Discrete Maths Research Group
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☯ A nice proof of Wei's Duality Theorem ☯

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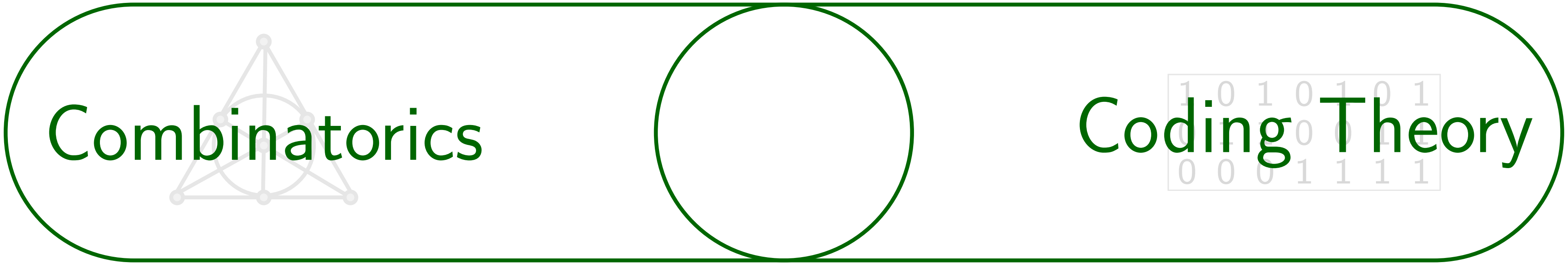
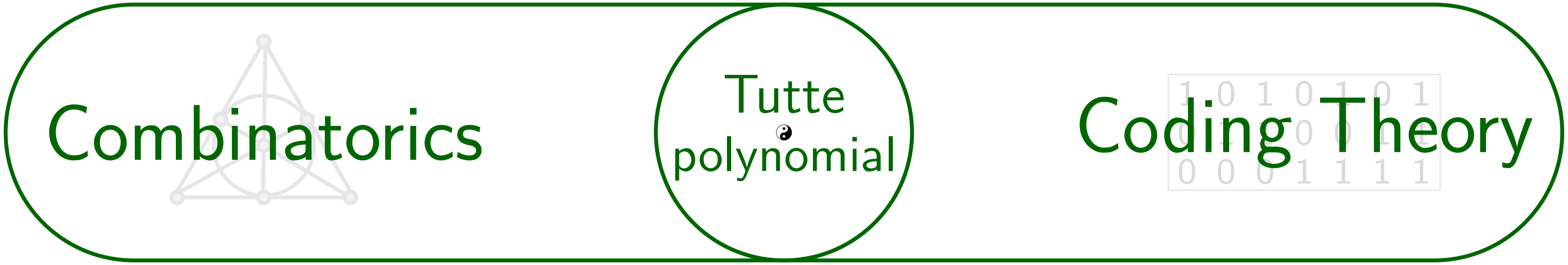
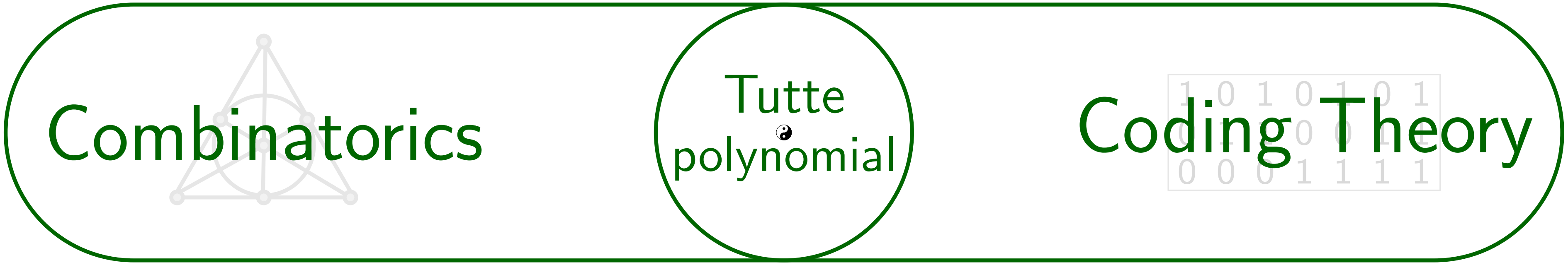
Combinatorics

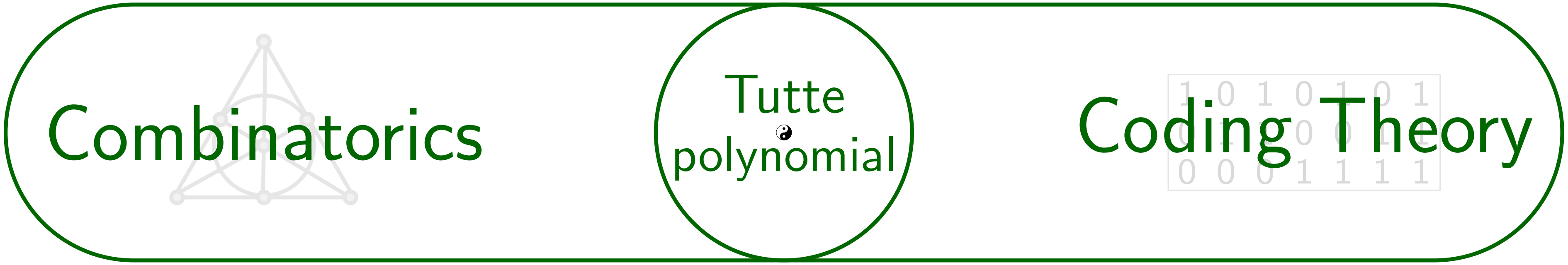
Coding Theory


Combinatorics
Coding Theory


Combinatorics
Tutte
polynomial
Coding Theory



Greene 1976 A nice proof of the MacWilliams Identity



Greene 1976 — A nice proof of the MacWilliams Identity

Duursma 2004 — A nice proof of Wei's Duality Theorem

linear code

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

0	0	0	0	0
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1
1	1	0	0	0
1	0	1	1	1
0	1	1	1	1
1	1	0	1	1

linear code

weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

0	0	0	0	0
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1
1	1	0	0	0
1	0	1	1	1
0	1	1	1	1
1	1	0	1	1

linear code

weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

Codewords	weights
0 0 0 0 0	0
1 0 1 0 0	
0 1 1 0 0	
0 0 0 1 1	
1 1 0 0 0	
1 0 1 1 1	
0 1 1 1 1	
1 1 0 1 1	

linear code

weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

					weights
0	0	0	0	0	0
1	0	1	0	0	2
0	1	1	0	0	
0	0	0	1	1	
1	1	0	0	0	
1	0	1	1	1	
0	1	1	1	1	
1	1	0	1	1	

linear code

weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

					weights
0	0	0	0	0	0
1	0	1	0	0	2
0	1	1	0	0	2
0	0	0	1	1	
1	1	0	0	0	
1	0	1	1	1	
0	1	1	1	1	
1	1	0	1	1	

linear code

weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

					weights
0	0	0	0	0	0
1	0	1	0	0	2
0	1	1	0	0	2
0	0	0	1	1	2
1	1	0	0	0	
1	0	1	1	1	
0	1	1	1	1	
1	1	0	1	1	

linear code | weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

Codeword	Weight
0 0 0 0 0	0
1 0 1 0 0	2
0 1 1 0 0	2
0 0 0 1 1	2
1 1 0 0 0	2
1 0 1 1 1	
0 1 1 1 1	
1 1 0 1 1	

linear code | weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

					weights
0	0	0	0	0	0
1	0	1	0	0	2
0	1	1	0	0	2
0	0	0	1	1	2
1	1	0	0	0	2
1	0	**1**	**1**	**1**	**4**
0	1	1	1	1	
1	1	0	1	1	

linear code

weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

Codeword					Weight
0	0	0	0	0	0
1	0	1	0	0	2
0	1	1	0	0	2
0	0	0	1	1	2
1	1	0	0	0	2
1	0	1	1	1	4
0	1	1	1	1	4
1	1	0	1	1	

linear code | **weights**

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

					weights
0	0	0	0	0	0
1	0	1	0	0	2
0	1	1	0	0	2
0	0	0	1	1	2
1	1	0	0	0	2
1	0	1	1	1	4
0	1	1	1	1	4
1	1	0	1	1	4

linear code | weights | supports

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

0 0 0 0 0	0	
1 0 1 0 0	2	
0 1 1 0 0	2	
0 0 0 1 1	2	
1 1 0 0 0	2	
1 0 1 1 1	4	
0 1 1 1 1	4	
1 1 0 1 1	4	

linear code | weights | supports

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

Codeword	weights	supports
0 0 0 0 0	0	$\varnothing$
1 0 1 0 0	2	
0 1 1 0 0	2	
0 0 0 1 1	2	
1 1 0 0 0	2	
1 0 1 1 1	4	
0 1 1 1 1	4	
1 1 0 1 1	4	

linear code				
1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

Codeword	weights	supports
0 0 0 0 0	0	$\varnothing$
1 0 1 0 0	2	13
0 1 1 0 0	2	
0 0 0 1 1	2	
1 1 0 0 0	2	
1 0 1 1 1	4	
0 1 1 1 1	4	
1 1 0 1 1	4	

linear code	weights	supports

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

Codewords	weights	supports
0 0 0 0 0	0	$\varnothing$
1 0 1 0 0	2	13
0 **1 1** 0 0	2	23
0 0 0 1 1	2	
1 1 0 0 0	2	
1 0 1 1 1	4	
0 1 1 1 1	4	
1 1 0 1 1	4	

linear code	weights	supports

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

Codeword	weights	supports
0 0 0 0 0	0	$\varnothing$
1 0 1 0 0	2	13
0 1 1 0 0	2	23
0 0 0 1 1	2	45
1 1 0 0 0	2	
1 0 1 1 1	4	
0 1 1 1 1	4	
1 1 0 1 1	4	

linear code					weights	supports

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

					weights	supports
0	0	0	0	0	0	$\varnothing$
1	0	1	0	0	2	13
0	1	1	0	0	2	23
0	0	0	1	1	2	45
1	1	0	0	0	2	12
1	0	1	1	1	4	
0	1	1	1	1	4	
1	1	0	1	1	4	

linear code — weights — supports

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

					weights	supports
0	0	0	0	0	0	$\varnothing$
1	0	1	0	0	2	13
0	1	1	0	0	2	23
0	0	0	1	1	2	45
1	1	0	0	0	2	12
1	0	1	1	1	4	1345
0	1	1	1	1	4	
1	1	0	1	1	4	

linear code | weights | supports

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

Codeword	weights	supports
0 0 0 0 0	0	$\varnothing$
1 0 1 0 0	2	13
0 1 1 0 0	2	23
0 0 0 1 1	2	45
1 1 0 0 0	2	12
1 0 1 1 1	4	1345
0 1 1 1 1	4	2345
1 1 0 1 1	4	

linear code					weights	supports
1	2	3	4	5		
1	0	1	0	0		
0	1	1	0	0		
0	0	0	1	1		

Codewords

0	0	0	0	0	0	$\varnothing$
1	0	1	0	0	2	13
0	1	1	0	0	2	23
0	0	0	1	1	2	45
1	1	0	0	0	2	12
1	0	1	1	1	4	1345
0	1	1	1	1	4	2345
1	1	0	1	1	4	1245

linear code	weights	dual code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

1	1	1	0	0
0	0	0	1	1

Codewords

Codewords					weights
0	0	0	0	0	0
1	0	1	0	0	2
0	1	1	0	0	2
0	0	0	1	1	2
1	1	0	0	0	2
1	0	1	1	1	4
0	1	1	1	1	4
1	1	0	1	1	4

linear code | weights | dual code

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

dual code

1	1	1	0	0
0	0	0	1	1

Codewords (linear code) and weights

					weights
0	0	0	0	0	0
1	0	1	0	0	2
0	1	1	0	0	2
0	0	0	1	1	2
1	1	0	0	0	2
1	0	1	1	1	4
0	1	1	1	1	4
1	1	0	1	1	4

Codewords (dual code)

0	0	0	0	0
1	1	1	0	0
0	0	0	1	1
1	1	1	1	1

linear code

weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

					weights
0	0	0	0	0	0
1	0	1	0	0	2
0	1	1	0	0	2
0	0	0	1	1	2
1	1	0	0	0	2
1	0	1	1	1	4
0	1	1	1	1	4
1	1	0	1	1	4

weight enumerator

$$A(z) = 1 + 4z^2 + 3z^4$$

linear code | weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

Codewords

Codeword					Weight
0	0	0	0	0	0
1	0	1	0	0	2
0	1	1	0	0	2
0	0	0	1	1	2
1	1	0	0	0	2
1	0	1	1	1	4
0	1	1	1	1	4
1	1	0	1	1	4

weight enumerator

$$A(z) = 1 + 4z^2 + 3z^4$$

$$W(x, y) = x^5 + 4x^3y^2 + 3xy^4$$

homogenous weight enumerator

MacWilliams Identity (1963)

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x + (q-1)y, x - y)$$

Codewords

0 0 0 0 0	0
1 0 1 0 0	2
0 1 1 0 0	2
0 0 0 1 1	2
1 1 0 0 0	2
1 0 1 1 1	4
0 1 1 1 1	4
1 1 0 1 1	4

weight enumerator

$$A(z) = 1 + 4z^2 + 3z^4$$

$$W(x,y) = x^5 + 4x^3y^2 + 3xy^4$$

homogenous weight enumerator

MacWilliams Identity (1963)

$$W_{C^\perp}(x, y) = \frac{1}{q^k} W_C(x + (q-1)y, x - y)$$

MacWilliams Identity (1963)

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x+(q-1)y, x-y)$$

C

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

MacWilliams Identity (1963)

$$W_{C^\perp}(x, y) = \frac{1}{q^k} W_C(x + (q-1)y, x - y)$$

C

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$C^\perp$

1	1	1	0	0
0	0	0	1	1

MacWilliams Identity (1963)

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x + (q-1)y, x - y)$$

C

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$W_C(x,y) = x^5 + 4x^3y^2 + 3xy^4$$

$C^\perp$

1	1	1	0	0
0	0	0	1	1

MacWilliams Identity (1963)

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x+(q-1)y, x-y)$$

C

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$W_C(x,y) = x^5 + 4x^3y^2 + 3xy^4$$

$C^\perp$

1	1	1	0	0
0	0	0	1	1

$$W_{C^\perp}(x,y) = x^5 + x^3y^2 + x^2y^3 + y^5$$

MacWilliams Identity (1963)

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x+(q-1)y, x-y)$$

C

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$W_C(x,y) = x^5 + 4x^3y^2 + 3xy^4$$

$C^\perp$

1	1	1	0	0
0	0	0	1	1

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x+(q-1)y, x-y)$$

MacWilliams Identity (1963)

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x + (q-1)y, x - y)$$

C

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$W_C(x,y) = x^5 + 4x^3y^2 + 3xy^4$$

$C^\perp$

1	1	1	0	0
0	0	0	1	1

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x + (q-1)y, x - y)$$

$$= \frac{1}{2^3}\big(1 + 4(x+y)^3(x-y)^2 + 4(x+y)(x-y)^4\big)$$

MacWilliams Identity (1963)

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x+(q-1)y, x-y)$$

C

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$W_C(x,y) = x^5 + 4x^3y^2 + 3xy^4$$

$C^\perp$

1	1	1	0	0
0	0	0	1	1

$$\begin{aligned} W_{C^\perp}(x,y) &= \frac{1}{q^k} W_C(x+(q-1)y, x-y) \\ &= \frac{1}{2^3}\big(1 + 4(x+y)^3(x-y)^2 + 4(x+y)(x-y)^4\big) \\ &= x^5 + x^3y^2 + x^2y^3 + y^5 \end{aligned}$$

linear code

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

linear code

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

linear code

rank function

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

linear code

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

rank function

$$\rho(45) = 1$$

linear code

rank function

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$\rho(12) = 2$$

linear code

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

rank function

$$\rho(123) = 2$$

linear code

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

rank function

$$\rho(124) = 3$$

linear code

rank function

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$\rho(124) = 3$$

Lemma

$$\rho_C(E) - \rho_C(A) = |E - A| - \rho_{C^\perp}(E - A)$$

linear code

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

dual code

1	2	3	4	5
1	1	1	0	0
0	0	0	1	1

Lemma

$$\rho_C(E) - \rho_C(A) = |E - A| - \rho_{C^\perp}(E - A)$$

linear code

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$\rho_C(123) = 2$

dual code

1	2	3	4	5
1	1	1	0	0
0	0	0	1	1

Lemma $\rho_C(E) - \rho_C(A) = |E - A| - \rho_{C^\perp}(E - A)$

linear code

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$\rho_C(123) = 2$

dual code

1	2	3	4	5
1	1	1	0	0
0	0	0	1	1

$\rho_{C^\perp}(45) = 1$

Lemma

$$\rho_C(E) - \rho_C(A) = |E - A| - \rho_{C^\perp}(E - A)$$

linear code

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$\rho_C(123) = 2$

A

dual code

1	2	3	4	5
1	1	1	0	0
0	0	0	1	1

$\rho_{C^\perp}(45) = 1$

$E - A$

Lemma

$$\rho_C(E) - \rho_C(A) = |E - A| - \rho_{C^\perp}(E - A)$$

$$3 - 2 = 2 - 1$$

linear code

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

rank function

$$\rho(124) = 3$$

Tutte polynomial

$$T(x+1, y+1) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)}$$

linear code

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

rank function

$$\rho(124) = 3$$

Tutte polynomial

$$T(x+1, y+1) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)}$$

$$= 6 + 9x + 5x^2 + x^3 + 5y + y^2 + 4xy + x^2y$$

linear code

rank function

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$\rho(124) = 3$

Tutte polynomial and rank generator function

$$T(x+1, y+1) = R(x, y) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)}$$

$$= 6 + 9x + 5x^2 + x^3 + 5y + y^2 + 4xy + x^2y$$

linear code	rank function

1	2	3	4	5
1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$\rho(124) = 3$$

Tutte polynomial and rank generator function

$$T(x+1, y+1) = R(x,y) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)}$$

$$= 6 + 9x + 5x^2 + x^3 + 5y + y^2 + 4xy + x^2y$$

... and coboundary polynomial

$$\overline{\chi}(xy, y+1, 1) = y^{\rho(E)} R(x,y)$$

Tutte polynomial and rank generator function

$$T(x+1, y+1) = R(x, y) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)}$$

... and coboundary polynomial

$$\overline{\chi}(xy, y+1, 1) = y^{\rho(E)} R(x, y)$$

Tutte polynomial and rank generator function

$$T(x+1, y+1) = R(x, y) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)}$$

... and **coboundary polynomial**

$$\overline{\chi}(xy, y+1, 1) = y^{\rho(E)} R(x, y)$$

Lemma

- $R_{C^\perp}(x, y) = R_C(y, x)$
- $T_{C^\perp}(x, y) = T_C(y, x)$
- $\overline{\chi}_{C^\perp}(\lambda, x, y) = \lambda^{-\rho_C(E)} \overline{\chi}_C(\lambda, x + (\lambda - 1)y, x - y)$

Tutte polynomial and rank generator function

$$T(x+1, y+1) = R(x, y) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)}$$

… and coboundary polynomial

$$\overline{\chi}(xy, y+1, 1) = y^{\rho(E)} R(x, y)$$

Lemma

- $R_{C^\perp}(x, y) = R_C(y, x)$
- $T_{C^\perp}(x, y) = T_C(y, x)$
- $\overline{\chi}_{C^\perp}(\lambda, x, y) = \lambda^{-\rho_C(E)} \overline{\chi}_C(\lambda, x + (\lambda - 1)y, x - y)$

Proof $\rho_C(E) - \rho_C(A) = |E - A| - \rho_{C^\perp}(E - A)$ $\square$

MacWilliams Identity (1963)

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x + (q-1)y, x - y)$$

C

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$W_C(x,y) = x^5 + 4x^3y^2 + 3xy^4$$

$C^\perp$

1	1	1	0	0
0	0	0	1	1

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x + (q-1)y, x - y)$$

$$= \frac{1}{2^3}\left(1 + 4(x+y)^3(x-y)^2 + 4(x+y)(x-y)^4\right)$$

$$= x^5 + x^3y^2 + x^2y^3 + y^5$$

MacWilliams Identity (1963)

$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x+(q-1)y, x-y)$$

C

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$W_C(x,y) = x^5 + 4x^3y^2 + 3xy^4$$

Greene's Theorem (1976)

$$W_C(x,y) = \overline{\chi}_C(q,x,y)$$

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$$W_{C^\perp}(x,y) = \frac{1}{q^k} W_C(x+(q-1)y, x-y)$$

C

1	0	1	0	0
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$$W_C(x,y) = \overline{\chi}(2,x,y) = x^5 + 4x^3y^2 + 3xy^4$$

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$$W_{C^\perp}(x,y) = \overline{\chi}_{C^\perp}(q,x,y)$$

$$= q^{-\rho(E)} \overline{\chi}_C(q, x+(q-1)y, x-y)$$

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Proof

$$\begin{aligned} W_{C^\perp}(x, y) &= \overline{\chi}_{C^\perp}(q, x, y) \\ &= q^{-\rho(E)} \overline{\chi}_C(q, x + (q-1)y, x - y) \\ &= \frac{1}{q^k} W_C(x + (q-1)y, x - y) \qquad \square \end{aligned}$$

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$C \quad = $ a linear code over a finite field $\mathbb{F}_q$

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CGB 2010, CGB 2013, BJM 2014, BSW 2015, BC	Related results

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

1 0 1 0 0   0 1 1 0 0   0 0 0 1 1   1 1 0 0 0   1 0 1 1 1   0 1 1 1 1   1 1 0 1 1

subcodes

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

1 0 1 0 0	0 1 1 0 0	0 0 0 1 1	1 1 0 0 0	1 0 1 1 1	0 1 1 1 1	1 1 0 1 1
2	2	2	2	4	4	4

subcodes

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

1 0 1 0 0	0 1 1 0 0	0 0 0 1 1	1 1 0 0 0	1 0 1 1 1	0 1 1 1 1	1 1 0 1 1
2	2	2	2	4	4	4
1 0 1 0 0 0 1 1 0 0	1 0 1 0 0 0 0 0 1 1	1 0 1 0 0 0 1 1 1 1	1 1 0 0 0 0 0 0 1 1	1 1 0 0 0 1 0 1 1 1	0 0 0 1 1 0 1 1 0 0	0 1 1 0 0 1 0 1 1 1

subcodes

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

1 0 1 0 0	0 1 1 0 0	0 0 0 1 1	1 1 0 0 0	1 0 1 1 1	0 1 1 1 1	1 1 0 1 1
2	2	2	2	4	4	4

1 0 1 0 0 0 1 1 0 0	1 0 1 0 0 0 0 0 1 1	1 0 1 0 0 0 1 1 1 1	1 1 0 0 0 0 0 0 1 1	1 1 0 0 0 1 0 1 1 1	0 0 0 1 1 0 1 1 0 0	0 1 1 0 0 1 0 1 1 1
3	4	5	4	5	4	5

subcodes

linear code

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix}$ 2 $\quad \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 2 $\quad \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 2 $\quad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}$ 2 $\quad \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 4 $\quad \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ 4 $\quad \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \end{bmatrix}$ 4

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 3 $\quad \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 4 $\quad \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ 5 $\quad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 4 $\quad \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 5 $\quad \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 4 $\quad \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 5

subcodes

linear code

higher weights

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

$$d_1 =$$
$$d_2 =$$
$$d_3 =$$

subcodes

1 0 1 0 0	0 1 1 0 0	0 0 0 1 1	1 1 0 0 0	1 0 1 1 1	0 1 1 1 1	1 1 0 1 1
2	2	2	2	4	4	4

1 0 1 0 0 0 1 1 0 0	1 0 1 0 0 0 0 0 1 1	1 0 1 0 0 0 1 1 1 1	1 1 0 0 0 0 0 0 1 1	1 1 0 0 0 1 0 1 1 1	0 0 0 1 1 0 1 1 0 0	0 1 1 0 0 1 0 1 1 1
3	4	5	4	5	4	5

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

5

linear code | **higher weights**

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

$d_1 = 2$
$d_2 =$
$d_3 =$

subcodes

1 0 1 0 0	0 1 1 0 0	0 0 0 1 1	1 1 0 0 0	1 0 1 1 1	0 1 1 1 1	1 1 0 1 1
2	2	2	2	4	4	4
1 0 1 0 0 0 1 1 0 0	1 0 1 0 0 0 0 0 1 1	1 0 1 0 0 0 1 1 1 1	1 1 0 0 0 0 0 0 1 1	1 1 0 0 0 1 0 1 1 1	0 0 0 1 1 0 1 1 0 0	0 1 1 0 0 1 0 1 1 1
3	4	5	4	5	4	5

1 0 1 0 0
0 1 1 0 0
0 0 0 1 1

5

linear code

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

higher weights

$d_1 = 2$

$d_2 = 3$

$d_3 =$

subcodes

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 1 & 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & 1 & 0 & 0 & 0 \end{matrix}$	$\begin{matrix} 1 & 0 & 1 & 1 & 1 \end{matrix}$	$\begin{matrix} 0 & 1 & 1 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & 1 & 0 & 1 & 1 \end{matrix}$
2	2	2	2	4	4	4

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{matrix}$
3	4	5	4	5	4	5

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

5

linear code

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

higher weights

$$d_1 = 2$$
$$d_2 = 3$$
$$d_3 = 5$$

subcodes

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 1 & 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & 1 & 0 & 0 & 0 \end{matrix}$	$\begin{matrix} 1 & 0 & 1 & 1 & 1 \end{matrix}$	$\begin{matrix} 0 & 1 & 1 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & 1 & 0 & 1 & 1 \end{matrix}$
2	2	2	2	4	4	4

$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$	$\begin{matrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{matrix}$	$\begin{matrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{matrix}$	$\begin{matrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{matrix}$
3	4	5	4	5	4	5

$$\begin{matrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{matrix}$$

5

linear code

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

higher weights

$d_1 = 2$ $\quad d_1^{\perp} = 2$

$d_2 = 3$ $\quad d_2^{\perp} = 5$

$d_3 = 5$

subcodes

$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \end{pmatrix}$ 2

$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \end{pmatrix}$ 2

$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 \end{pmatrix}$ 2

$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \end{pmatrix}$ 2

$\begin{pmatrix} 1 & 0 & 1 & 1 & 1 \end{pmatrix}$ 4

$\begin{pmatrix} 0 & 1 & 1 & 1 & 1 \end{pmatrix}$ 4

$\begin{pmatrix} 1 & 1 & 0 & 1 & 1 \end{pmatrix}$ 4

$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$ 3

$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$ 4

$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{pmatrix}$ 5

$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$ 4

$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{pmatrix}$ 5

$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$ 4

$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{pmatrix}$ 5

$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$ 5

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

higher weights

$d_1 = 2$ $\quad d_1^{\perp} = 2$

$d_2 = 3$ $\quad d_2^{\perp} = 5$

$d_3 = 5$

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

higher weights

$d_1 = 2 \quad d_1^{\perp} = 2$

$d_2 = 3 \quad d_2^{\perp} = 5$

$d_3 = 5$

$$U = \{d_1, \ldots, d_k\}$$

$$V = \{n + 1 - d_{n-k-1}^{\perp}, \ldots, n + 1 - d_1^{\perp}\}$$

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

higher weights

$d_1 = 2 \quad d_1^\perp = 2$

$d_2 = 3 \quad d_2^\perp = 5$

$d_3 = 5$

$$U = \{d_1, \ldots, d_k\} = \{2, 3, 5\}$$

$$V = \{n + 1 - d_{n-k-1}^\perp, \ldots, n + 1 - d_1^\perp\} = \{1, 4\}$$

linear code | higher weights

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$d_1 = 2 \quad d_1^\perp = 2$

$d_2 = 3 \quad d_2^\perp = 5$

$d_3 = 5$

$$U = \{d_1, \ldots, d_k\} = \{2, 3, 5\}$$

$$V = \{n + 1 - d^\perp_{n-k-1}, \ldots, n + 1 - d_1^\perp\} = \{1, 4\}$$

Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

Wei-type theorems

Wei-type theorems

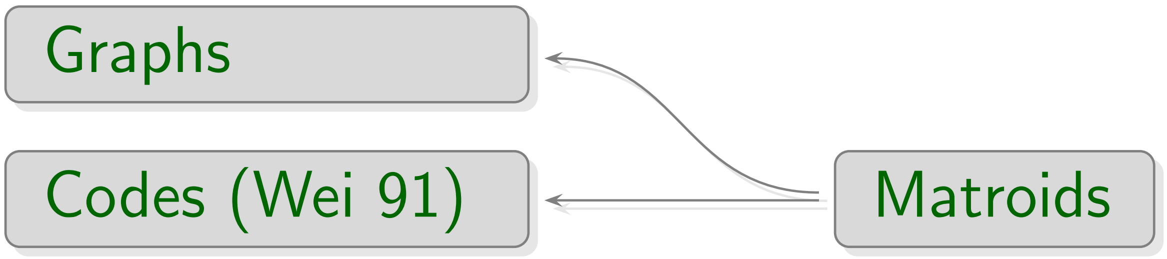
Codes (Wei 91)

Wei-type theorems

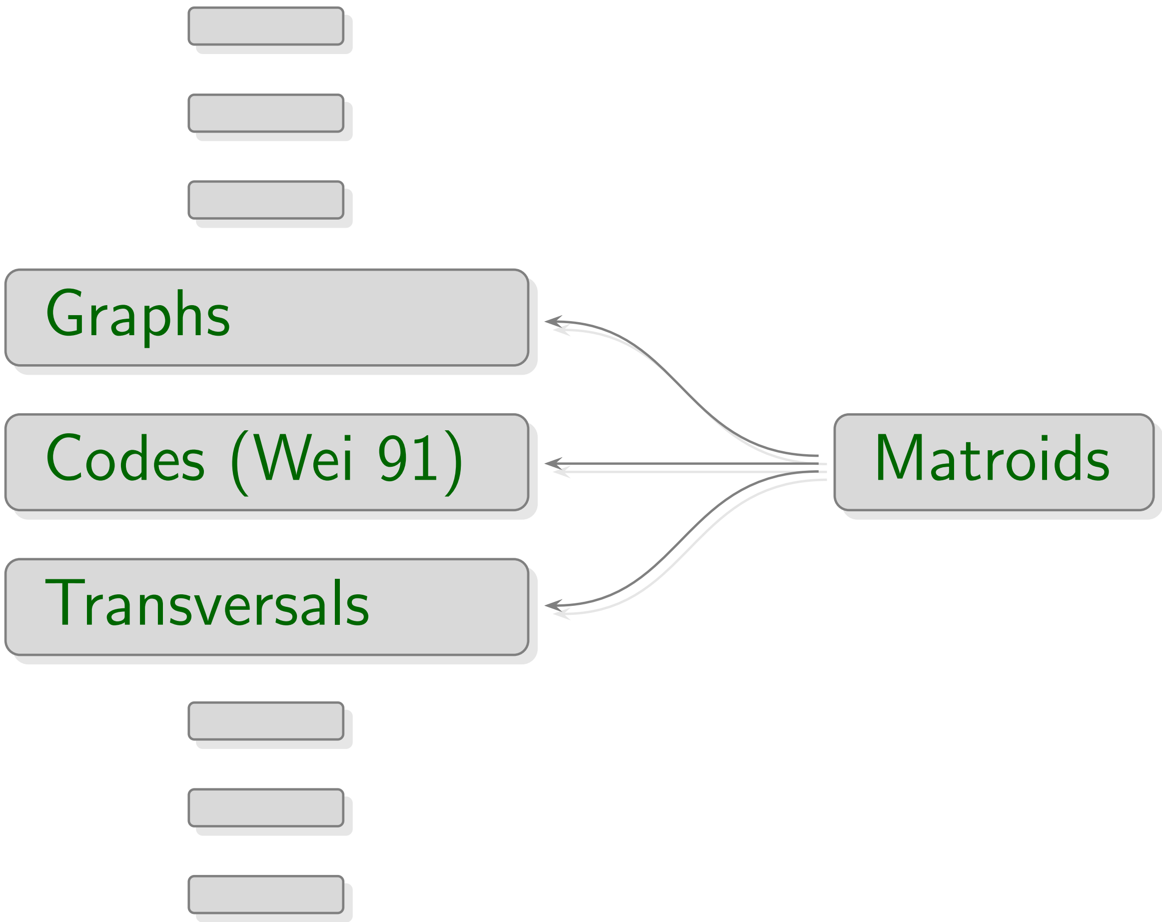
Graphs

Codes (Wei 91)

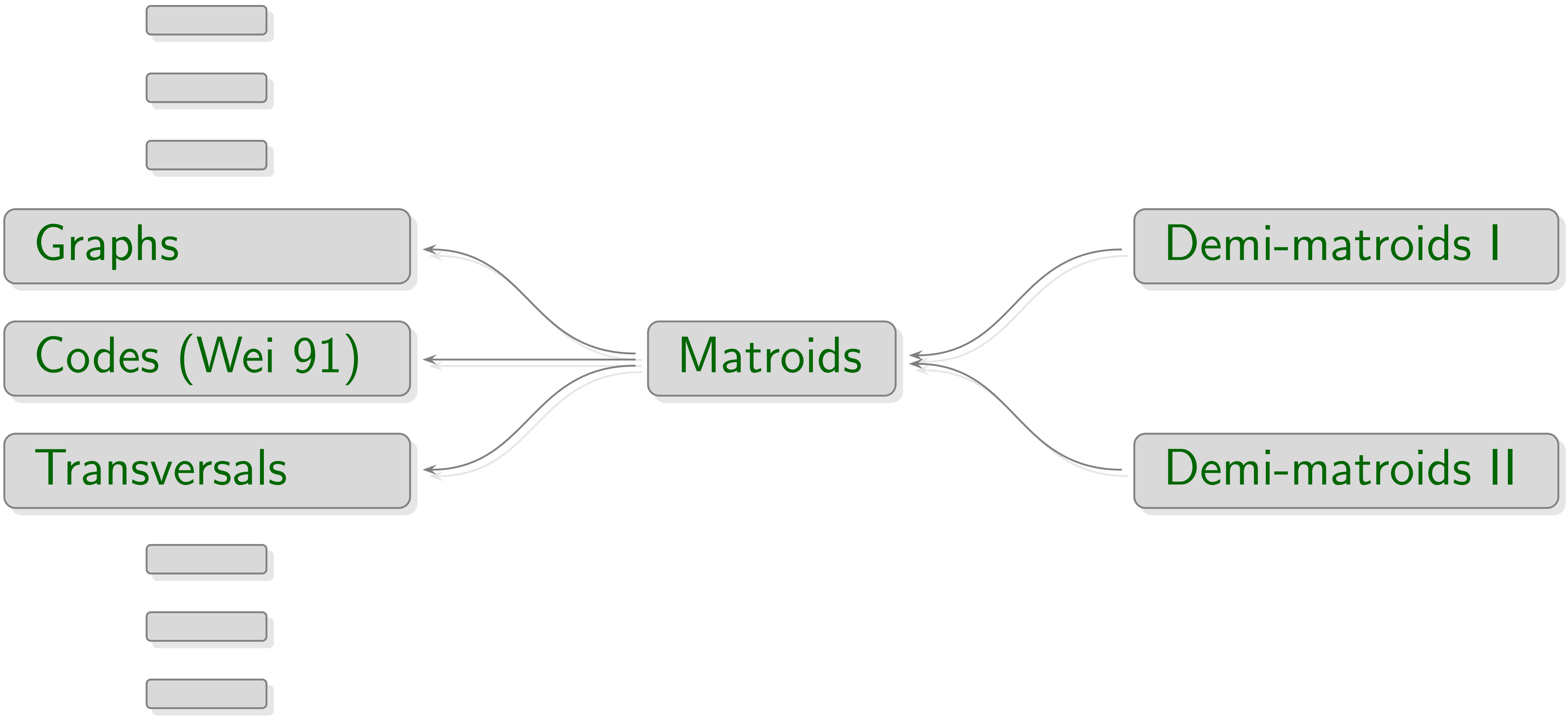
Wei-type theorems



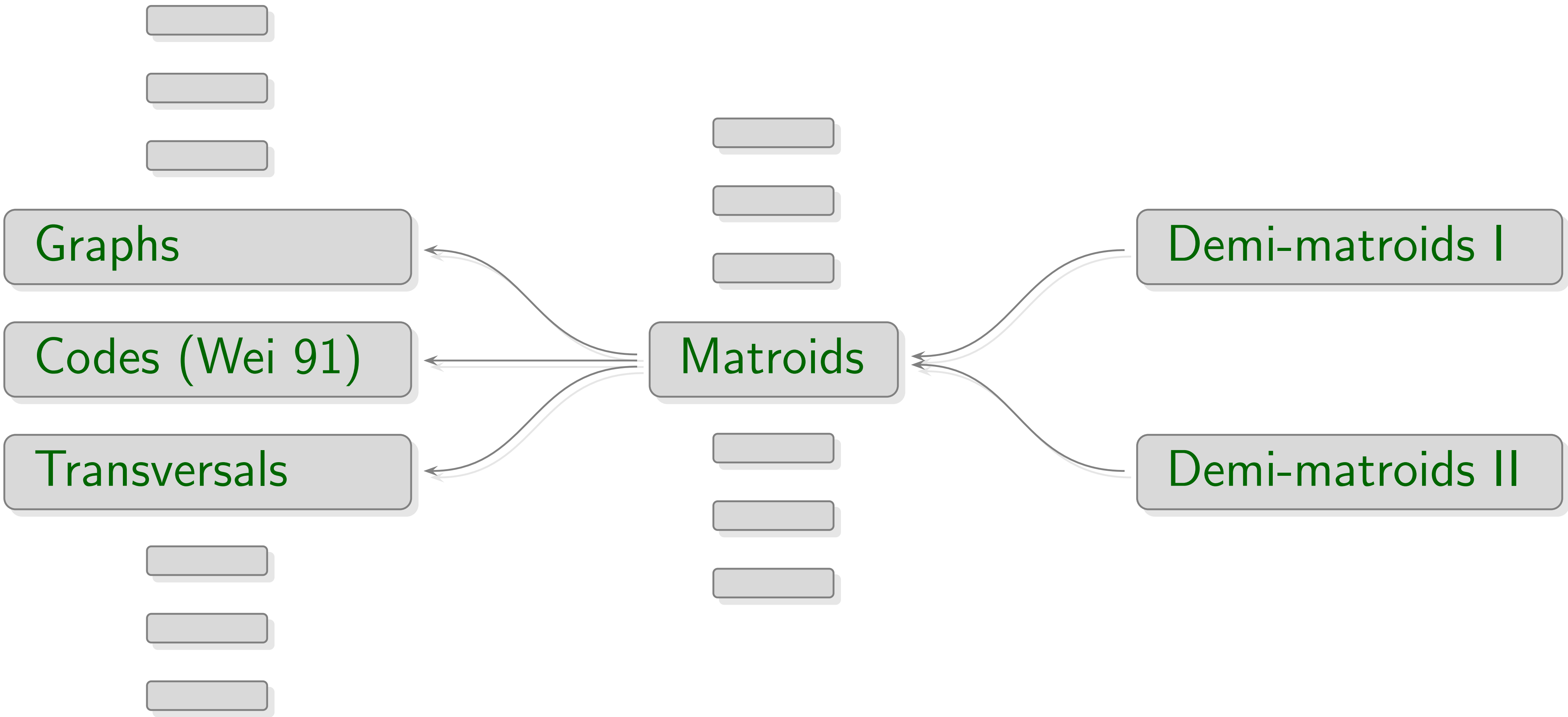
Wei-type theorems

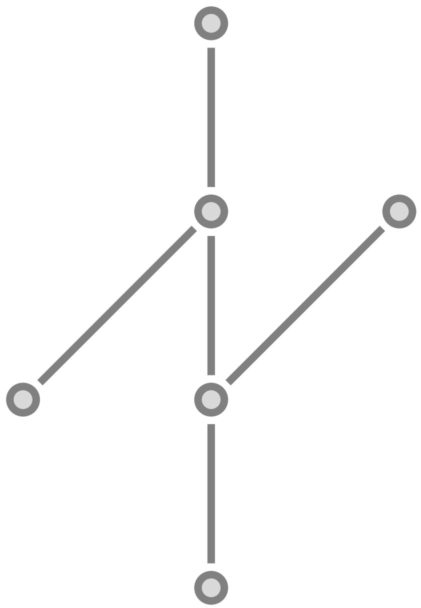


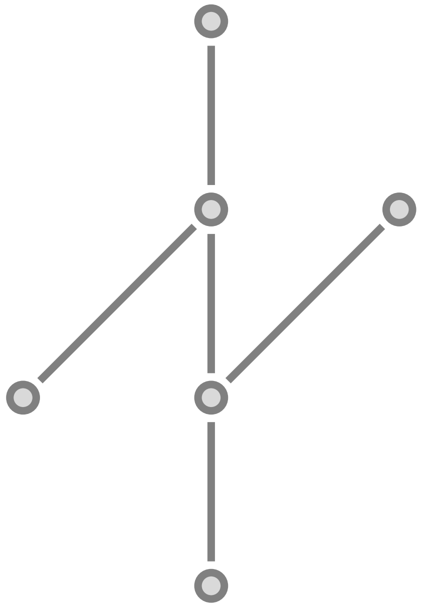
Wei-type theorems



Wei-type theorems

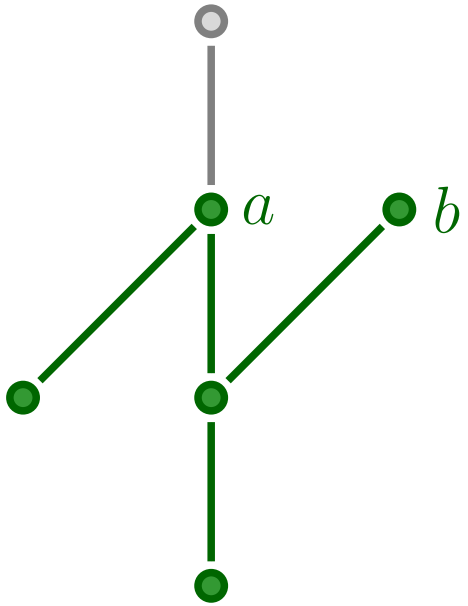






$\langle A \rangle$ = all elements beneath A

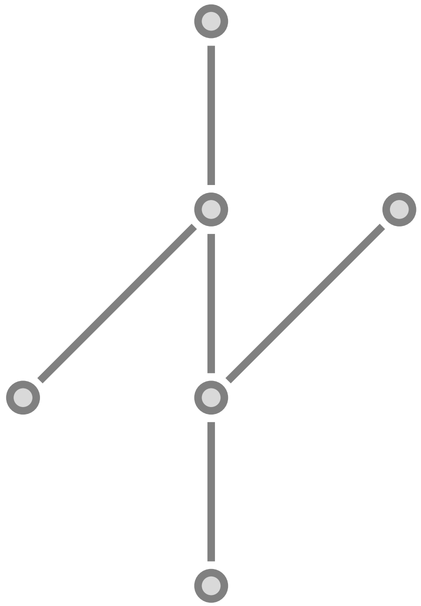
ideal

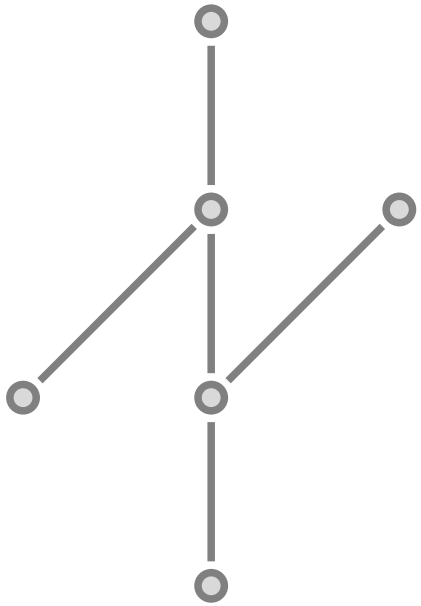


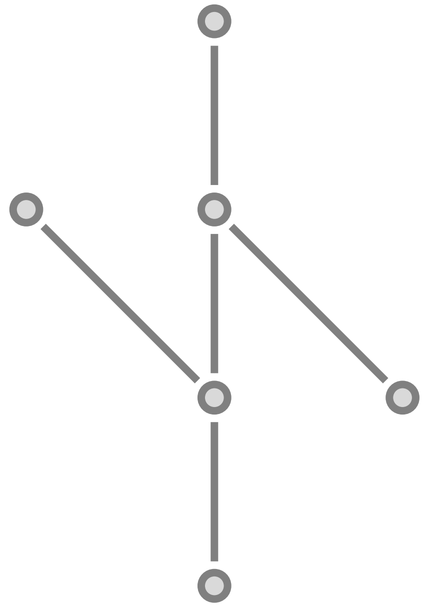
$\langle \{a, b\} \rangle$

$\langle A \rangle$ = all elements beneath A

ideal

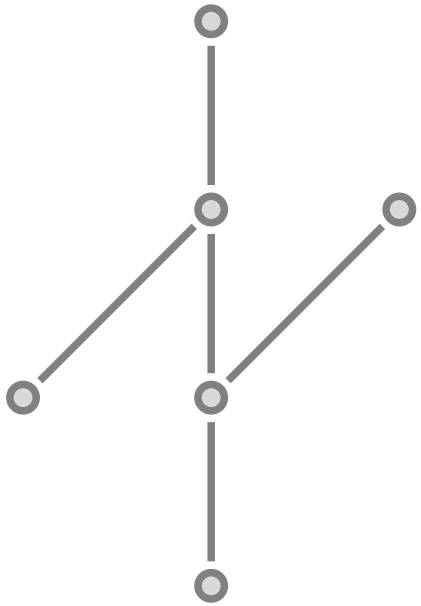




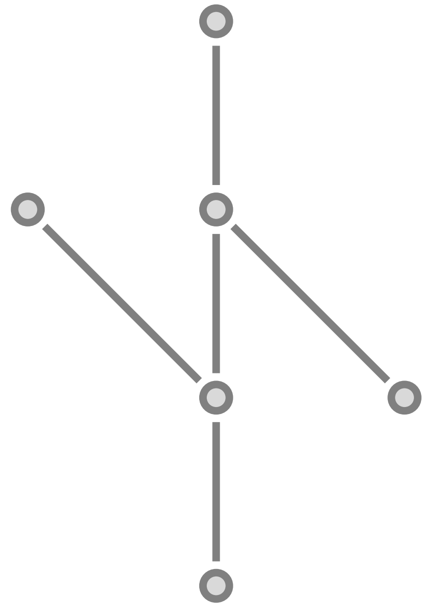


dual

P

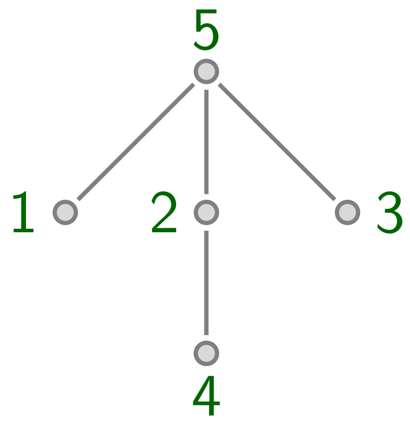


$\overline{P}$

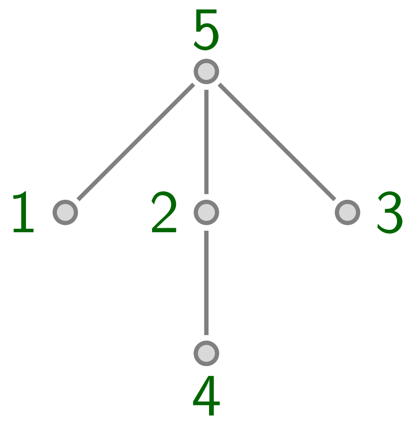


dual

linear code



1	0	1	0	0
0	1	1	0	0
0	0	0	1	1



linear code

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

subcodes

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix}$ 2

$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 3

$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}$ 3

$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 4

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ 5

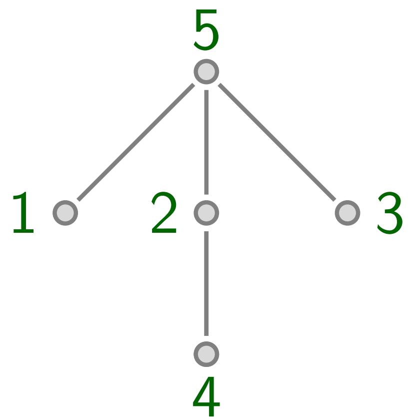
$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 5

$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 5



linear code

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

higher weights

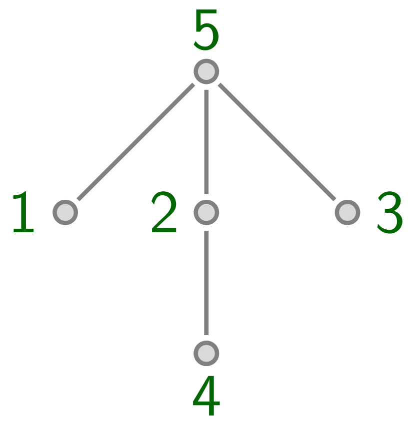
$d_1^P =$

$d_2^P =$

$d_3^P =$

subcodes

Subcode	Weight
1 0 1 0 0	2
0 1 1 0 0	3
0 0 0 1 1	5
1 1 0 0 0	3
1 0 1 1 1	5
0 1 1 1 1	5
1 1 0 1 1	5
1 0 1 0 0 / 0 1 1 0 0	4
1 0 1 0 0 / 0 0 0 1 1	5
1 0 1 0 0 / 0 1 1 1 1	5
1 1 0 0 0 / 0 0 0 1 1	5
1 1 0 0 0 / 1 0 1 1 1	5
0 0 0 1 1 / 0 1 1 0 0	5
0 1 1 0 0 / 1 0 1 1 1	5
1 0 1 0 0 / 0 1 1 0 0 / 0 0 0 1 1	5



linear code

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

higher weights

$d_1^P = 2$

$d_2^P =$

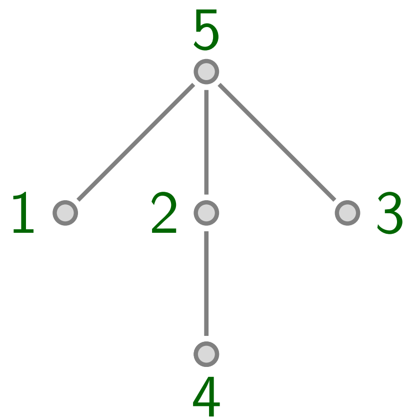
$d_3^P =$

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \end{bmatrix}$
2	3	5	3	5	5	5
$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$
4	5	5	5	5	5	5

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

5

subcodes



linear code

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

higher weights

$$d_1^P = 2$$
$$d_2^P = 4$$
$$d_3^P =$$

subcodes

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \end{bmatrix}$ 2

$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 3

$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \end{bmatrix}$ 3

$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 4

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}$ 5

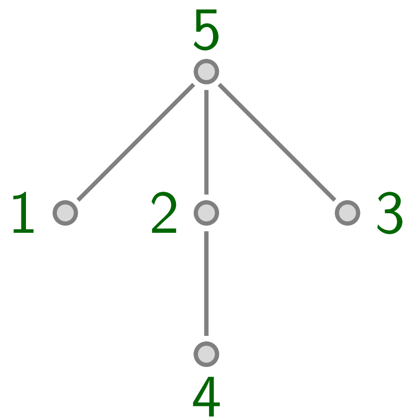
$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$ 5

$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \end{bmatrix}$ 5

$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$ 5



linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

higher weights

$$d_1^P = 2$$
$$d_2^P = 4$$
$$d_3^P = 5$$

subcodes

1	0	1	0	0

2

0	1	1	0	0

3

0	0	0	1	1

5

1	1	0	0	0

3

1	0	1	1	1

5

0	1	1	1	1

5

1	1	0	1	1

5

1	0	1	0	0
0	1	1	0	0

4

1	0	1	0	0
0	0	0	1	1

5

1	0	1	0	0
0	1	1	1	1

5

1	1	0	0	0
0	0	0	1	1

5

1	1	0	0	0
1	0	1	1	1

5

0	0	0	1	1
0	1	1	0	0

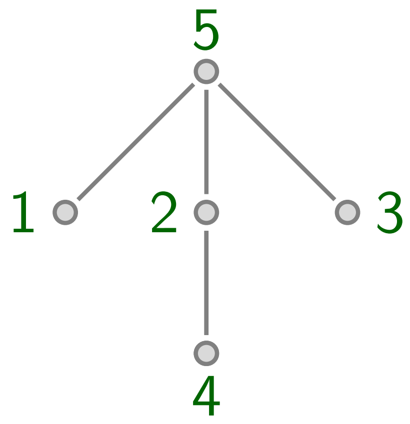
5

0	1	1	0	0
1	0	1	1	1

5

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

5



linear code

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

higher weights

$d_1^P = 2$ $\quad d_1^{\perp,\overline{P}} = 3$

$d_2^P = 4$ $\quad d_2^{\perp,\overline{P}} = 5$

$d_3^P = 5$

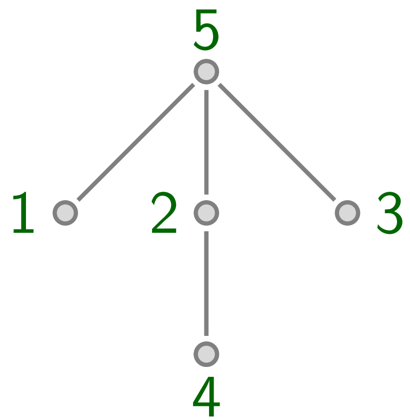
subcodes

1 0 1 0 0	0 1 1 0 0	0 0 0 1 1	1 1 0 0 0	1 0 1 1 1	0 1 1 1 1	1 1 0 1 1
2	3	5	3	5	5	5

1 0 1 0 0 0 1 1 0 0	1 0 1 0 0 0 0 0 1 1	1 0 1 0 0 0 1 1 1 1	1 1 0 0 0 0 0 0 1 1	1 1 0 0 0 1 0 1 1 1	0 0 0 1 1 0 1 1 0 0	0 1 1 0 0 1 0 1 1 1
4	5	5	5	5	5	5

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

5



linear code

$$\begin{pmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

higher weights

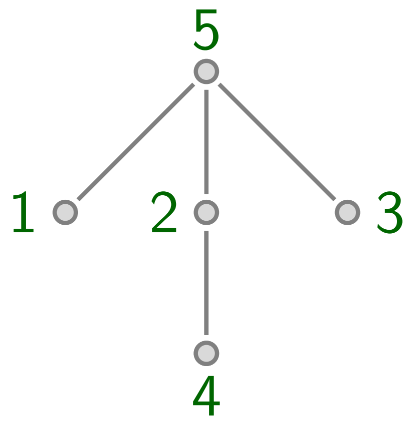
$d_1^P = 2 \quad d_1^{\perp,\overline{P}} = 3$

$d_2^P = 4 \quad d_2^{\perp,\overline{P}} = 5$

$d_3^P = 5$

$$U = \{d_1^P, \ldots, d_k^P\} = \{2, 4, 5\}$$

$$V = \{n+1-d_{n-k-1}^{\perp,\overline{P}}, \ldots, n+1-d_1^{\perp,\overline{P}}\} = \{1, 3\}$$



linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

higher weights

$d_1^P = 2$ $\quad d_1^{\perp,\overline{P}} = 3$

$d_2^P = 4$ $\quad d_2^{\perp,\overline{P}} = 5$

$d_3^P = 5$

$$U = \{d_1^P, \ldots, d_k^P\} = \{2, 4, 5\}$$

$$V = \{n + 1 - d_{n-k-1}^{\perp,\overline{P}}, \ldots, n + 1 - d_1^{\perp,\overline{P}}\} = \{1, 3\}$$

Poset Code Wei Duality

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

Moura & Firer 2010
BJMS 2012
Barg & Purkayastha 2012

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$U = \{d_1, \ldots, d_k\} = \{2, 3, 5\}$$

$$V = \{n + 1 - d^{\perp}_{n-k-1}, \ldots, n + 1 - d^{\perp}_1\} = \{1, 4\}$$

Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

linear code $R_C(x,y) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)}$

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$U = \{d_1, \ldots, d_k\} = \{2,3,5\}$$

$$V = \{n+1-d^{\perp}_{n-k-1}, \ldots, n+1-d^{\perp}_1\} = \{1,4\}$$

Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

linear code $R_C(x,y) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)} = \sum_{i=0}^{k} \sum_{j=0}^{n-k} r_{ij}\, x^i y^j$

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$U = \{d_1, \ldots, d_k\} = \{2,3,5\}$$

$$V = \{n+1-d^{\perp}_{n-k-1}, \ldots, n+1-d^{\perp}_1\} = \{1,4\}$$

Wei's Duality Theorem (1991)

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linear code $R_C(x,y) = \sum_{A \subseteq E} x^{\rho(E)-\rho(A)} y^{|A|-\rho(A)} = \sum_{i=0}^{k} \sum_{j=0}^{n-k} r_{ij}\, x^i y^j$

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$= 6 + 9x + 5x^2 + x^3 + 5y + y^2 + 4xy + x^2y$$

$$U = \{d_1, \ldots, d_k\} = \{2, 3, 5\}$$

$$V = \{n+1-d^{\perp}_{n-k-1}, \ldots, n+1-d^{\perp}_1\} = \{1, 4\}$$

Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

linear code $R_C(x,y) = \sum_{i,j} r_{ij}\, x^i y^j = 6 + 9x + 5x^2 + x^3 + 5y + 4xy + x^2y + y^2$

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$U = \{d_1, \ldots, d_k\} = \{2, 3, 5\}$$

$$V = \{n+1-d^{\perp}_{n-k-1}, \ldots, n+1-d^{\perp}_1\} = \{1, 4\}$$

Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

linear code $R_C(x,y) = \sum_{i,j} r_{ij}\, x^i y^j = 6 + 9x + 5x^2 + x^3 + 5y + 4xy + x^2y + y^2$

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$R_{C^\perp}(x,y) = \sum_{i,j} r^\perp_{ij}\, x^i y^j = 6 + 9y + 5y^2 + y^3 + 5x + 4xy + xy^2 + x^2$

$U = \{d_1, \ldots, d_k\} = \{2,3,5\}$

$V = \{n+1-d^\perp_{n-k-1}, \ldots, n+1-d^\perp_1\} = \{1,4\}$

Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

linear code

$$R_C(x,y) = \sum_{i,j} r_{ij}\, x^i y^j = 6 + 9x + 5x^2 + x^3 + 5y + 4xy + x^2y + y^2$$

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$R_{C^\perp}(x,y) = \sum_{i,j} r^\perp_{ij}\, x^i y^j = 6 + 9y + 5y^2 + y^3 + 5x + 4xy + xy^2 + x^2$$

linear code

$$R_C(x,y) = \sum_{i,j} r_{ij}\, x^i y^j = 6 + 9x + 5x^2 + x^3 + 5y + 4xy + x^2y + y^2$$

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$R_{C^\perp}(x,y) = \sum_{i,j} r^\perp_{ij}\, x^i y^j = 6 + 9y + 5y^2 + y^3 + 5x + 4xy + xy^2 + x^2$$

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$$M = [r_{ij}]$$

linear code

$$R_C(x,y) = \sum_{i,j} r_{ij}\, x^i y^j = 6 + 9x + 5x^2 + x^3 + 5y + 4xy + x^2y + y^2$$

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$R_{C^\perp}(x,y) = \sum_{i,j} r^\perp_{ij}\, x^i y^j = 6 + 9y + 5y^2 + y^3 + 5x + 4xy + xy^2 + x^2$$

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$$M = [r_{ij}]$$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$$M^\perp = [r^\perp_{ij}]$$

linear code

$$R_C(x,y) = \sum_{i,j} r_{ij}\, x^i y^j = 6 + 9x + 5x^2 + x^3 + 5y + 4xy + x^2y + y^2$$

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

$$R_{C^\perp}(x,y) = \sum_{i,j} r^\perp_{ij}\, x^i y^j = 6 + 9y + 5y^2 + y^3 + 5x + 4xy + xy^2 + x^2$$

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$$M = [r_{ij}]$$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$$M^\perp = [r^\perp_{ij}] = [r_{ji}] = M^T$$

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$$M = [r_{ij}]$$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$$M^{\perp} = [r_{ij}^{\perp}] = [r_{ji}] = M^T$$

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$M = [r_{ij}]$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$M^\perp = [r^\perp_{ij}] = [r_{ji}] = M^T$

Lemma

The non-zero entries of M and $M^\perp$ form Ferrers shapes.

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$M = [r_{ij}]$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$M^{\perp} = [r_{ij}^{\perp}] = [r_{ji}] = M^T$

Lemma

The non-zero entries of M and $M^{\perp}$ form Ferrers shapes.

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$M = [r_{ij}]$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$M^{\perp} = [r^{\perp}_{ij}] = [r_{ji}] = M^T$

Lemma

- $d_i - i \; = \; \#$ 0s in row i of M
- $d^{\perp}_j - j \; = \; \#$ 0s in row j of $M^{\perp}$

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$$M = [r_{ij}]$$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$$M^\perp = [r^\perp_{ij}] = [r_{ji}] = M^T$$

Lemma

- $d_i - i \; = \; \#\,0$s in row i of M
- $d^\perp_j - j \; = \; \#\,0$s in row j of $M^\perp \; = \; \#\,0$s in column j of M

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$d_1 - 1 = 1$

$M = [r_{ij}]$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$M^{\perp} = [r^{\perp}_{ij}] = [r_{ji}] = M^T$

Lemma

- $d_i - i \;=\; \#\,$0s in row i of M
- $d^{\perp}_j - j \;=\; \#\,$0s in row j of $M^{\perp} \;=\; \#\,$0s in column j of M

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$$d_1 - 1 = 1$$
$$d_2 - 2 = 1$$

$$M = [r_{ij}]$$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$$M^{\perp} = [r^{\perp}_{ij}] = [r_{ji}] = M^T$$

Lemma

- $d_i - i \;=\; \#$ 0s in row i of M
- $d^{\perp}_j - j \;=\; \#$ 0s in row j of $M^{\perp} \;=\; \#$ 0s in column j of M

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$d_1 - 1 = 1$
$d_2 - 2 = 1$
$d_3 - 3 = 2$

$M = [r_{ij}]$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$M^\perp = [r_{ij}^\perp] = [r_{ji}] = M^T$

Lemma

- $d_i - i \; = \; \#\,$0s in row i of M
- $d_j^\perp - j \; = \; \#\,$0s in row j of $M^\perp \; = \; \#\,$0s in column j of M

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$d_1 - 1 = 1$

$d_2 - 2 = 1$

$d_3 - 3 = 2$

$M = [r_{ij}]$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$d_1^{\perp} - 1 = 1$

$M^{\perp} = [r_{ij}^{\perp}] = [r_{ji}] = M^T$

Lemma

- $d_i - i = \#$ 0s in row i of M
- $d_j^{\perp} - j = \#$ 0s in row j of $M^{\perp}$ $= \#$ 0s in column j of M

linear code

1	0	1	0	0
0	1	1	0	0
0	0	0	1	1

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$d_1 - 1 = 1$

$d_2 - 2 = 1$

$d_3 - 3 = 2$

$M = [r_{ij}]$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$d_1^\perp - 1 = 1$

$d_2^\perp - 2 = 3$

$M^\perp = [r_{ij}^\perp] = [r_{ji}] = M^T$

Lemma

- $d_i - i$ = # 0s in row i of M
- $d_j^\perp - j$ = # 0s in row j of $M^\perp$ = # 0s in column j of M

linear code

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

	0	1	2
0	6	5	1
1	9	4	0
2	5	1	0
3	1	0	0

$$d_1 - 1 = 1$$
$$d_2 - 2 = 1$$
$$d_3 - 3 = 2$$

$$M = [r_{ij}]$$

	0	1	2	3
0	6	9	5	1
1	5	4	1	0
2	1	0	0	0

$$d_1^{\perp} - 1 = 1$$
$$d_2^{\perp} - 2 = 3$$

$$M^{\perp} = [r_{ij}^{\perp}] = [r_{ji}] = M^T$$

$$d_1 = 2$$
$$d_2 = 3$$
$$d_3 = 5$$

$$d_1^{\perp} = 2$$
$$d_2^{\perp} = 5$$

higher weights

Lemma

- $d_i - i \; = \; \#$ 0s in row i of M
- $d_j^{\perp} - j \; = \; \#$ 0s in row j of $M^{\perp} \; = \; \#$ 0s in column j of M

Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

$U = \{d_i\}$

$V = \{n + 1 - d_j^{\perp}\}$

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Proof

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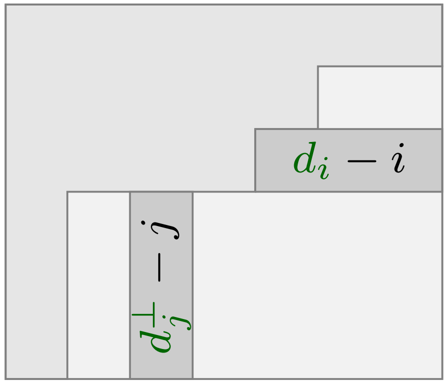
Proof



Case I

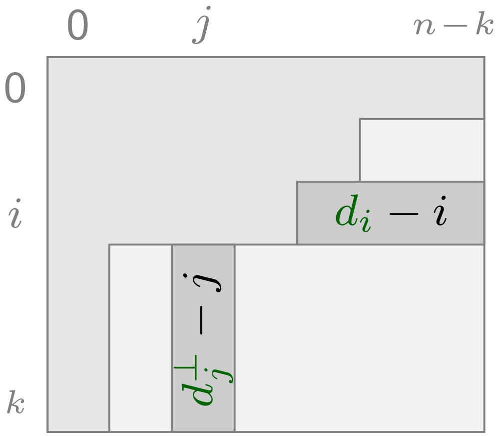
Wei's Duality Theorem (1991)

$U = \{d_i\}$

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

$V = \{n + 1 - d_j^{\perp}\}$

Proof



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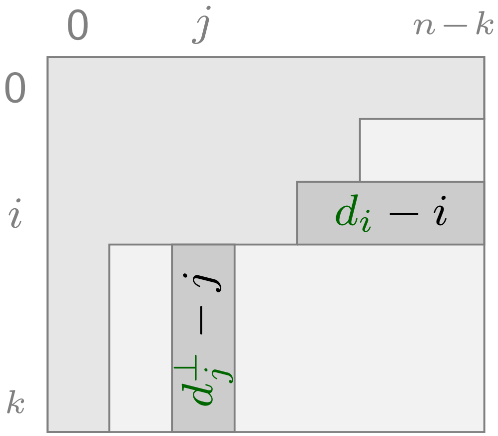

Case I

$$d_i + d_j^{\perp} = (d_i - i) + (d_j^{\perp} - j) + i + j$$

Wei's Duality Theorem (1991)

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$U = \{d_i\}$

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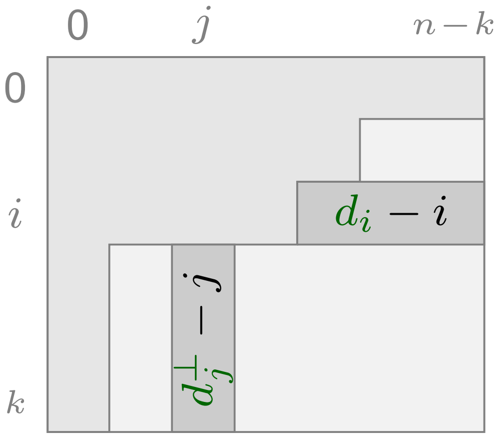
Proof



Case I

$$\begin{aligned} d_i + d_j^\perp &= (d_i - i) + (d_j^\perp - j) + i + j \\ &\leq (n - k + 1) - (j + 1) \\ &\quad + (k + 1) - (i + 1) + i + j \end{aligned}$$

Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

$U = \{d_i\}$

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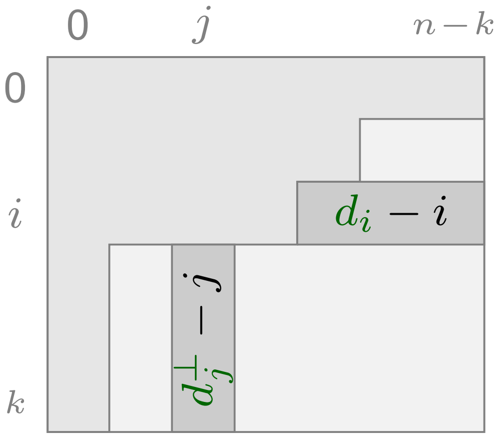
Proof



Case I

$$
\begin{aligned}
d_i + d_j^\perp &= (d_i - i) + (d_j^\perp - j) + i + j \\
&\le (n - k + 1) - (j + 1) \\
&\qquad + (k + 1) - (i + 1) + i + j = n
\end{aligned}
$$

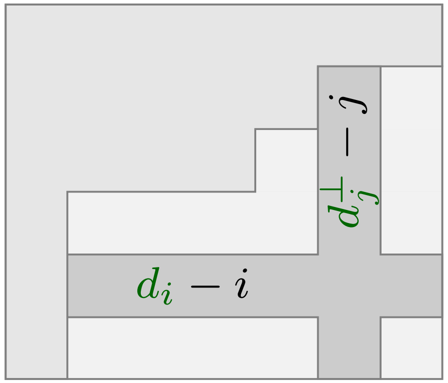
Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

$U = \{d_i\}$

$V = \{n + 1 - d_j^\perp\}$

Proof



Case II

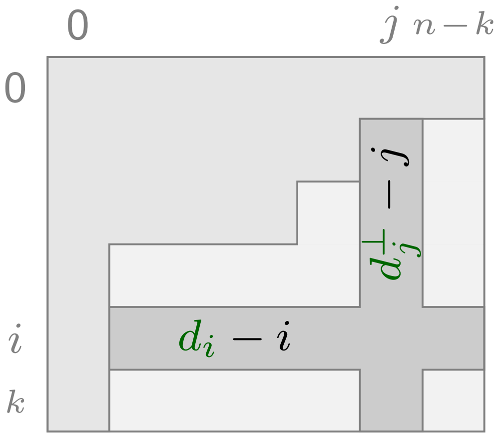
Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

$U = \{d_i\}$

$V = \{n + 1 - d_j^{\perp}\}$

Proof



Case II

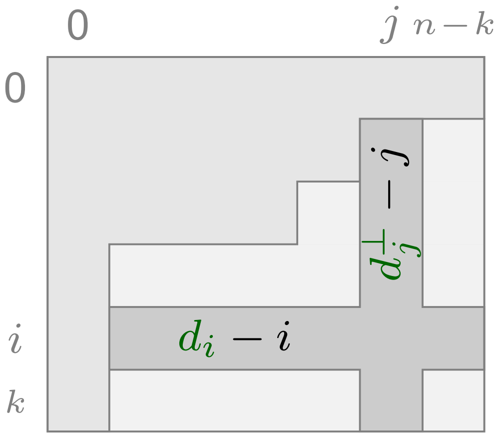
Wei's Duality Theorem (1991)

$U = \{d_i\}$

$V = \{n+1-d_j^\perp\}$

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

Proof



Case II

$$d_i + d_j^\perp = (d_i - i) + (d_j^\perp - j) + i + j$$

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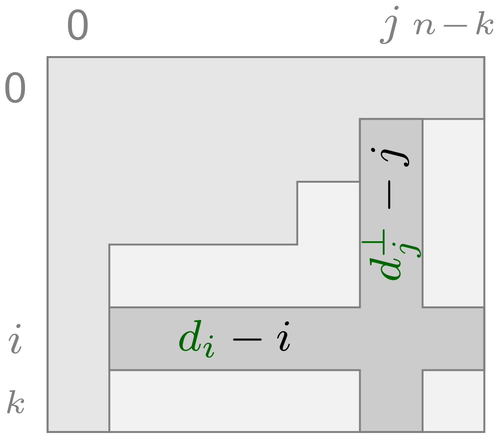
Proof



Case II

$$d_i + d_j^{\perp} = (d_i - i) + (d_j^{\perp} - j) + i + j$$
$$\geq (n - k + 1 - j) + (k + 1 - i) + i + j$$

Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

$U = \{d_i\}$

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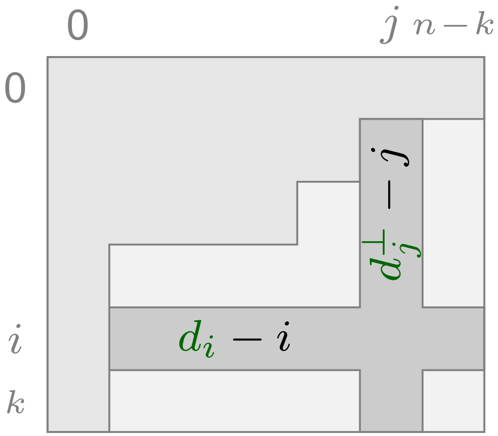
Proof



Case II

$$\begin{aligned} d_i + d_j^\perp &= (d_i - i) + (d_j^\perp - j) + i + j \\ &\geq (n - k + 1 - j) + (k + 1 - i) + i + j \\ &= n + 2 \end{aligned}$$

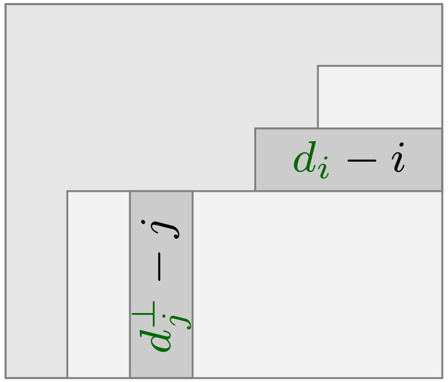
Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

$U = \{d_i\}$

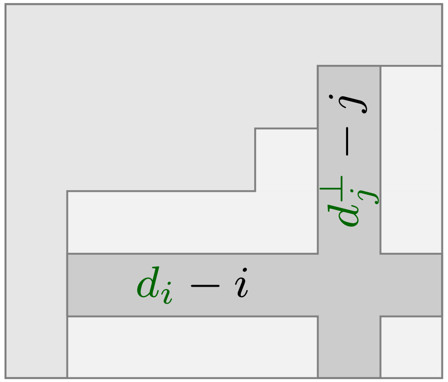
$V = \{n + 1 - d_j^\perp\}$

Proof



Case I

$d_i + d_j^\perp \leq n$



Case II

$d_i + d_j^\perp \geq n + 2$

Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

$U = \{d_i\}$

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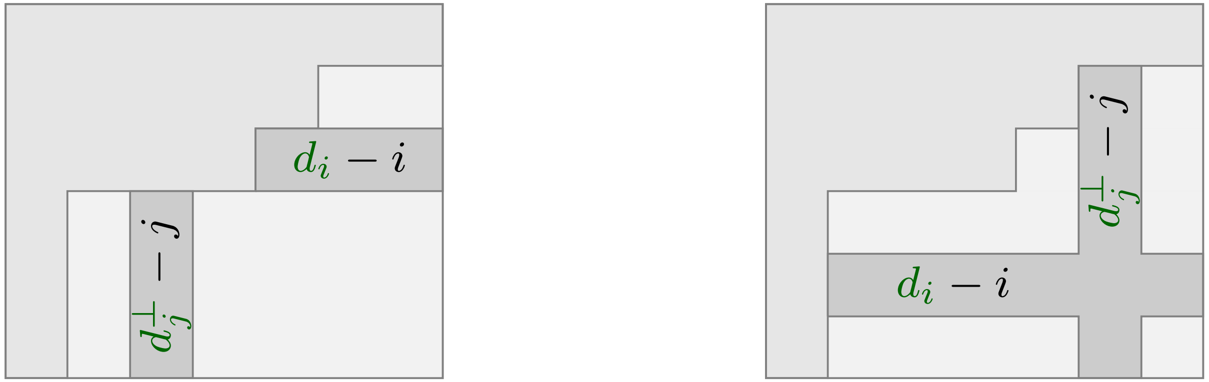
Proof



Case I

$d_i + d_j^\perp \leq n$

Case II

$d_i + d_j^\perp \geq n + 2$

In either case, $d_i + d_j^\perp \neq n + 1$

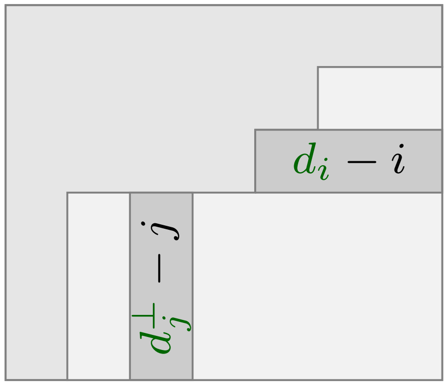
Wei's Duality Theorem (1991)

$U = \{d_i\}$

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

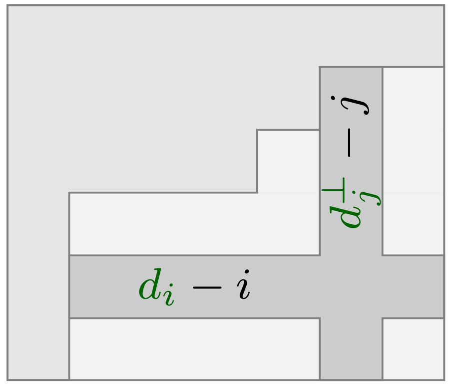
$V = \{n + 1 - d_j^\perp\}$

Proof



Case I

$d_i + d_j^\perp \leq n$



Case II

$d_i + d_j^\perp \geq n + 2$

In either case, $d_i + d_j^\perp \neq n + 1$ — so $U \cap V = \varnothing$. $\square$

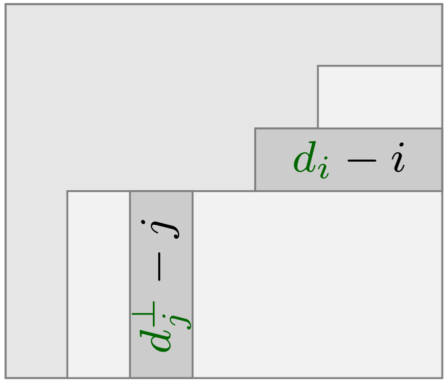
Wei's Duality Theorem (1991)

$U \cup V = \{1, \ldots, n\}$ and $U \cap V = \varnothing$

$U = \{d_i\}$

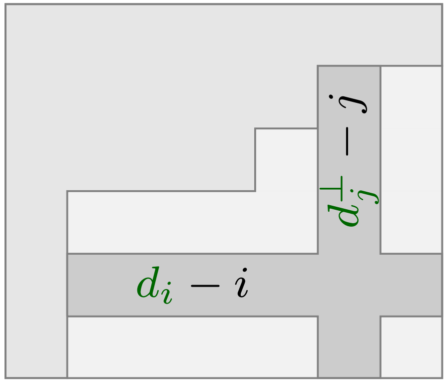
$V = \{n + 1 - d_j^{\perp}\}$

Proof



Case I

$d_i + d_j^{\perp} \leq n$



Case II

$d_i + d_j^{\perp} \geq n + 2$

In either case, $d_i + d_j^{\perp} \neq n + 1$ — so $U \cap V = \varnothing$. □

☯ **THANK YOU!** ☯