{matrix}$ — 2

$\begin{matrix} 0 & 0 & 0 & 1 & 1 \end{matrix}$ — 2

$\begin{matrix} 1 & 1 & 0 & 0 & 0 \end{matrix}$ — 2

$\begin{matrix} 1 & 0 & 1 & 1 & 1 \end{matrix}$ — 4

$\begin{matrix} 0 & 1 & 1 & 1 & 1 \end{matrix}$ — 4

$\begin{matrix} 1 & 1 & 0 & 1 & 1 \end{matrix}$ — 4

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{matrix}$ — 3

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$ — 4

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{matrix}$ — 5

$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$ — 4

$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{matrix}$ — 5

$\begin{matrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{matrix}$ — 4

$\begin{matrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{matrix}$ — 5

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$ — 5

## linear code

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

## higher weights

$$d_1 = 2$$
$$d_2 = 3$$
$$d_3 = 5$$

## subcodes

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 0 1 0 0 | 0 1 1 0 0 | 0 0 0 1 1 | 1 1 0 0 0 | 1 0 1 1 1 | 0 1 1 1 1 | 1 1 0 1 1 |
| 2 | 2 | 2 | 2 | 4 | 4 | 4 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 0 1 0 0<br>0 1 1 0 0 | 1 0 1 0 0<br>0 0 0 1 1 | 1 0 1 0 0<br>0 1 1 1 1 | 1 1 0 0 0<br>0 0 0 1 1 | 1 1 0 0 0<br>1 0 1 1 1 | 0 0 0 1 1<br>0 1 1 0 0 | 0 1 1 0 0<br>1 0 1 1 1 |
| 3 | 4 | 5 | 4 | 5 | 4 | 5 |

| |
|---|
| 1 0 1 0 0<br>0 1 1 0 0<br>0 0 0 1 1 |
| 5 |

## linear code

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

## higher weights

$d_1 = 2$ $\quad d_1^{\perp} = 2$

$d_2 = 3$ $\quad d_2^{\perp} = 5$

$d_3 = 5$

subcodes

| 1 0 1 0 0 | 0 1 1 0 0 | 0 0 0 1 1 | 1 1 0 0 0 | 1 0 1 1 1 | 0 1 1 1 1 | 1 1 0 1 1 |
|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 4 | 4 | 4 |

| 1 0 1 0 0<br>0 1 1 0 0 | 1 0 1 0 0<br>0 0 0 1 1 | 1 0 1 0 0<br>0 1 1 1 1 | 1 1 0 0 0<br>0 0 0 1 1 | 1 1 0 0 0<br>1 0 1 1 1 | 0 0 0 1 1<br>0 1 1 0 0 | 0 1 1 0 0<br>1 0 1 1 1 |
|---|---|---|---|---|---|---|
| 3 | 4 | 5 | 4 | 5 | 4 | 5 |

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

5

## linear code

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

## higher weights

$d_1 = 2 \quad d_1^{\perp} = 2$

$d_2 = 3 \quad d_2^{\perp} = 5$

$d_3 = 5$

linear code | higher weights

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

$d_1 = 2 \quad d_1^\perp = 2$

$d_2 = 3 \quad d_2^\perp = 5$

$d_3 = 5$

$$U = \{d_1, \ldots, d_k\}$$
$$V = \{n + 1 - d^\perp_{n-k-1}, \ldots, n + 1 - d^\perp_1\}$$

linear code

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

higher weights

$d_1 = 2 \quad d_1^{\perp} = 2$

$d_2 = 3 \quad d_2^{\perp} = 5$

$d_3 = 5$

$$U = \{d_1, \ldots, d_k\} = \{2, 3, 5\}$$

$$V = \{n + 1 - d^{\perp}_{n-k-1}, \ldots, n + 1 - d^{\perp}_1\} = \{1, 4\}$$

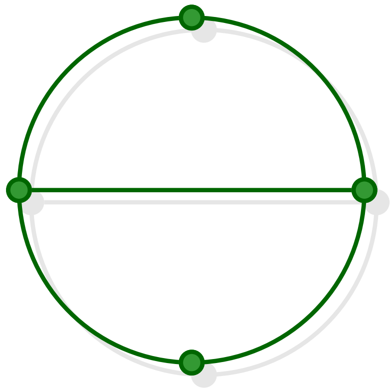
| linear code | higher weights |
|---|---|
| $\begin{matrix} 1&0&1&0&0\\ 0&1&1&0&0\\ 0&0&0&1&1 \end{matrix}$ | $\begin{matrix} d_1 = 2 & d_1^\perp = 2 \\ d_2 = 3 & d_2^\perp = 5 \\ d_3 = 5 & \end{matrix}$ |

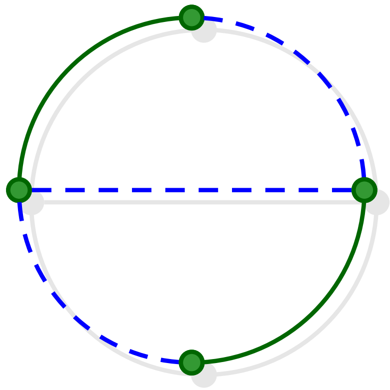
$$U = \{d_1, \ldots, d_k\} = \{2, 3, 5\}$$

$$V = \{n + 1 - d^\perp_{n-k-1}, \ldots, n + 1 - d^\perp_1\} = \{1, 4\}$$

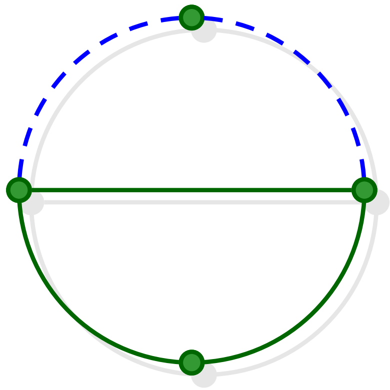
**Wei's Duality Theorem (Wei '91)**

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

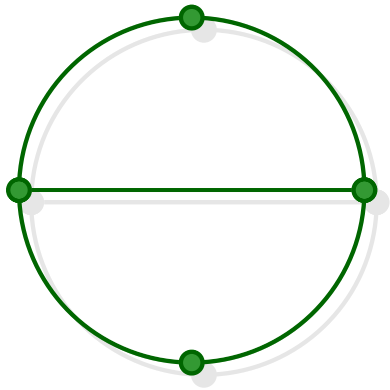




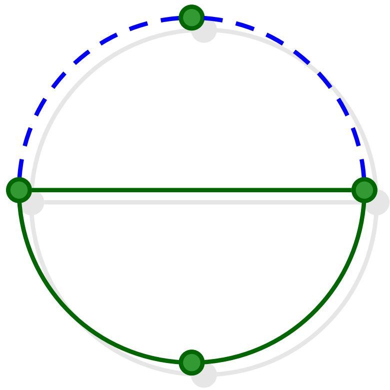
Bond = minimal cutset



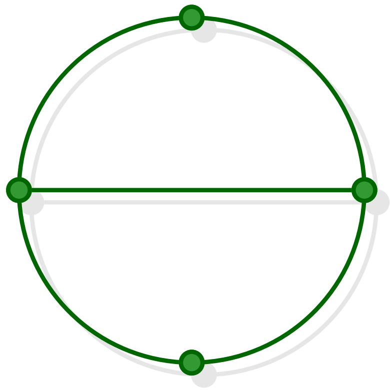
Bond = minimal cutset



$b_1$ = minimal size of a bond

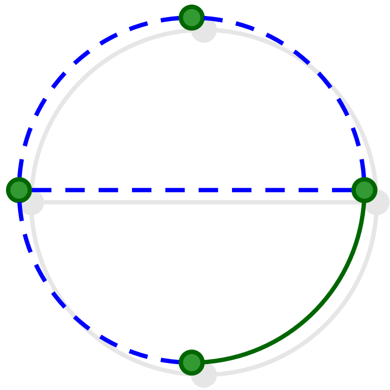


$b_1$ = minimal size of a bond = 2



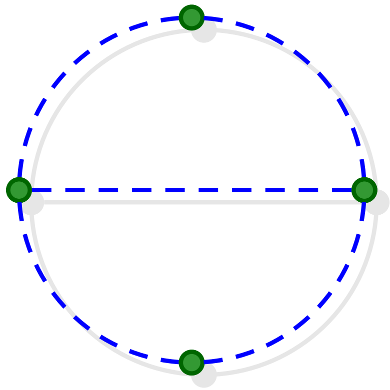
$b_1$ = minimal size of a bond = 2

$b_2$ = min. # edges in 2 distinct bonds =



$b_1$ = minimal size of a bond = 2

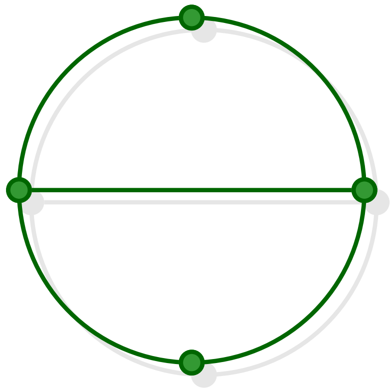
$b_2$ = min. # edges in 2 distinct bonds = 4



$b_1 =$ minimal size of a bond $= 2$

$b_2 =$ min. # edges in 2 distinct bonds $= 4$

$b_3 =$ min. # edges in 3 distinct bonds $B_1, B_2, B_3,\ \ B_3 \not\subseteq B_1 \cup B_2 \ = 5$

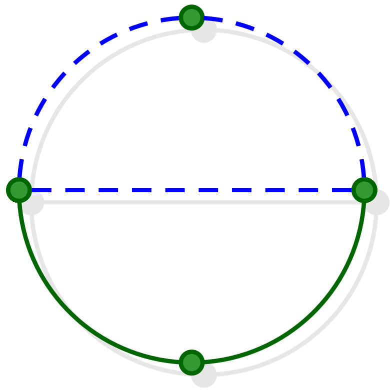


$b_1$ = minimal size of a bond = 2

$b_2$ = min. # edges in 2 distinct bonds = 4

$b_3$ = min. # edges in 3 distinct bonds $B_1, B_2, B_3,\ \ B_3 \not\subseteq B_1 \cup B_2$ = 5

$c_1$ = minimal size of a cycle =

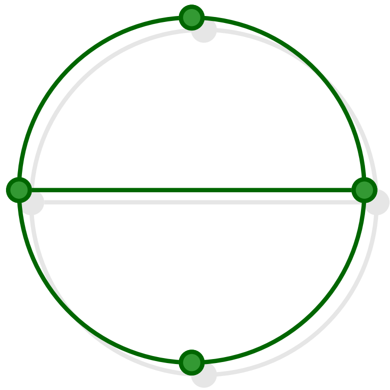


$b_1 =$ minimal size of a bond $=$ 2

$b_2 =$ min. # edges in 2 distinct bonds $=$ 4

$b_3 =$ min. # edges in 3 distinct bonds $B_1, B_2, B_3,\ \ B_3 \not\subseteq B_1 \cup B_2 \ \ =$ 5

$c_1 =$ minimal size of a cycle $=$ 3



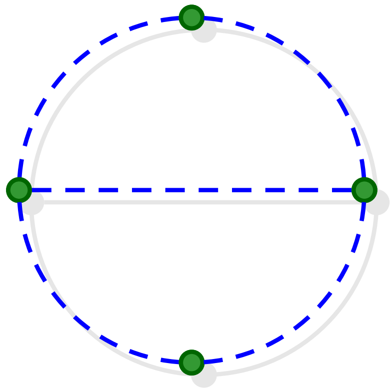
$b_1$ = minimal size of a bond = 2

$b_2$ = min. # edges in 2 distinct bonds = 4

$b_3$ = min. # edges in 3 distinct bonds $B_1, B_2, B_3,\ \ B_3 \not\subseteq B_1 \cup B_2$ = 5

$c_1$ = minimal size of a cycle = 3

$c_2$ = min. # edges in 2 distinct cycles =



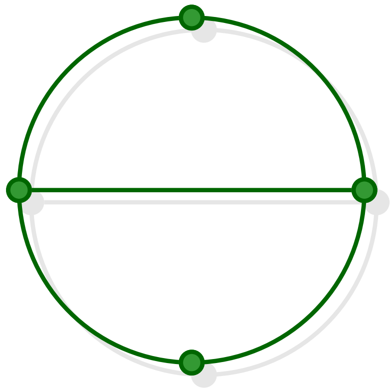
$b_1$ = minimal size of a bond = 2

$b_2$ = min. # edges in 2 distinct bonds = 4

$b_3$ = min. # edges in 3 distinct bonds $B_1, B_2, B_3,\ \ B_3 \not\subseteq B_1 \cup B_2$ = 5

$c_1$ = minimal size of a cycle = 3

$c_2$ = min. # edges in 2 distinct cycles = 5



$b_1 =$ minimal size of a bond $= 2$

$b_2 =$ min. # edges in 2 distinct bonds $= 4$

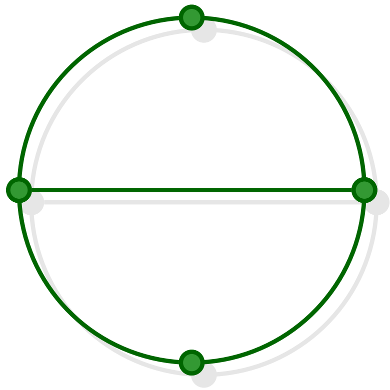
$b_3 =$ min. # edges in 3 distinct bonds $B_1, B_2, B_3,\ \ B_3 \not\subseteq B_1 \cup B_2 \ \ = 5$

$c_1 =$ minimal size of a cycle $= 3$

$c_2 =$ min. # edges in 2 distinct cycles $= 5$

Set $U = \{b_1, b_2, b_3\} = \{2, 4, 5\}$

and $V = \{5 + 1 - c_2, 5 + 1 - c_1\} = \{1, 3\}$.



$b_1$ = minimal size of a bond = 2

$b_2$ = min. # edges in 2 distinct bonds = 4

$b_3$ = min. # edges in 3 distinct bonds $B_1, B_2, B_3,\ \ B_3 \not\subseteq B_1 \cup B_2$ = 5

$c_1$ = minimal size of a cycle = 3

$c_2$ = min. # edges in 2 distinct cycles = 5

Set $U = \{b_1, b_2, b_3\} = \{2, 4, 5\}$

and $V = \{5 + 1 - c_2, 5 + 1 - c_1\} = \{1, 3\}$.

$U \cup V = \{1, 2, 3, 4, 5\}$ and $U \cap V = \emptyset$

$G$ = a multigraph on $n$ edges

Define

$k$ = # edges in a spanning forest of $G$

$b_i$ = min. # edges in $i$ bonds, none contained in the union of the others

$c_j$ = min. # edges in $j$ cycles, none contained in the union of the others

$$U = \{b_1, \ldots, b_k\}$$

$$V = \{n+1-c_{n-k}, \ldots, n+1-c_1\}.$$

Britz 2007: $U \cup V = \{1, \ldots, n\}$ and $U \cap V = \emptyset$

$M$ = a matroid of rank $k$ on $n$ elements

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Define

$$f_i = \max\{\, |X| \;:\; \rho_M(X) = i\}$$

$$f_j^* = \max\{\, |X| \;:\; \rho_{M^*}(X) = j\}$$

$M$ = a matroid of rank $k$ on $n$ elements

Define

$$f_i = \max\{\, |X| \;:\; \rho_M(X) = i\}$$

$$f_j^* = \max\{\, |X| \;:\; \rho_{M^*}(X) = j\}$$

$$U = \{f_0 + 1, \ldots, f_{k-1} + 1\}$$

$$V = \{n - f_{n-k-1}^*, \ldots, n - f_0^*\}.$$

$M$ = a matroid of rank $k$ on $n$ elements

Define

$$f_i = \max\{\,|X| \,:\, \rho_M(X) = i\}$$

$$f^*_j = \max\{\,|X| \,:\, \rho_{M^*}(X) = j\}$$

$$U = \{f_0 + 1, \ldots, f_{k-1} + 1\}$$

$$V = \{n - f^*_{n-k-1}, \ldots, n - f^*_0\}.$$

$$U \cup V = \{1, \ldots, n\} \quad \text{and} \quad U \cap V = \emptyset$$

$M$ = a matroid of rank $k$ on $n$ elements

Define

$$f_i = \max\{\,|X| \,:\, \rho_M(X) = i\}$$

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$$U = \{f_0 + 1, \ldots, f_{k-1} + 1\}$$

$$V = \{n - f_{n-k-1}^*, \ldots, n - f_0^*\}.$$

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \emptyset$

BJMS 2012
Larsen 2005

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Define

$$f_i = \max\{\, |X| \;:\; \rho_M(X) = i\}$$

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$$U = \{f_0 + 1, \ldots, f_{k-1} + 1\}$$

$$V = \{n - f_{n-k-1}^*, \ldots, n - f_0^*\}.$$

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \emptyset$

BJMS 2012
Larsen 2005

**Proof**. Assume that the theorem is false.

Then $f_i + 1 = n - f_j^*$ for some $i, j$.

Let $A \subseteq E$ satisfy $|A| = f_j^*$ and $r_{M^*}(A) = j$.

Then $|E - A| = f_i + 1$, so $r_M(E - A) \geq i + 1$.

Since $|E - A| + r_{M^*}(A) - r(M^*) = r_M(E - A)$,

$$-f_j^* + j + r \geq i + 1.$$

Similarly,

$$n - f_i + i - r \geq j + 1.$$

Hence, $1 = n - f_i - f_j^* \geq 2$, a contradiction. ■

$E$ = a set

$s, t$ = integer functions on $2^E$

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$s, t$ = integer functions on $2^E$

$D = (E, s, t)$ is a demi-matroid if and only if, for all $X \subseteq Y \subseteq E$,

- $0 \leq s(X) \leq s(Y) \leq |Y|$
- $0 \leq t(X) \leq t(Y) \leq |Y|$
- $|E - X| - s(E - X) = t(E) - t(X)$

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- $|E - X| - s(E - X) = t(E) - t(X)$

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- $0 \leq s(X) \leq s(Y) \leq |Y|$
- $0 \leq t(X) \leq t(Y) \leq |Y|$
- $|E - X| - s(E - X) = t(E) - t(X)$

$E$ = a set on $n$ elements

$s, t$ = integer functions on $2^E$ with $k = s(E)$

$D = (E, s, t)$ is a demi-matroid if and only if, for all $X \subseteq Y \subseteq E$,

- $0 \leq s(X) \leq s(Y) \leq |Y|$
- $0 \leq t(X) \leq t(Y) \leq |Y|$
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$s_i = \max\{\, |X| : s(X) = i\}$

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$E$ = a set on $n$ elements

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- $0 \leq t(X) \leq t(Y) \leq |Y|$
- $|E - X| - s(E - X) = t(E) - t(X)$

$s_i = \max\{\, |X| \,:\, s(X) = i\}$

$t_j = \max\{\, |X| \,:\, t(X) = j\}$

$\sigma_i = \min\{\, |X| \,:\, s(X) = i\}$

$\tau_j = \min\{\, |X| \,:\, t(X) = j\}$

$E$ = a set on $n$ elements

$s, t$ = integer functions on $2^E$ with $k = s(E)$

$D = (E, s, t)$ is a demi-matroid if and only if, for all $X \subseteq Y \subseteq E$,

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- $0 \leq t(X) \leq t(Y) \leq |Y|$
- $|E - X| - s(E - X) = t(E) - t(X)$

$s_i = \max\{ |X| : s(X) = i\}$ $\qquad$ $S = \{n - s_{k-1}, \ldots, n - s_0\}$

$t_j = \max\{ |X| : t(X) = j\}$ $\qquad$ $T = \{t_0 + 1, \ldots, t_{n-k-1} + 1\}$

$\sigma_i = \min\{ |X| : s(X) = i\}$ $\qquad$ $U = \{\sigma_1, \ldots, \sigma_k\}$

$\tau_j = \min\{ |X| : t(X) = j\}$ $\qquad$ $V = \{n + 1 - \tau_{n-k}, \ldots, n + 1 - \tau_1\}$

$E$ = a set on $n$ elements

$s, t$ = integer functions on $2^E$ with $k = s(E)$

$D = (E, s, t)$ is a demi-matroid if and only if, for all $X \subseteq Y \subseteq E$,

- $0 \leq s(X) \leq s(Y) \leq |Y|$
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- $|E - X| - s(E - X) = t(E) - t(X)$

$s_i = \max\{\,|X| : s(X) = i\}$

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$\sigma_i = \min\{\,|X| : s(X) = i\}$

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$S = \{n - s_{k-1}, \ldots, n - s_0\}$

$T = \{t_0 + 1, \ldots, t_{n-k-1} + 1\}$

$U = \{\sigma_1, \ldots, \sigma_k\}$

$V = \{n + 1 - \tau_{n-k}, \ldots, n + 1 - \tau_1\}$

BJMS 2012 : $S \cup T = \{1, \ldots, n\}$ and $S \cap T = \emptyset$

BJMS 2012 : $U \cup V = \{1, \ldots, n\}$ and $U \cap V = \emptyset$

# Wei-type theorems

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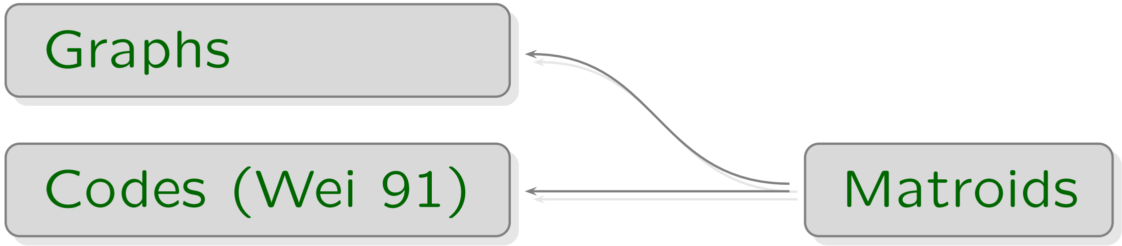
Codes (Wei 91)

# Wei-type theorems

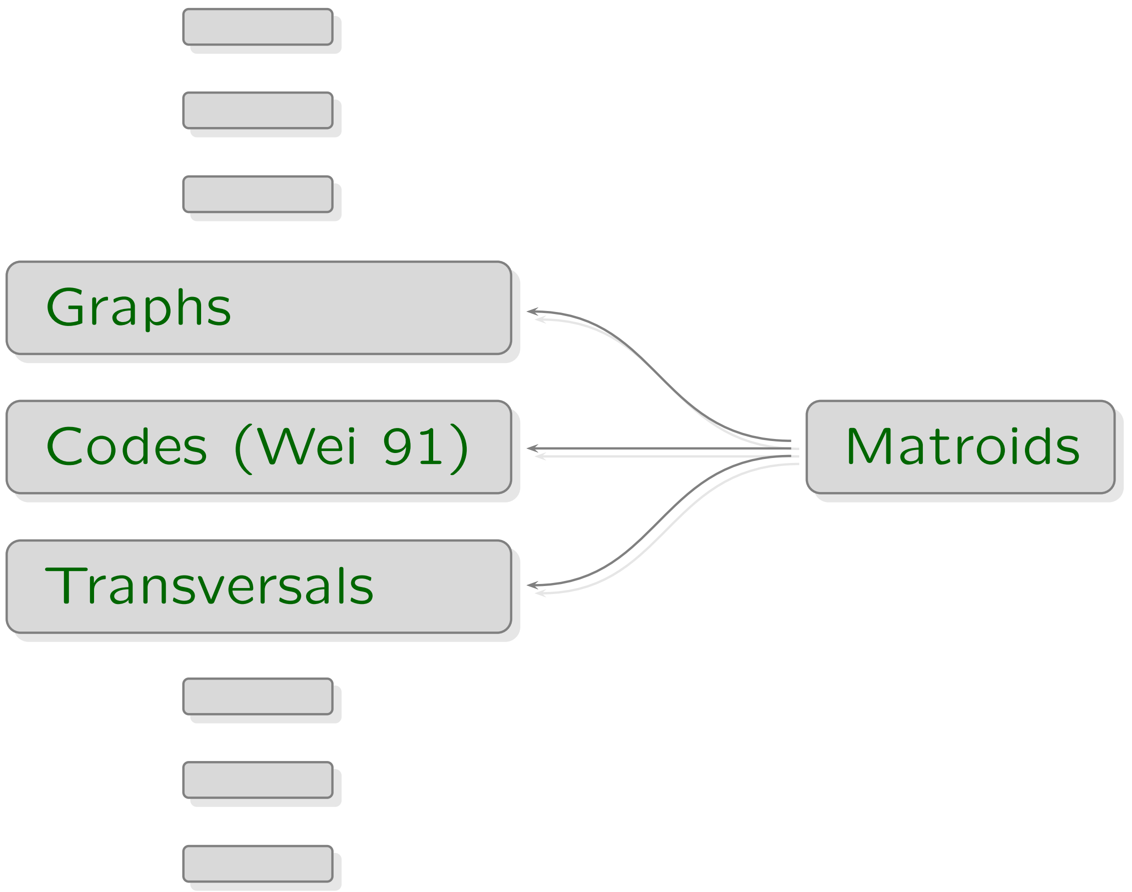
Graphs

Codes (Wei 91)

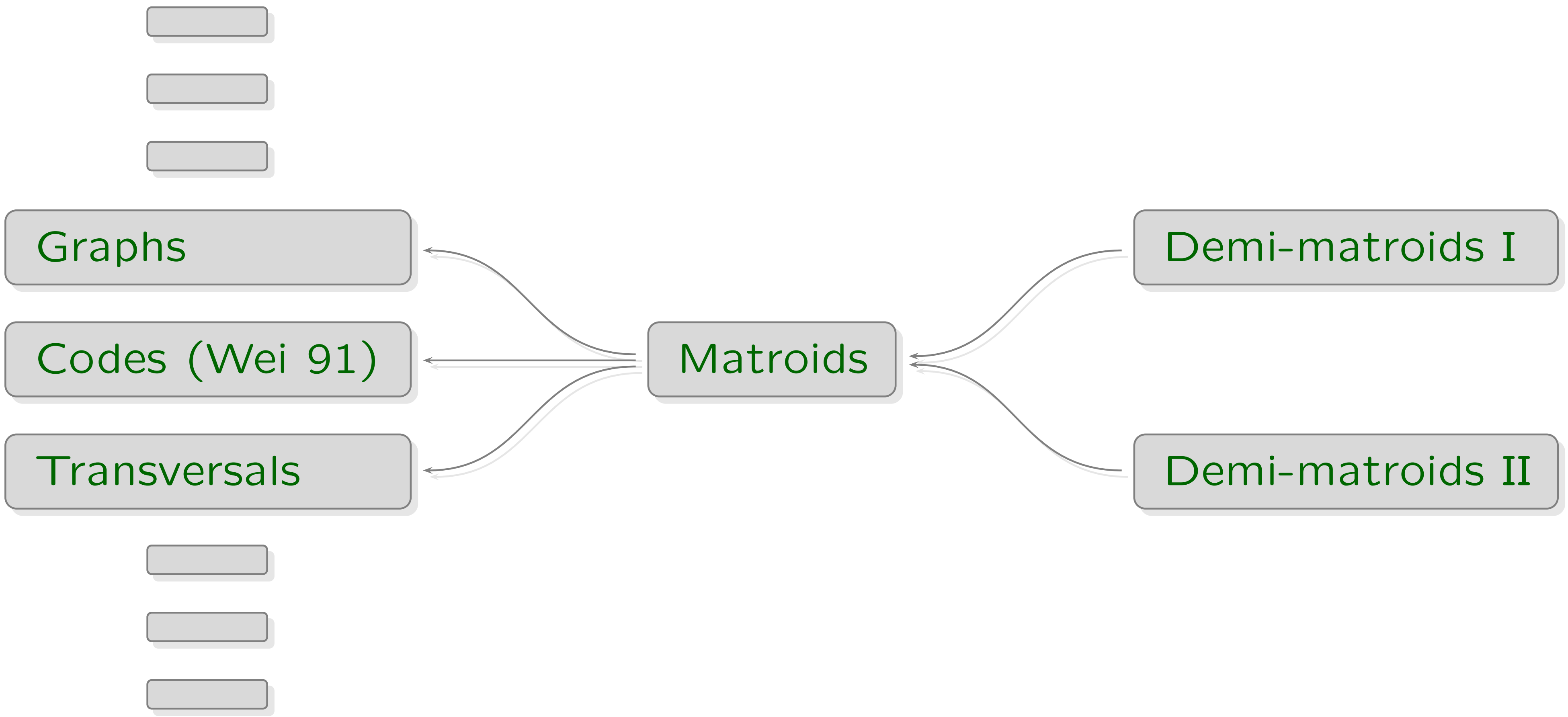
# Wei-type theorems



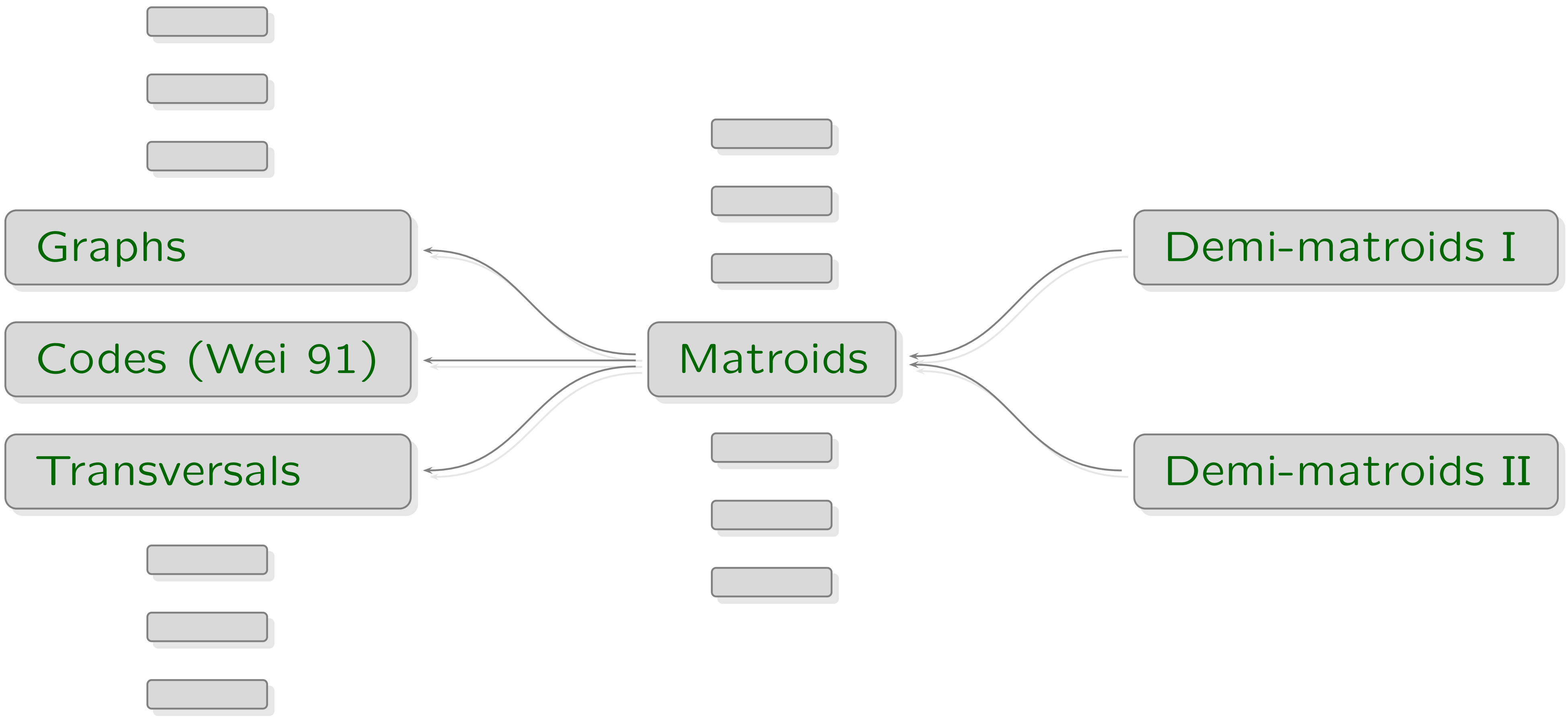
# Wei-type theorems

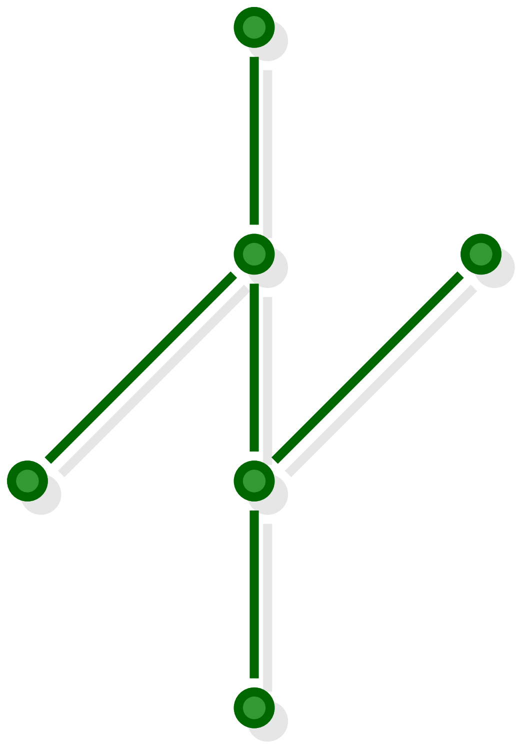


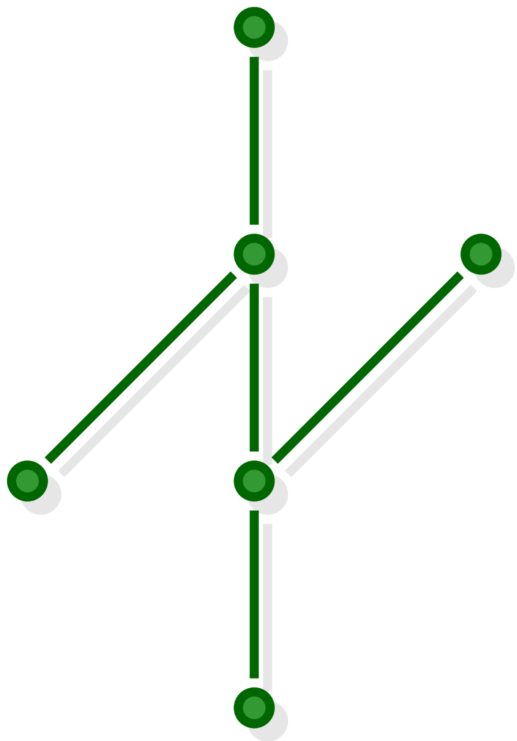
# Wei-type theorems



# Wei-type theorems

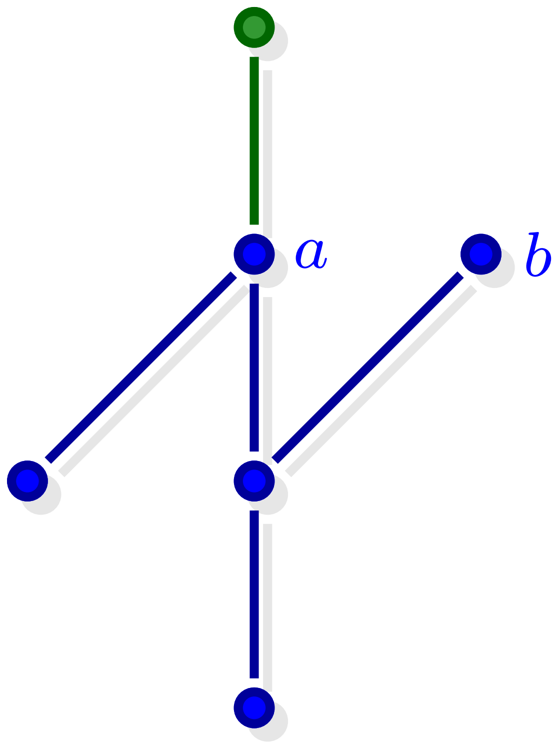






$\langle A \rangle$ = all elements beneath $A$

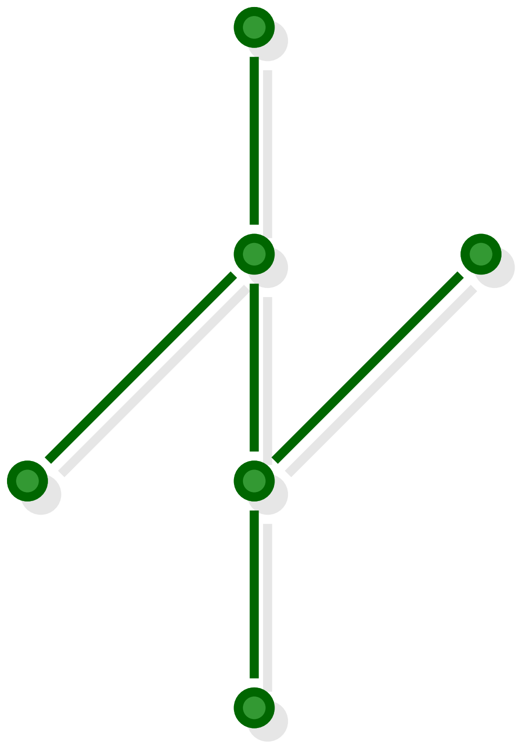
ideal

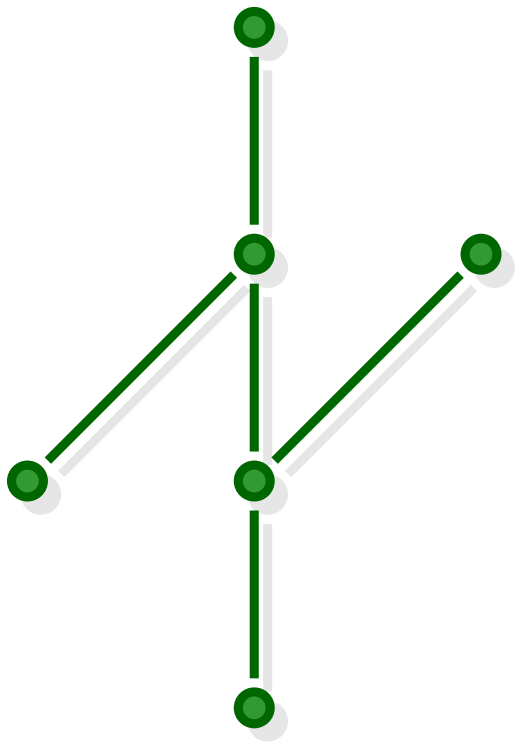


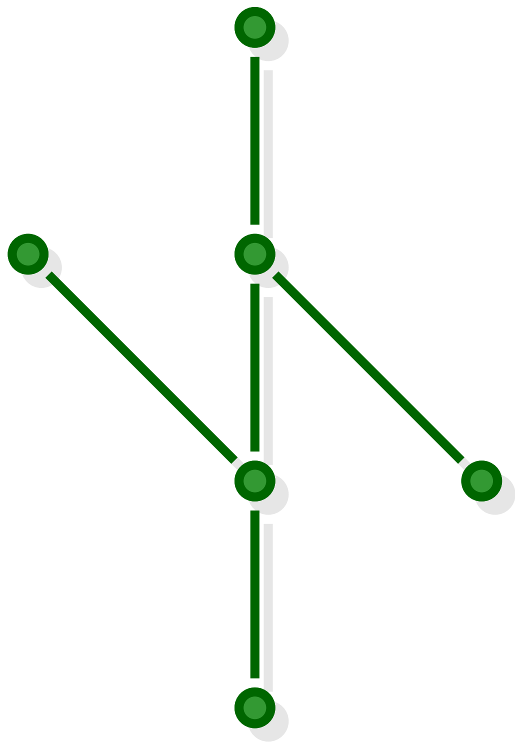

$\langle\{a, b\}\rangle$

$\langle A \rangle$ = all elements beneath $A$

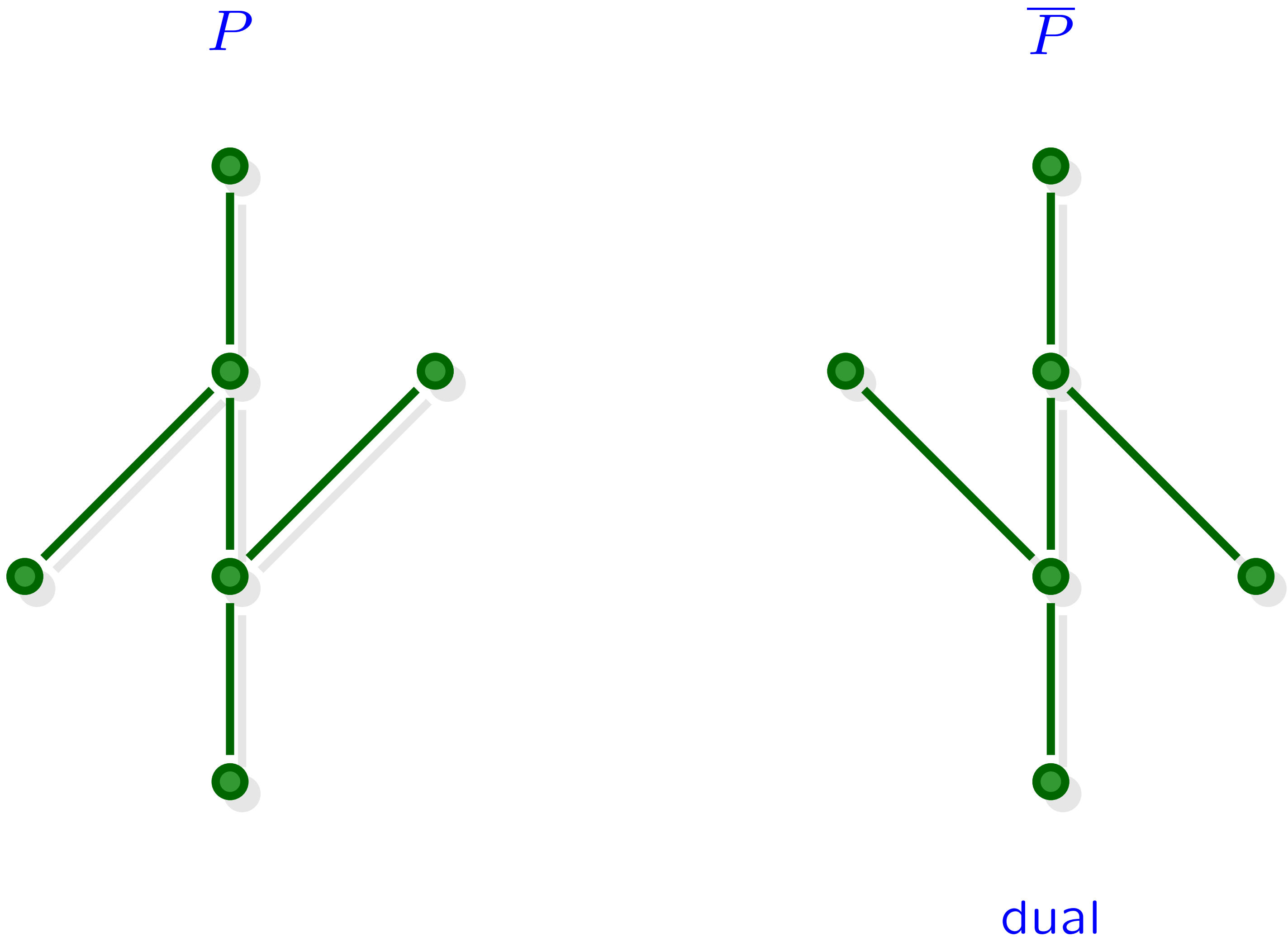
ideal







dual



$P$
$\overline{P}$
dual

$E$ = a set

$D$ = a demi-matroid $(E, s, t)$

$E$ = a set

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$P$ = a poset on $E$

$E$ = a set

$D$ = a demi-matroid $(E, s, t)$

$P$ = a poset on $E$

$$\sigma_i^P = \min\{\, |\langle X \rangle_P| \, : s(X) = i\}$$

$$\tau_j^{\overline{P}} = \min\{\, |\langle X \rangle_{\overline{P}}| \, : t(X) = j\}$$

$E$ = a set

$D$ = a demi-matroid $(E, s, t)$

$P$ = a poset on $E$

$$s_i^P = \max\{\, |E - \langle E - X \rangle_P| \ : s(X) = i\}$$

$$t_j^{\overline{P}} = \max\{\, |E - \langle E - X \rangle_{\overline{P}}| \ : t(X) = j\}$$

$$\sigma_i^P = \min\{\, |\langle X \rangle_P| \ : s(X) = i\}$$

$$\tau_j^{\overline{P}} = \min\{\, |\langle X \rangle_{\overline{P}}| \ : t(X) = j\}$$

$E$ = a set on $n$ elements

$D$ = a demi-matroid $(E, s, t)$ with $k = s(E)$

$P$ = a poset on $E$

$$s_i^P = \max\{\,|E - \langle E - X\rangle_P| \,:\, s(X) = i\} \qquad S^P = \{n - s_{k-1}^P, \ldots, n - s_0^P\}$$

$$t_j^{\overline{P}} = \max\{\,|E - \langle E - X\rangle_{\overline{P}}| \,:\, t(X) = j\} \qquad T^{\overline{P}} = \{t_0^{\overline{P}} + 1, \ldots, t_{n-k-1}^{\overline{P}} + 1\}$$

$$\sigma_i^P = \min\{\,|\langle X\rangle_P| \,:\, s(X) = i\} \qquad U^P = \{\sigma_1^P, \ldots, \sigma_k^P\}$$

$$\tau_j^{\overline{P}} = \min\{\,|\langle X\rangle_{\overline{P}}| \,:\, t(X) = j\} \qquad V^{\overline{P}} = \{n + 1 - \tau_{n-k}^{\overline{P}}, \ldots, n + 1 - \tau_1^{\overline{P}}\}$$

$E$ = a set on $n$ elements

$D$ = a demi-matroid $(E, s, t)$ with $k = s(E)$

$P$ = a poset on $E$

$s_i^P = \max\{\, |E - \langle E - X \rangle_P| \; : s(X) = i\}$ $\quad$ $S^P = \{n - s_{k-1}^P, \ldots, n - s_0^P\}$

$t_j^{\overline{P}} = \max\{\, |E - \langle E - X \rangle_{\overline{P}}| \; : t(X) = j\}$ $\quad$ $T^{\overline{P}} = \{t_0^{\overline{P}} + 1, \ldots, t_{n-k-1}^{\overline{P}} + 1\}$

$\sigma_i^P = \min\{\, |\langle X \rangle_P| \; : s(X) = i\}$ $\quad$ $U^P = \{\sigma_1^P, \ldots, \sigma_k^P\}$

$\tau_j^{\overline{P}} = \min\{\, |\langle X \rangle_{\overline{P}}| \; : t(X) = j\}$ $\quad$ $V^{\overline{P}} = \{n + 1 - \tau_{n-k}^{\overline{P}}, \ldots, n + 1 - \tau_1^{\overline{P}}\}$

BJMS 2012 : $S^P \cup T^{\overline{P}} = \{1, \ldots, n\}$ and $S^P \cap T^{\overline{P}} = \emptyset$

BJMS 2012 : $U^P \cup V^{\overline{P}} = \{1, \ldots, n\}$ and $U^P \cap V^{\overline{P}} = \emptyset$

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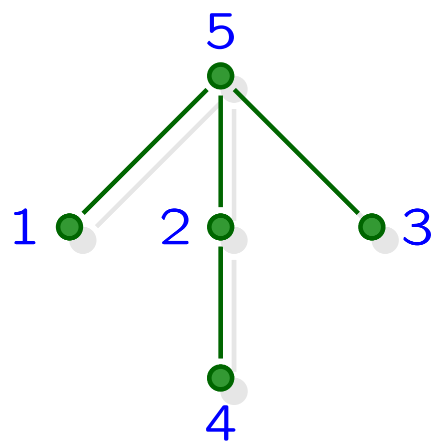
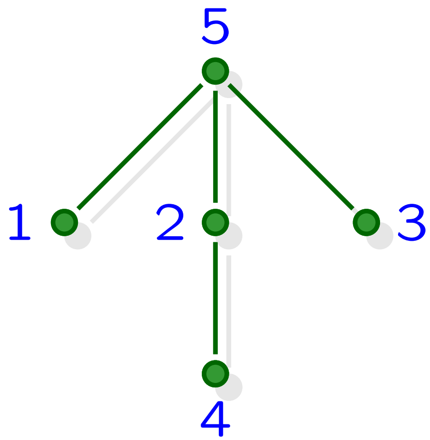
Poset generalisation of Wei's Duality Theorem

Moura & Firer 2010
Barg & Purkayastha
BJMS 2012




| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

$$d_2^P =$$




| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

$d_2^P =$

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |

4

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

5

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |

5

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

5

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |

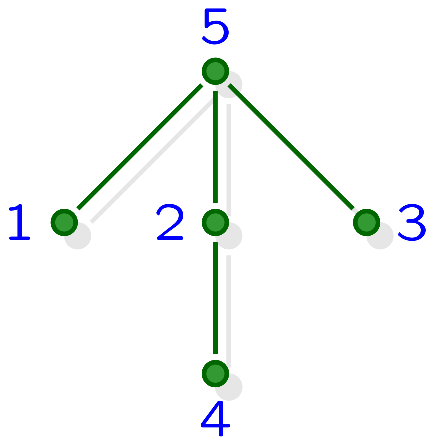
5

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |

5

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |

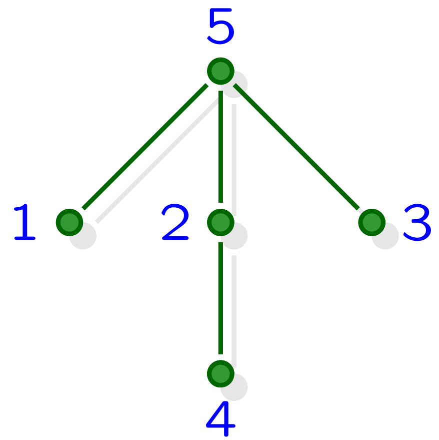
5




$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$d_2^P = 4$$

| | | | | | | |
|---|---|---|---|---|---|---|
| $\begin{matrix} 1&0&1&0&0 \\ 0&1&1&0&0 \end{matrix}$ | $\begin{matrix} 1&0&1&0&0 \\ 0&0&0&1&1 \end{matrix}$ | $\begin{matrix} 1&0&1&0&0 \\ 0&1&1&1&1 \end{matrix}$ | $\begin{matrix} 1&1&0&0&0 \\ 0&0&0&1&1 \end{matrix}$ | $\begin{matrix} 1&1&0&0&0 \\ 1&0&1&1&1 \end{matrix}$ | $\begin{matrix} 0&0&0&1&1 \\ 0&1&1&0&0 \end{matrix}$ | $\begin{matrix} 0&1&1&0&0 \\ 1&0&1&1&1 \end{matrix}$ |
| 4 | 5 | 5 | 5 | 5 | 5 | 5 |




| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

$d_1^P = 2$ $\quad d_1^{\overline{P},\perp} = 3$

$d_2^P = 4$ $\quad d_2^{\overline{P},\perp} = 5$

$d_3^P = 5$

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |

4

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

5

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |

5

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |

5

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |

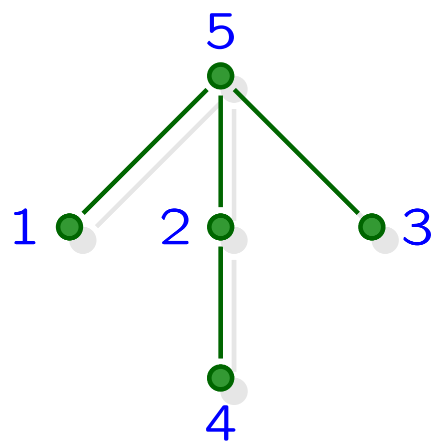
5

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |

5

| | | | | |
|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 |

5




$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$d_1^P = 2$ $\quad d_1^{\overline{P},\perp} = 3$

$d_2^P = 4$ $\quad d_2^{\overline{P},\perp} = 5$

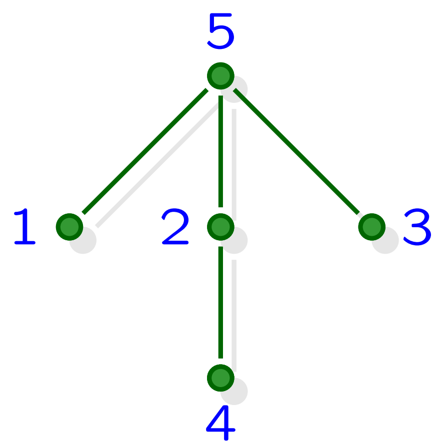
$d_3^P = 5$

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{pmatrix}$$

4 5 5 5 5 5 5

$$U = \{d_1^P, d_2^P, d_3^P\} = \{2, 4, 5\}$$

$$V = \{5 + 1 - d_2^{\overline{P},\perp}, \ldots, 5 + 1 - d_1^{\overline{P},\perp}\} = \{1, 3\}$$




$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$d_1^P = 2$ $\quad d_1^{\overline{P},\perp} = 3$

$d_2^P = 4$ $\quad d_2^{\overline{P},\perp} = 5$

$d_3^P = 5$

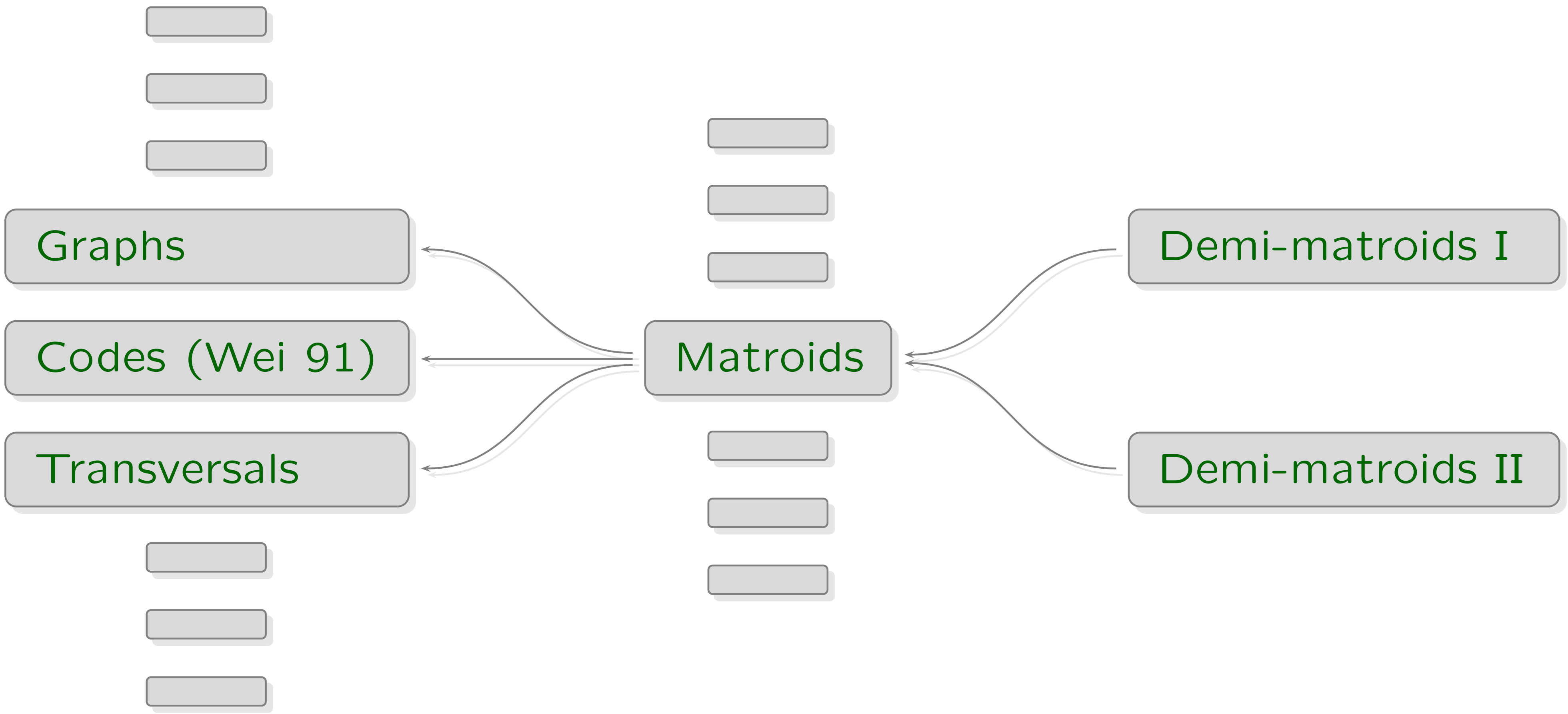
| | | | | | | |
|---|---|---|---|---|---|---|
| $\begin{matrix} 1&0&1&0&0 \\ 0&1&1&0&0 \end{matrix}$ | $\begin{matrix} 1&0&1&0&0 \\ 0&0&0&1&1 \end{matrix}$ | $\begin{matrix} 1&0&1&0&0 \\ 0&1&1&1&1 \end{matrix}$ | $\begin{matrix} 1&1&0&0&0 \\ 0&0&0&1&1 \end{matrix}$ | $\begin{matrix} 1&1&0&0&0 \\ 1&0&1&1&1 \end{matrix}$ | $\begin{matrix} 0&0&0&1&1 \\ 0&1&1&0&0 \end{matrix}$ | $\begin{matrix} 0&1&1&0&0 \\ 1&0&1&1&1 \end{matrix}$ |
| 4 | 5 | 5 | 5 | 5 | 5 | 5 |

$$U = \{d_1^P, d_2^P, d_3^P\} = \{2, 4, 5\}$$

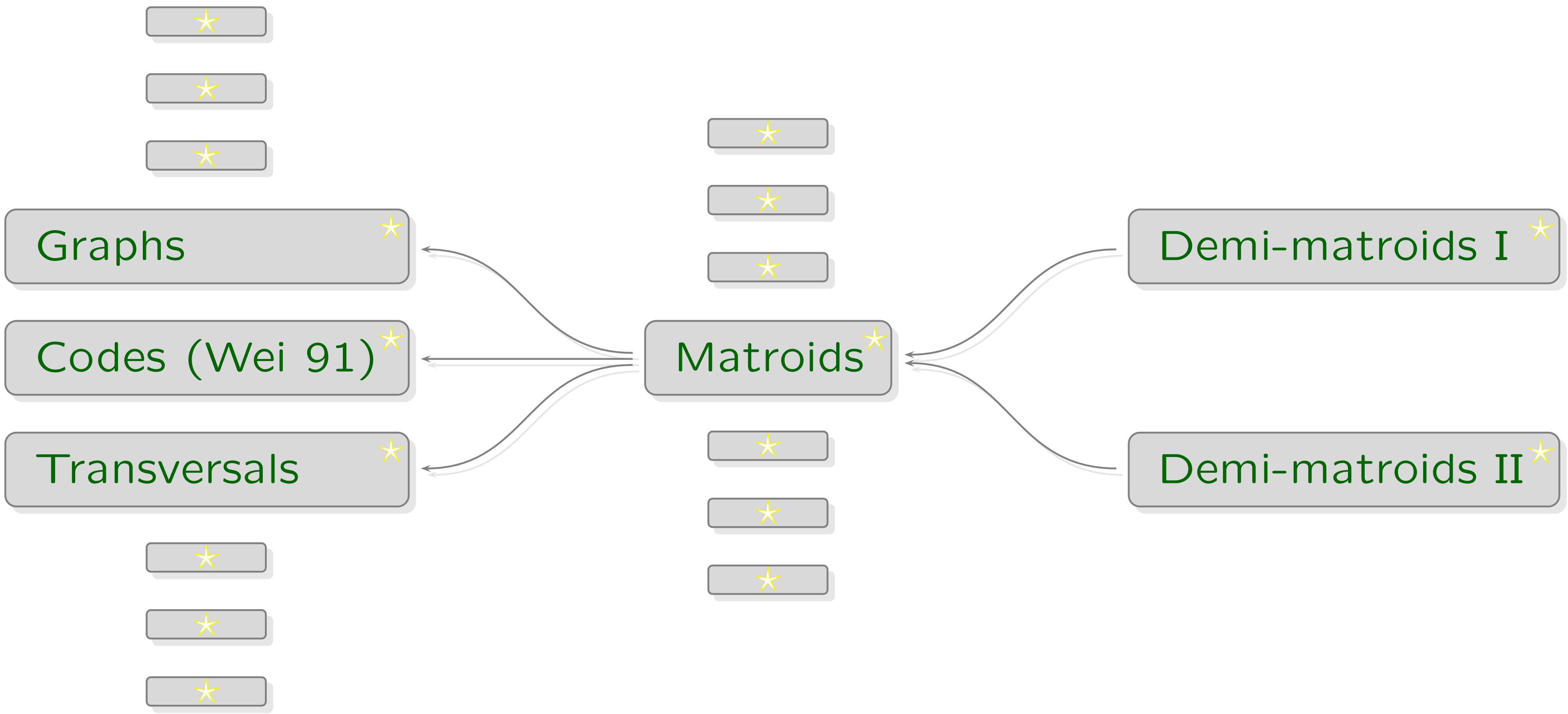
$$V = \{5 + 1 - d_2^{\overline{P},\perp}, \ldots, 5 + 1 - d_1^{\overline{P},\perp}\} = \{1, 3\}$$

$U \cup V = \{1, 2, 3, 4, 5\}$ and $U \cap V = \emptyset$

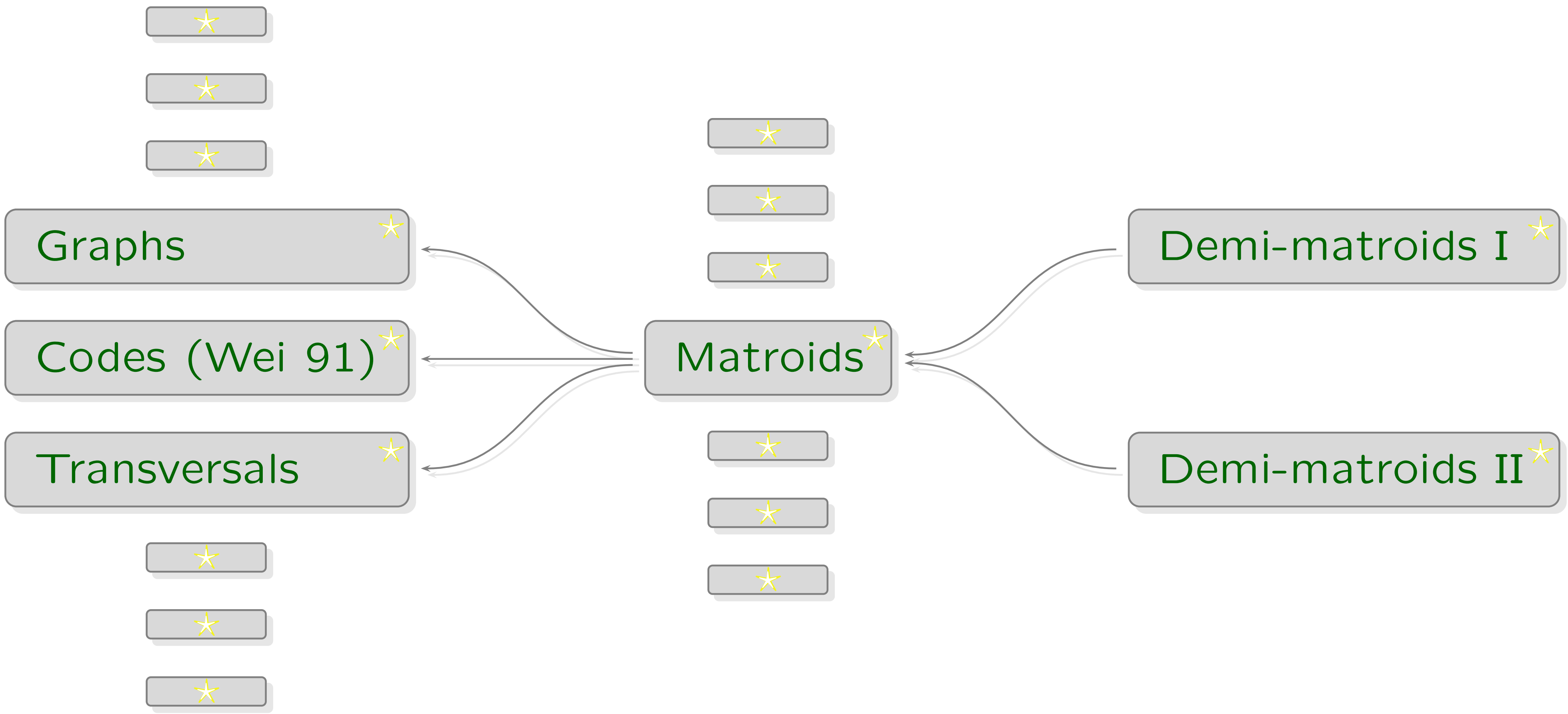
# Wei-type theorems



# Wei-type theorems



# Wei-type theorems




*Thank you!*