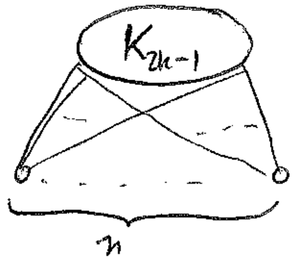
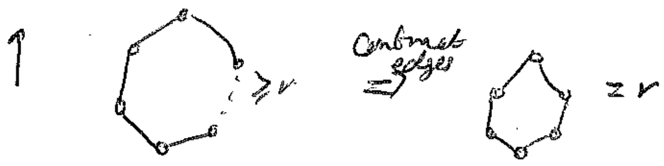
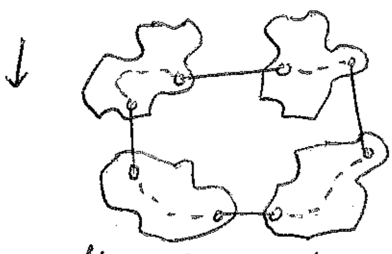
P1

$K^*_{2k-1,n}$




P2

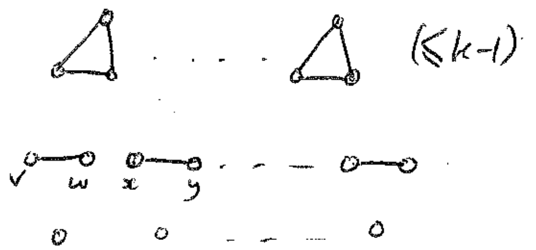


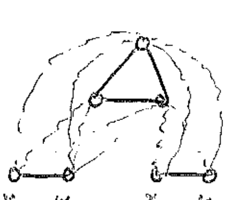
(branch sets - sets you contract down to get $r$-cycle minor)

These vertices with dotted lines could be same vertex, or could have path between them (which increase cycle size as a subgraph)).
(Subgraph cycle has at least $r$-vertices).

P3



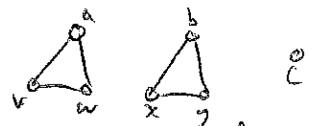

P4




$p_i + q_i = 5$
$\Rightarrow$ WLOG
$p_i = 3,\ q_i = 2$

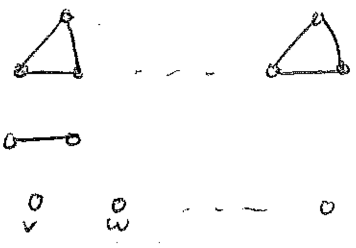
Pick $a \neq b \in K_3$ s.t. $vwa$, $xyb$ are $\Delta$s.

$\Rightarrow$

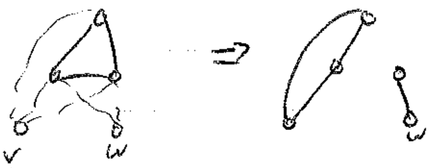



Swap $K_3$, $2 \times K_2$ for $2 \times K_3$, $K_1$.
Contradicts choice of partition.

P5

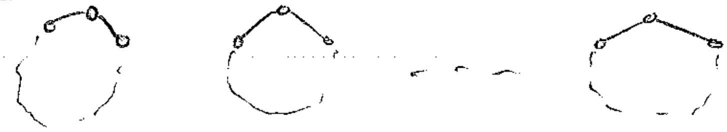



Might have one neighbour in $K_2$.
$\therefore$ $2k-1$ neighbours in $K_3$'s (like before) ($K_2$'s had $2k-1$ c.n. in $K_3$'s each)
$\therefore$ Calculations are exactly the same.

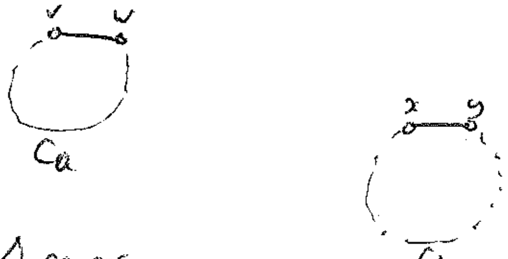



Pick a neighbour of $w$, get a $K_2$.
Other two neighbours, together with $v$ make a new $K_3$.
Swap $K_3$, $2 \times K_1$ for $K_3$, $K_2$.

P6

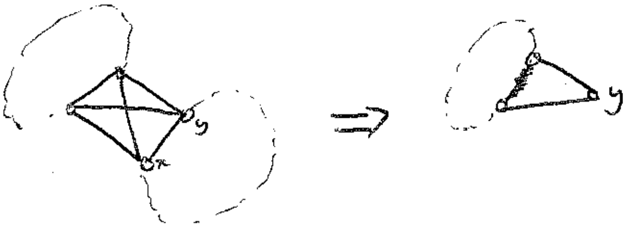


cycles of length r.

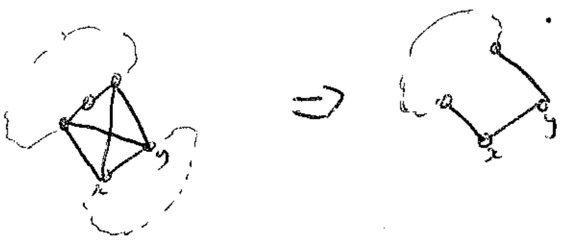


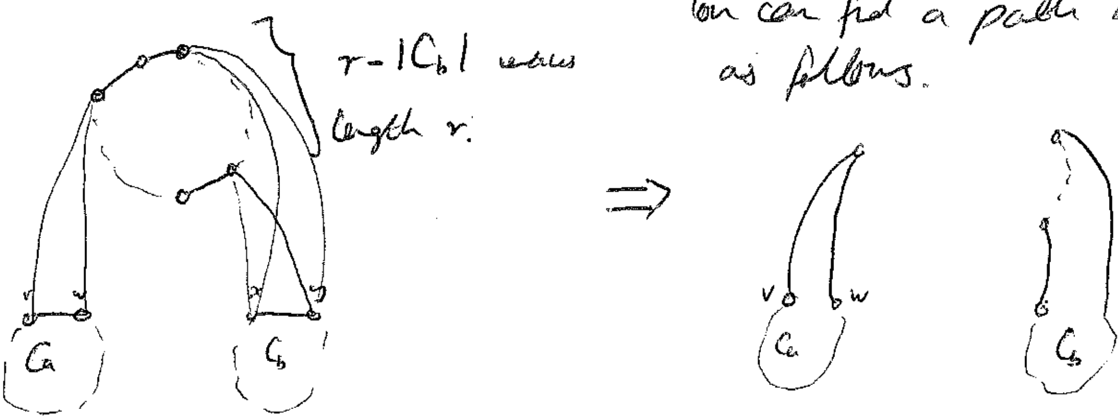

Where can common neighbours go?
Can be lost internally, (cycles could be cliques)
Could also have $\frac{1}{3}|C_a|$ common neighbours of xy wrt $C_a$

Note a cycle plus $\Delta$ across an edge gives slightly larger cycle




The rest are in ~~copies of~~ cycles of length r.

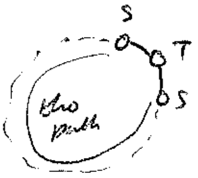


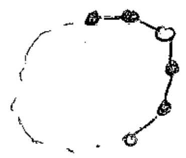

You can find a path & cadres as follows.

Swap $r, |C_a|, |C_b|$ for $|C_a|+1,\ |C_b|+r-|C_b| = r.$

(P7) Say $q = r-1$




However, could get



• in $S \cap T$

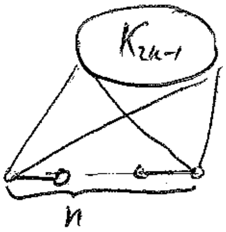
∘ in nothing

if $|S| = |T| = \frac{2}{3}r$.

for all cycles.

(Can avoid this problem if we place an upper bound on acceptable $q$ – we need to do this in one specialised case).

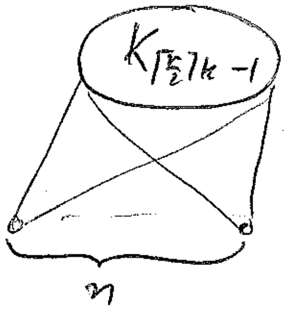
(P8)




← matching not indep set.

Still need 2 vertices from clique for each $\geq 4$ cycle.
$\therefore$ No $k$ 4-cycles here.
But $\delta = 2k$, and $d(G) \to 4k-1$ as $n \to \infty$.

(P9)




Any cycle has half its vertices (rounded up) in clique, or else has more than half in indep set; but $> \frac{1}{2}|C|$ vertices include two adjacent.
$\delta = \lceil \frac{r}{2} \rceil k - 1$, $\quad d(G) \to 2\lceil \frac{r}{2} \rceil k - 2$ as $n \to \infty$.

$2\lceil \frac{r}{2} \rceil = r$ if even, $r+1$ if odd

(P9 is generalisation of P1, better than P8 generalisation of clique instead of $K_2$'s).