Cops and Robbers on Graphs

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Overview

Cops, Robbers and Loops
  Rules of the Game
  Up, Down and around the Loop
  Cop Number and Loops

Cops, Robbers and Algebraic Topology
  Homomorphisms
  Homotopy Invariance
Game of Cops and Robbers

- Given a graph $G$:

- The cop chooses his starting position on a vertex of $G$.

- The robber chooses his starting point.

- They move each in turn from one vertex to an adjacent vertex.

- They can see each other at all times.

- Can the cop catch the robber?
Theorem (Characterisation of Copwin Graphs)

A graph is copwin if and only if it is dismantlable, i.e. if it can be reduced to a single vertex by successively removing vertices where the robber can be trapped. (Quilliot, 1978)

Theorem (Bounded Capture Time)

If $G$ has $n$ vertices, $n \geq 7$, then the capture time $ct(G)$ satisfies $ct(G) \leq n - 4$. (Gavenčiak, 2010)
The Impact of Loops

Figure: Partially looped $2 \times n$ grid
Cop moving away from the Robber

Figure: Graph $G$
Loops can help the Robber

Figure: Graph $H_1$

Figure: Graph $H_2$
Loops can also help the Cops

\[ x \equiv a \mod i \]

\[ v_{p^4, a} \quad 0 \leq a < p^4 \]

\[ v_{p^4 + 1, a} \quad 0 \leq a < p^4 + 1 \]

\[ v_{p^4 + p^2, a} \quad 0 \leq a < p^4 + p^2 \]
Cop Number and Loops

Given a graph $G$, let $G^+$ and $G^-$ be the graphs obtained by adding or removing loops on every vertex respectively.

**Proposition (Hahn et al.)**
$c(G^+) \leq c(G^-) + 1$

**Proposition**
$c(G^-) \leq 2c(G^+)$

**Proposition**
$\forall n, \exists G_n : c(G_n^+) = n$ and $c(G^-) = 2n - 1$

**Conjecture**
$c(G^+) < 2c(G^-)$
$f : X \rightarrow Y$

cops & robber here

cops' images chase robber's image
\[ f : X \rightarrow Y \]

- cops & robber here
- cops' images chase robber's image
$f : X \rightarrow Y$

- robber here
- cops there
- chasing robber’s image
Theorem (Homotopy Invariance)

If two homomorphisms are homotopic, they have the same cop number and their capture times differ by the homotopic distance at most.

Theorem (Characterisation of Copwin Graphs)

A graph is copwin if and only if it is contractible.
Thank you for your attention!

No cops or robbers were harmed in the making of this presentation.