

Switching techniques for edge decompositions of graphs

Daniel Horsley

Monash University, Australia

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Darryn Bryant, Barbara Maenhaut

Definitions

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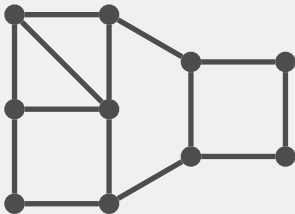
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A set $\{G_1, \dots, G_t\}$ of subgraphs of a graph G such that $\{E(G_1), \dots, E(G_t)\}$ is a partition of $E(G)$.

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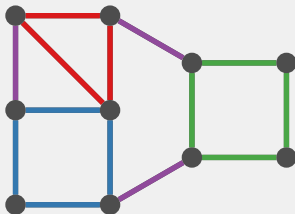


A graph G

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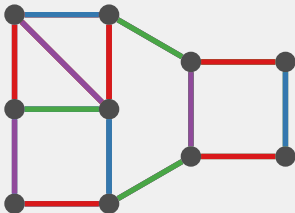


A decomposition of G into cycles and a matching

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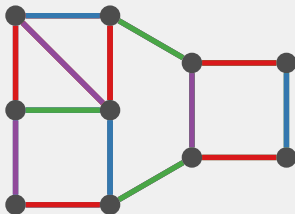


A decomposition of G into matchings

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A decomposition of G into matchings

proper t -edge colouring

A decomposition of a graph into t matchings.

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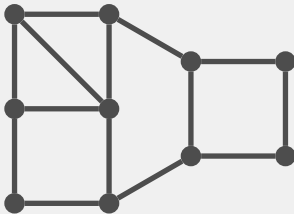
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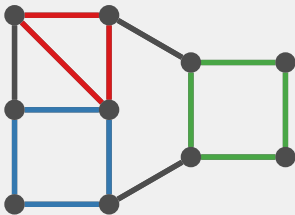


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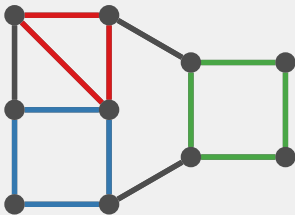


A packing of G with cycles

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A packing of G with cycles

Switching technique

A method for locally modifying edge decompositions that feels kind of switchy.

Warm up:
Proper edge colourings

Switching in proper edge colorings

Switching in proper edge colorings

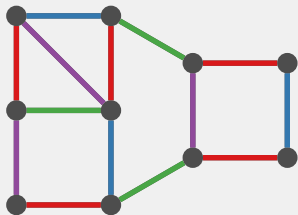
Theorem

If a graph G has a proper t -edge colouring, then it has a proper t -edge colouring such that the sizes of any two colour classes differ by at most one.

Switching in proper edge colorings

Theorem

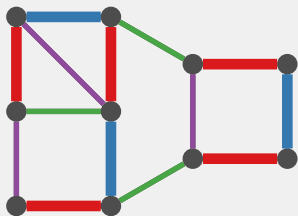
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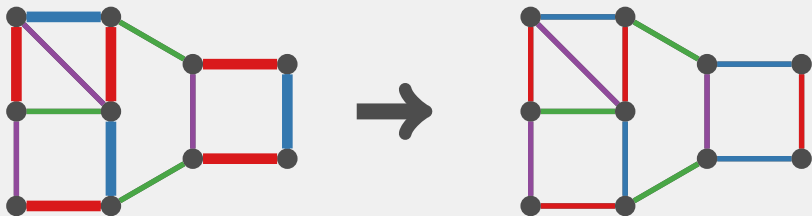
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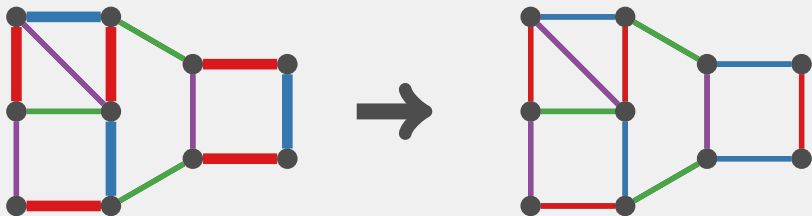
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Switching in proper edge colorings

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Theorem (Vizing 1964)

Every graph G has a proper $(\Delta(G) + 1)$ -edge colouring.

Feels kind of switchy

Is reminiscent of the argument on the last slide.

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This talk is about applying these kinds of switching techniques to edge decompositions of graphs *other than* edge colourings.

Part 1:

Embedding partial Steiner triple systems

Steiner triple systems

Steiner triple systems

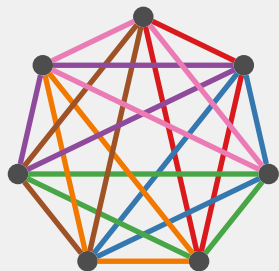
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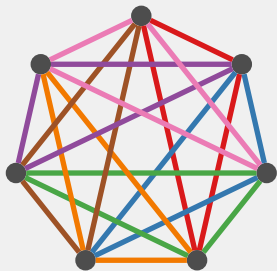


An STS(7)

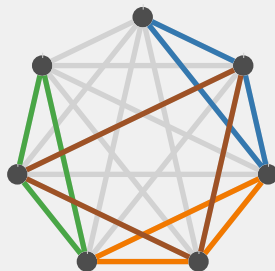
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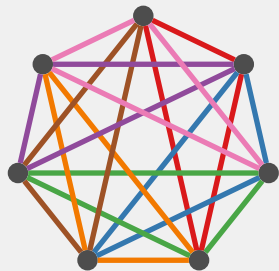


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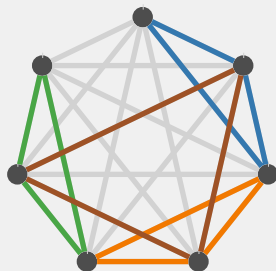
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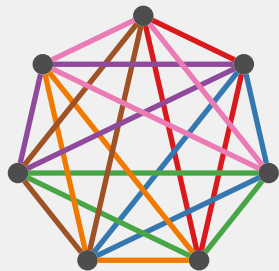
Theorem (Kirkman 1847)

An STS(v) exists if and only if $v \equiv 1, 3 \pmod{6}$.

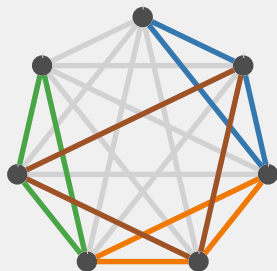
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Call such orders *admissible*.

Embedding PSTSs

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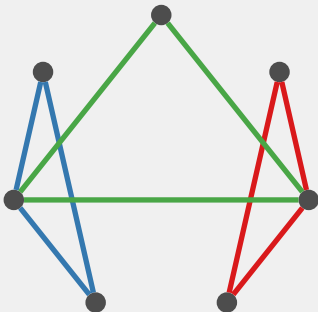
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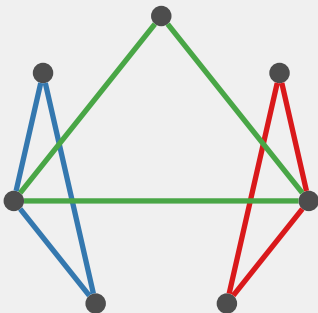


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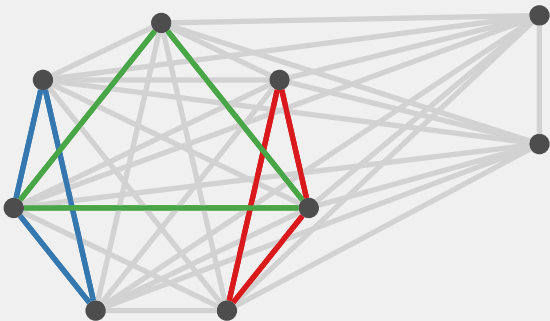
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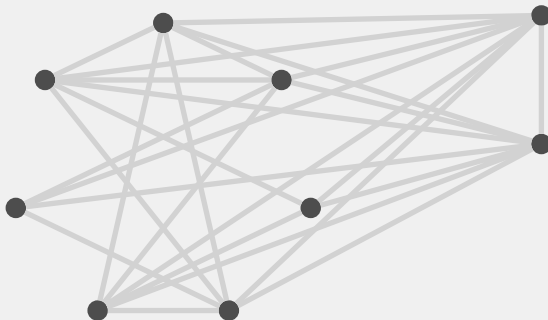


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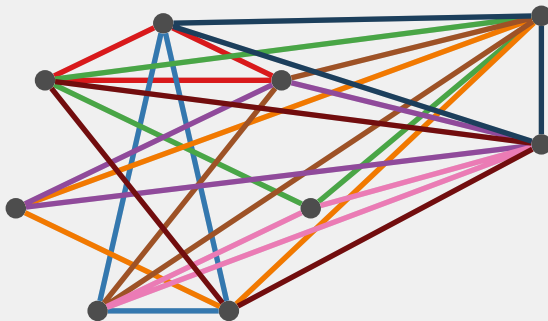


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An embedding of the $\text{PSTS}(7)$ of order 9

History

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Treash (1971): Every PSTS(u) has an embedding (of order at most 2^{2u}).

Lindner (1975): Every PSTS(u) has an embedding of order $6u + 3$.

Conjecture (Lindner 1977)

Every PSTS(u) has an embedding of order v for each admissible $v \geq 2u + 1$.

Andersen, Hilton, Mendelsohn (1980): Every PSTS(u) has an embedding of order v for each admissible $v \geq 4u + 1$.

Bryant (2004): Every PSTS(u) has an embedding of order v for each admissible $v \geq 3u - 2$.

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Work on embeddings of PTS(v, λ)s and quasigroup variants by Andersen, Colbourn, Hamm, Hao, Hoffman, Lindner, Mendelsohn, Raines, Rodger, Rosa, Stubbs, Wallis in the 1970s, 80s, 90s and 00s.

Switching in triangle packings

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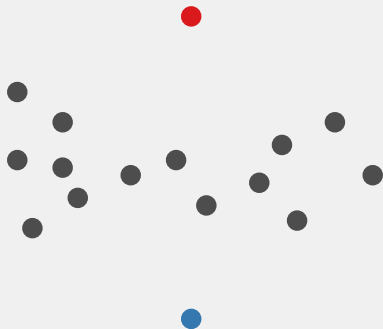
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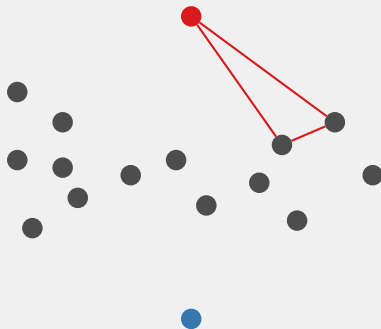
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in our case they will both be “new” vertices

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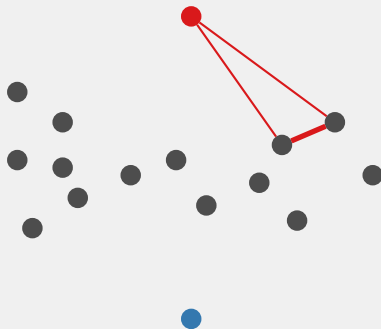
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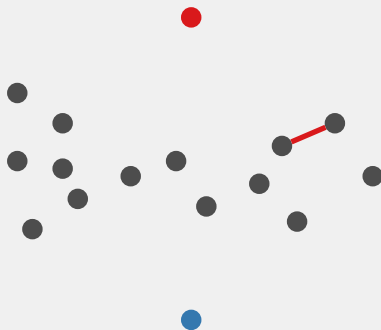
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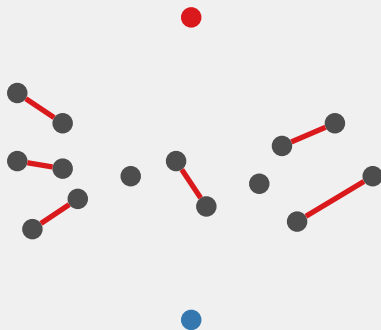
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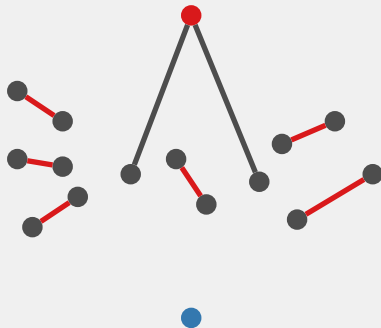
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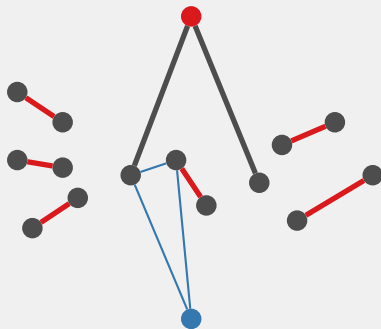
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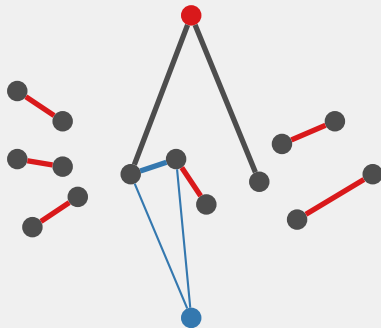
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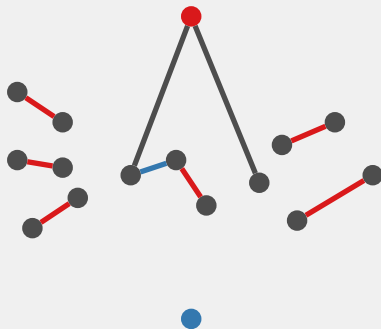
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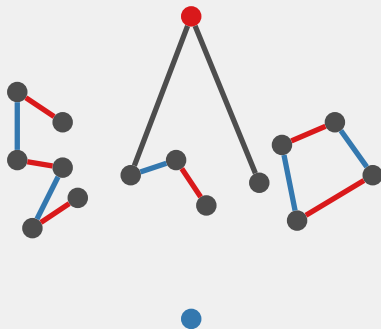
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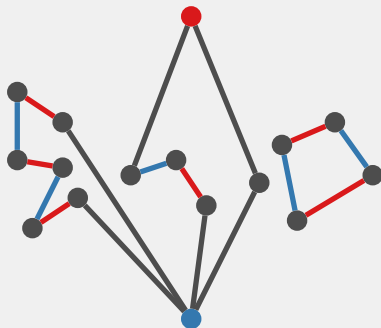
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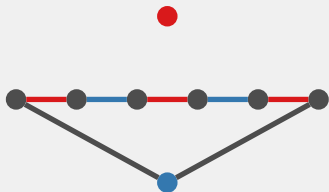
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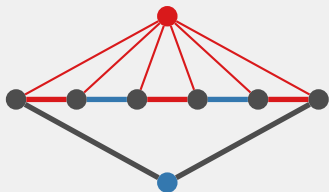
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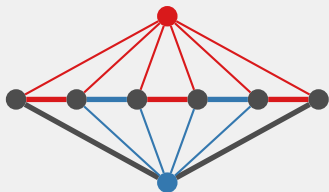
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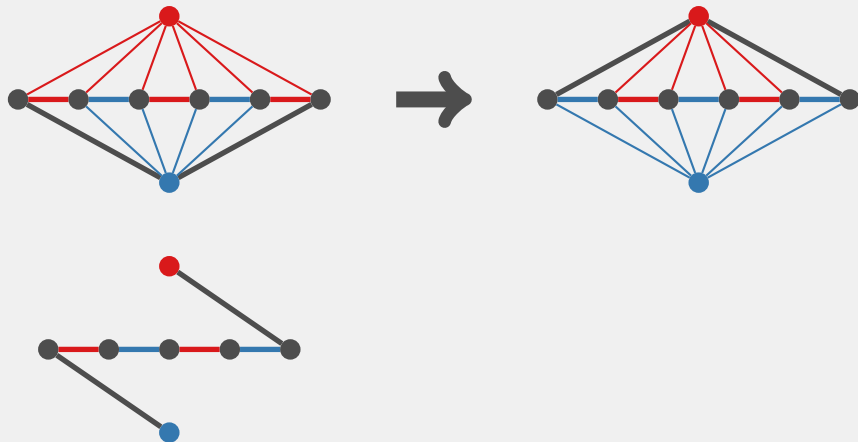
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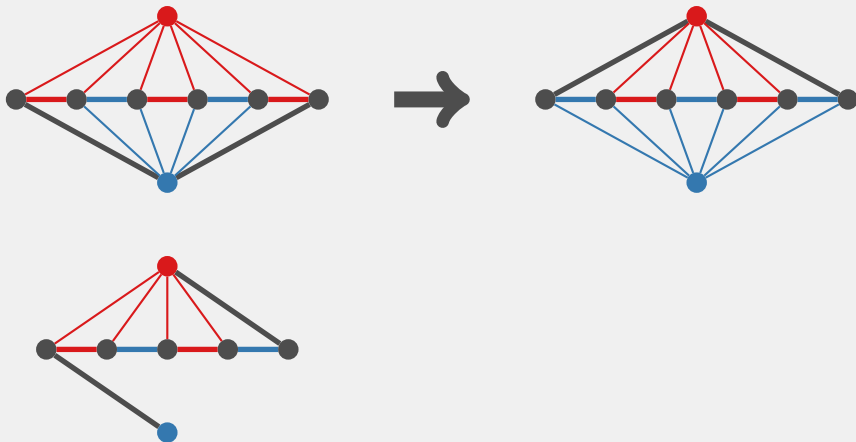
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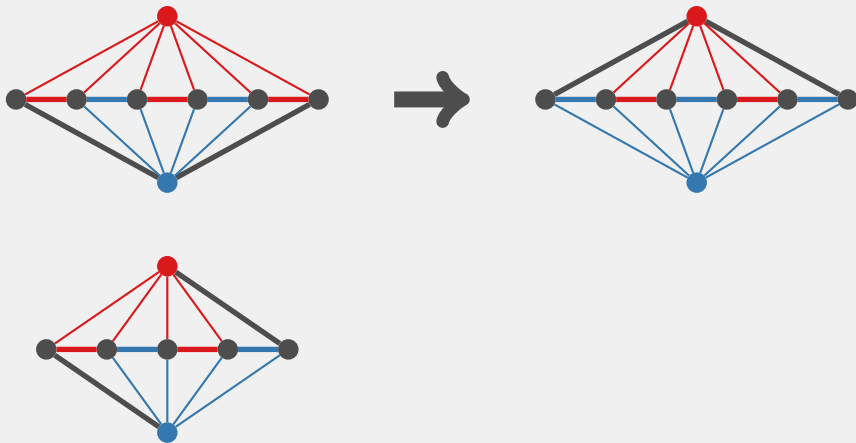
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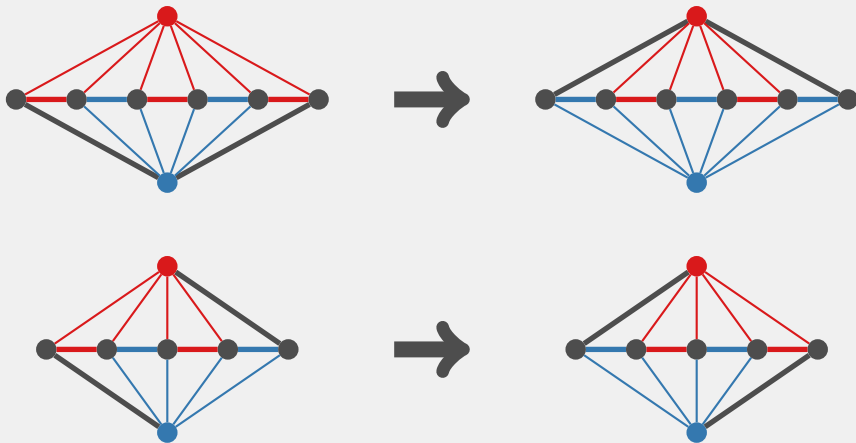
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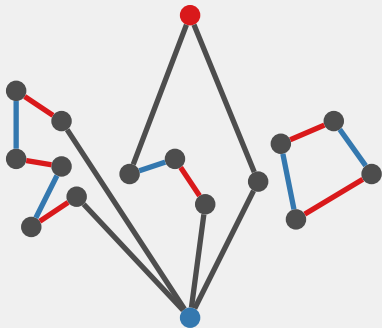


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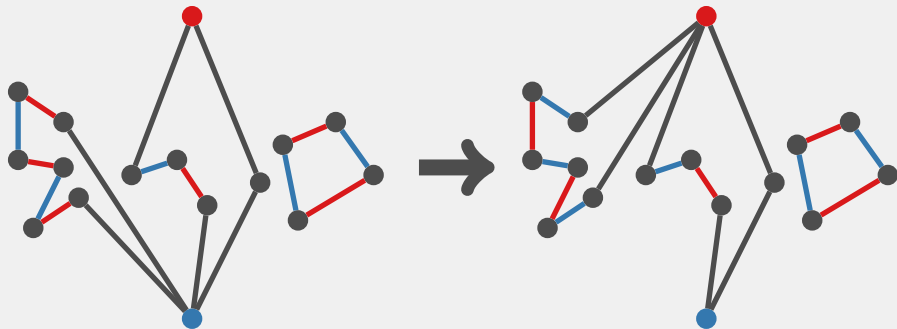


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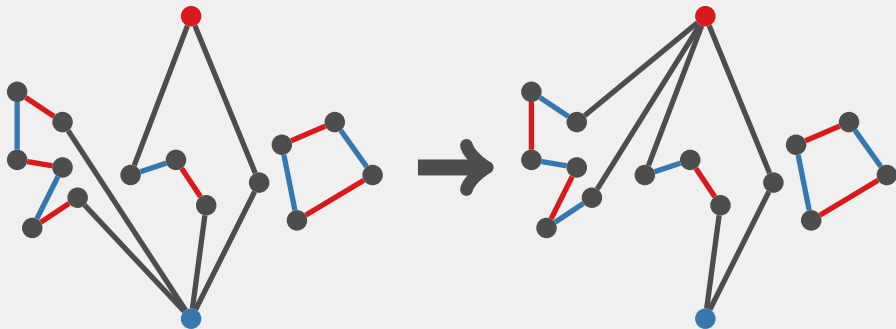
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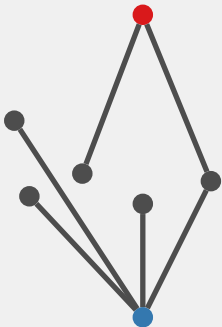


Lemma (Andersen, Hilton, Mendelsohn 1980)

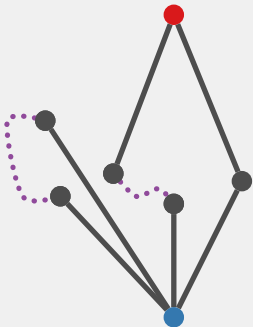
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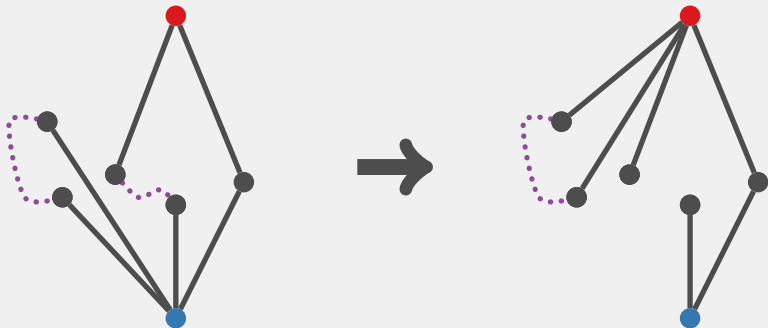
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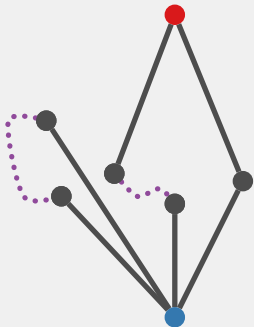
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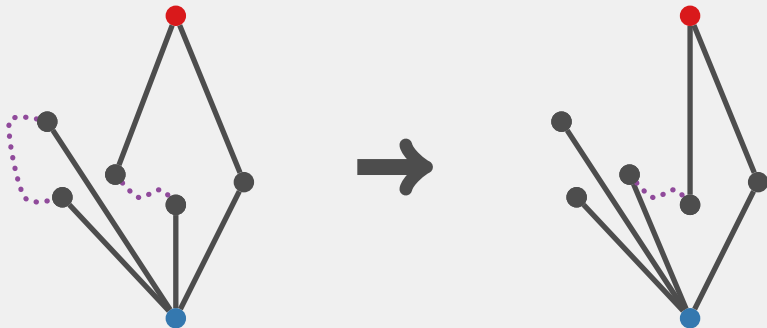
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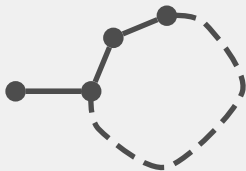
Say we wish to add another triangle to a partial embedding.

The leave of our packing has no triangles, but does contain a “lasso” on the new vertices.

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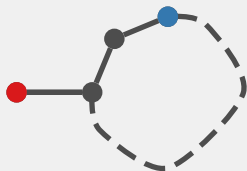
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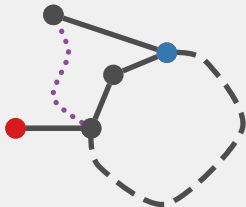
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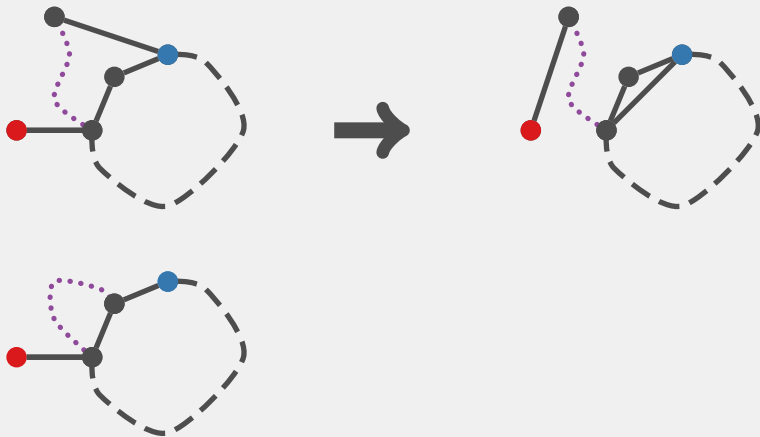
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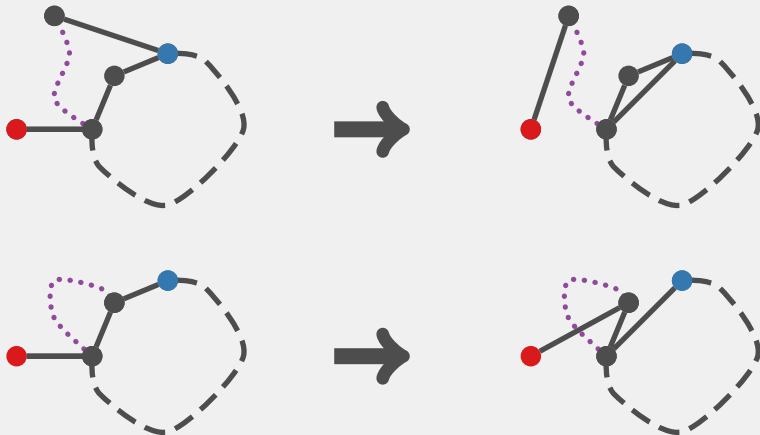
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Theorem (Bryant, H. 2009)

Every $\text{PSTS}(u)$ has an embedding of order v for each admissible $v \geq 2u + 1$.

More switching-assisted results on embeddings

Bryant, Buchanan (2007): Every partial totally symmetric quasigroup of order u has an embedding of order v for each even $v \geq 2u + 4$.

Bryant, Martin (2012): For $u \geq 28$, every $\text{PTS}(u, \lambda)$ has an embedding triple of order v for each admissible $v \geq 2u + 1$.

Martin, McCourt (2012): Any partial 5-cycle system of order $u \geq 255$ has an embedding of order at most $\frac{1}{4}(9u + 146)$.

H. (2014): “Half” of the possible embeddings of order less than $2u + 1$ for $\text{PSTS}(u)$ s with $\Delta(L) \leq \frac{1}{4}(u - 9)$ and $|E(L)| < \frac{1}{32}(u - 5)(u - 11) + 2$ exist.

H. (2014): Any $\text{PSTS}(u)$ with at most $\frac{1}{50}u^2 + o(u)$ triples has an embedding for each admissible order $v \geq \frac{1}{5}(8u + 17)$.

Part 2:

Cycle decompositions

Cycle decompositions of complete graphs

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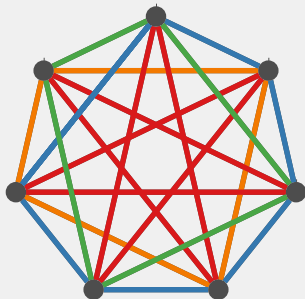
$$K_n \rightsquigarrow m_1, \dots, m_t$$

“There is a decomposition of K_n into cycles of lengths m_1, \dots, m_t .”

Cycle decompositions of complete graphs

$$K_n \rightsquigarrow m_1, \dots, m_t$$

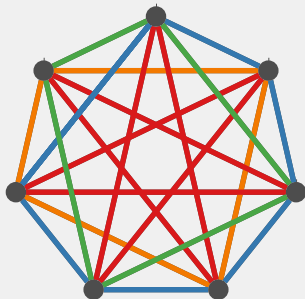
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$$K_7 \rightsquigarrow 7, 6, 4, 4$$

If $K_n \rightsquigarrow m_1, \dots, m_t$ then

(1) n is odd;

(2) $n \geq m_1, \dots, m_t \geq 3$; and

(3) $m_1 + \dots + m_t = \binom{n}{2}$.

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Alspach's cycle decomposition problem (1981)

Prove (1), (2) and (3) are sufficient for $K_n \rightsquigarrow m_1, \dots, m_t$.

History (highlights)

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When does $K_n \rightsquigarrow m, \dots, m$?

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- ▶ Work on limited sets of cycle lengths from Adams, Bryant, Heinrich, Horák, Khodkar, Maehaut, Rosa in the 1980s, 90s and 00s.
- ▶ A more general result from Balister in 2001.
- ▶ A reduction from Bryant, H. in 2009–2010.
- ▶ Solved by Bryant, H., Pettersson in 2014.

Switching in cycle packings

Switching in cycle packings



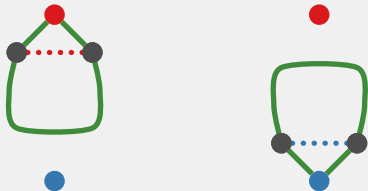
Switching in cycle packings



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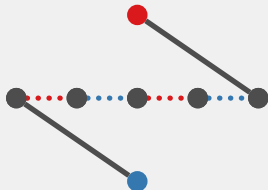
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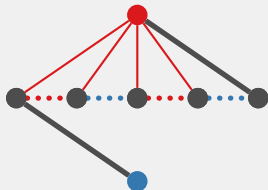
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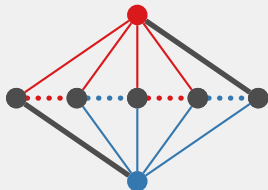
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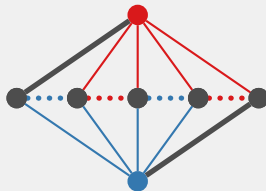
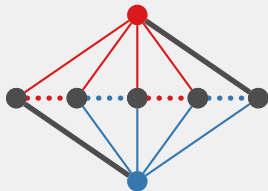
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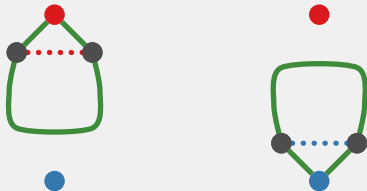
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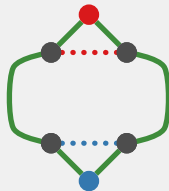
Theorem (Raines, Szaniszló 1999)

For $m \in \{4, 5\}$, if there is a packing of K_n with t m -cycles, then there is a packing of K_n with t m -cycles such that the numbers of cycles on any two vertices differ by at most one.

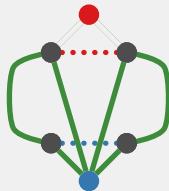
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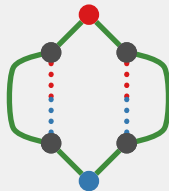
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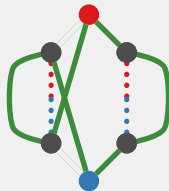
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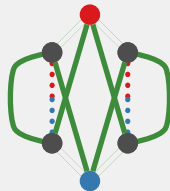
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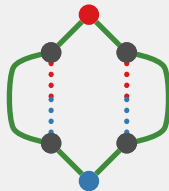
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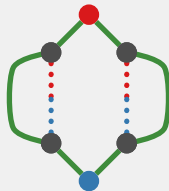
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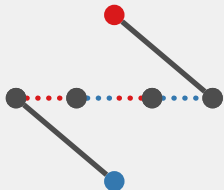
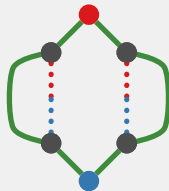
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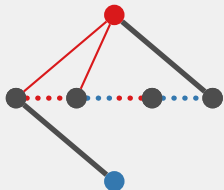
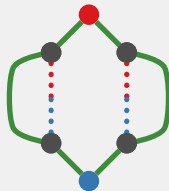
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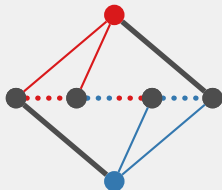
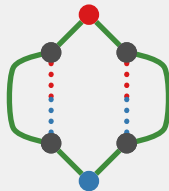
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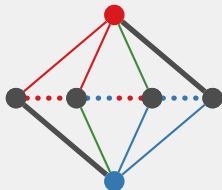
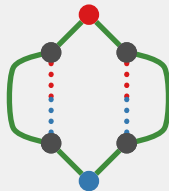
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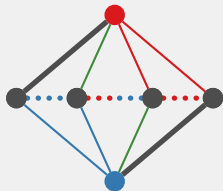
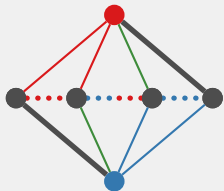
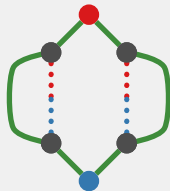
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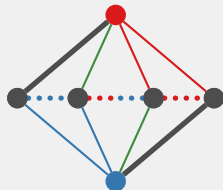
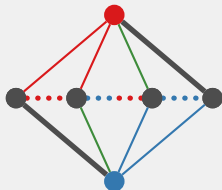
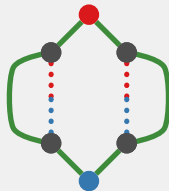
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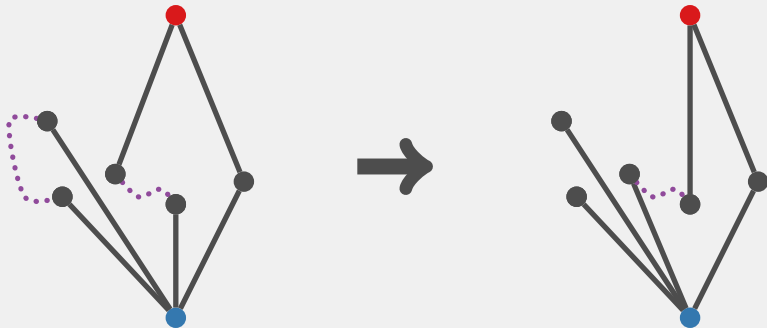
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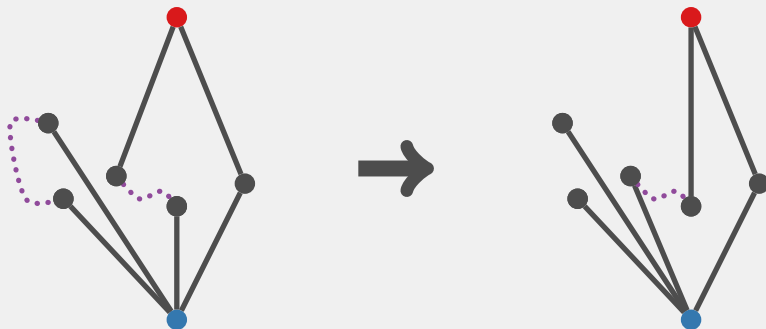
(Bryant, H., Maenhaut 2005)

Switching in cycle packings

Switching in cycle packings



Switching in cycle packings



this works for packings with cycles of arbitrary lengths

- and • must be twin vertices in the underlying graph
(for a packing of K_n this is trivial)

Using switching in cycle packings

Using switching in cycle packings

Equalising lemma (Bryant, H.)

$K_n \rightsquigarrow (m_1, m_2, \dots, m_t, x, y) \implies K_n \rightsquigarrow (m_1, m_2, \dots, m_t, x + 1, y - 1)$
when $x < y$ and $x + y \geq n + 2$.

Merging lemma (Bryant, H.)

$K_n \rightsquigarrow (m_1, m_2, \dots, m_t, c, x, y) \implies K_n \rightsquigarrow (m_1, m_2, \dots, m_t, c, x + y)$
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Reduction (Bryant, H.)

To solve Alspach's problem for K_n it suffices to solve it for lists of the form

$$3, 3, \dots, 3, 4, 4, \dots, 4, 5, 5, \dots, 5, k, n, n, \dots, n.$$

Theorem (Bryant, H., Pettersson 2014)

There is an (m_1, \dots, m_t) -decomposition of K_n if and only if

- (1) n is odd;
- (2) $n \geq m_1, \dots, m_t \geq 3$; and
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Switching-assisted cycle decomposition results

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Bryant (2010): Characterisation of when λK_n has a decomposition into paths of lengths m_1, \dots, m_t .

H. (2012): Partial results on when a complete multipartite graph has a decomposition into cycles of length m .

H., Hoyte (2016, 2017): Partial results on when $K_n - K_h$ has a decomposition into cycles of lengths m_1, \dots, m_t .

Asplund, Chaffee, Hammer (2017+): Partial results on when $\lambda K_{a,b}$ has a decomposition into cycles of lengths m_1, \dots, m_t .

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Note the underlying graphs in these results have large sets of pairwise twin vertices.

Part 3:

Almost regular decompositions

Regularising improper edge colourings

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In an improper edge colouring of K_n , we want to make the colour classes “as regular as possible” (without changing their sizes).

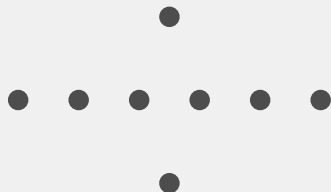
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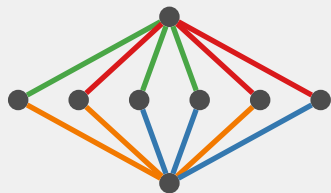
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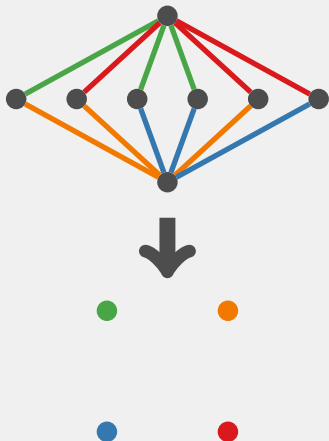
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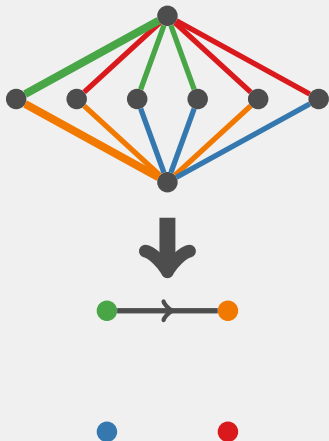
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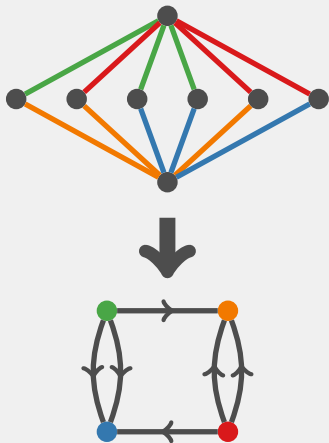
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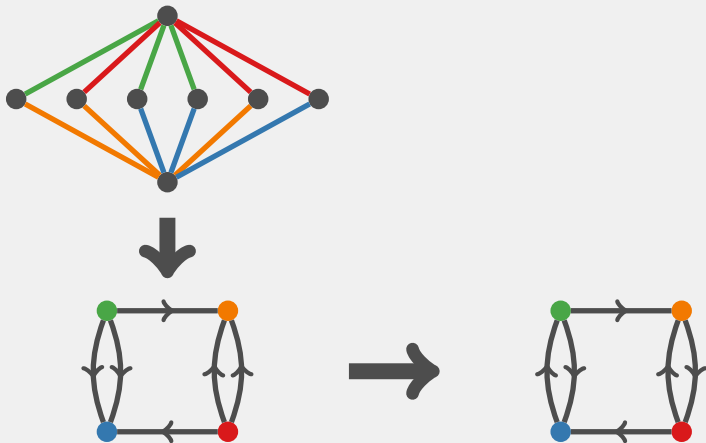
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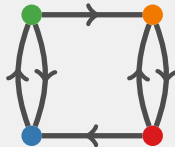
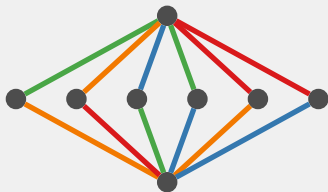
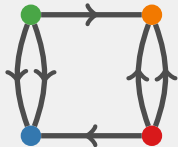
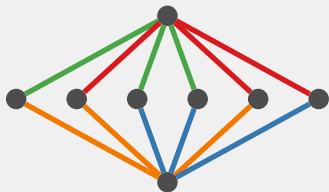
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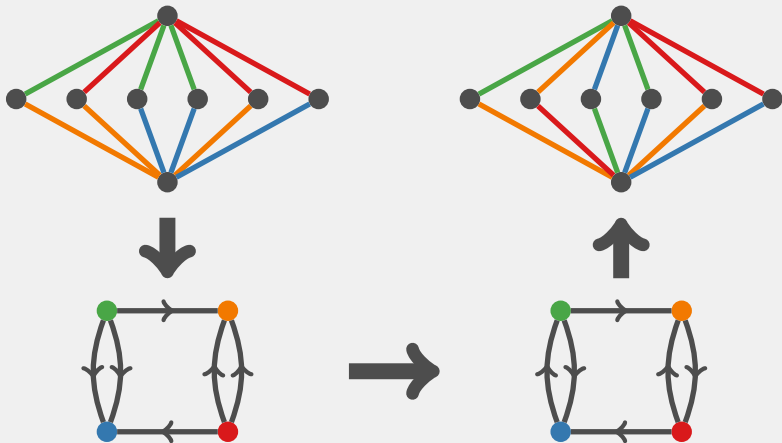
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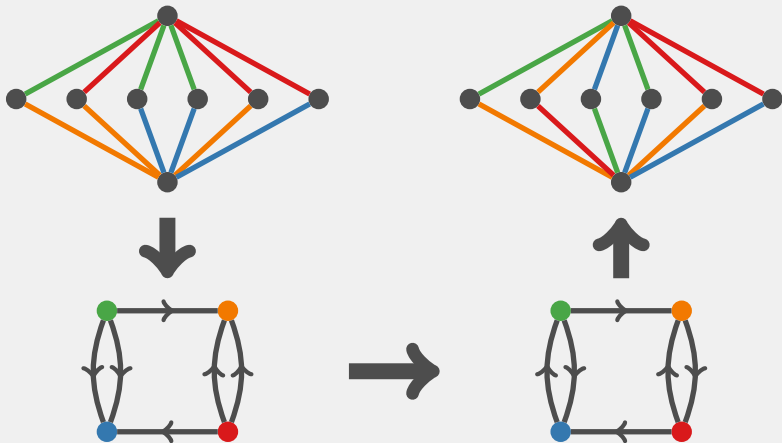
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(Bryant, Maenhaut 2008)

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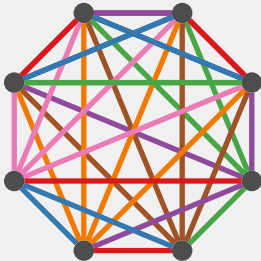


(Bryant, Maenhaut 2008)

Our previous switching techniques can also be viewed in this framework.

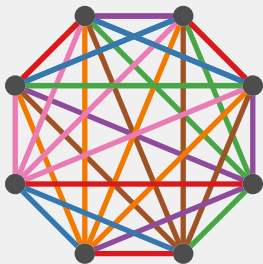
Applications

Applications

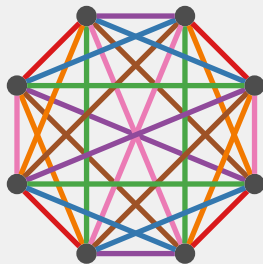


4 edges of each colour, arbitrary

Applications

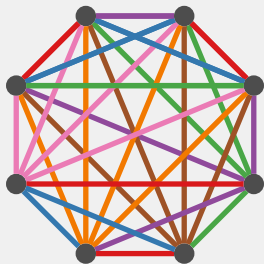


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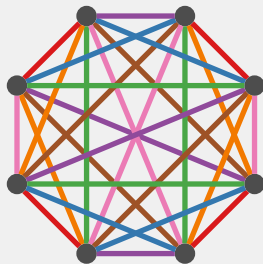


1-factorisation

Applications



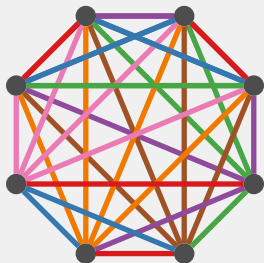
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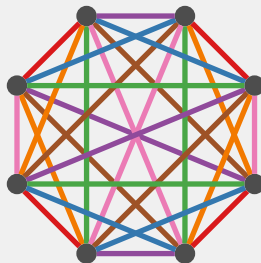
1-factorisation

This argument can be extended to give a neat proof of [Cruse \(1974\)](#) and [Andersen and Hilton \(1980\)](#) that characterise when an (improper) edge colouring of K_u can be extended to a k -factorisation of K_v .

Applications



4 edges of each colour, arbitrary



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[Bryant](#) recently extended these arguments to hypergraphs, where they give elegant proofs of many generalisations of Baranyai's theorem.

Future directions

Future directions

Can hypergraph switching be used in a more sophisticated way?

[Keevash](#) and [Barber, Csaba, Glock, Kühn, Lo, Osthus, Treglown](#) have recently obtained very strong results on edge decomposition of dense graphs. Can switching be usefully applied in this setting?

Can switching be usefully applied to fractional decomposition of graphs?

That's all