Switching techniques for edge decompositions of graphs

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Definitions

**Edge decomposition**

A set \(\{G_1, \ldots, G_t\}\) of subgraphs of a graph \(G\) such that \(\{E(G_1), \ldots, E(G_t)\}\) is a partition of \(E(G)\).

**Proper \(t\)-edge colouring**

A decomposition of a graph into \(t\) matchings.
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A decomposition of \( G \) into cycles and a matching
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A decomposition of \( G \) into matchings

![Graph diagram showing an edge decomposition of a graph into matchings](image-url)
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Definitions

Packing

A decomposition of $G$ into graphs $G_1, \ldots, G_t$ and a leave graph $L$.

Switching technique

A method for locally modifying edge decompositions that feels kind of switchy.
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A graph $G$
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Packing
A decomposition of $G$ into graphs $G_1, \ldots, G_t$ and a *leave* graph $L$.

A packing of $G$ with cycles

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A method for locally modifying edge decompositions that feels kind of *switchy*.
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Switching technique
A method for locally modifying edge decompositions that feels kind of switchy.
Warm up:
Proper edge colourings
Switching in proper edge colorings

Theorem

If a graph $G$ has a proper $t$-edge colouring, then it has a proper $t$-edge colouring such that the sizes of any two colour classes differ by at most one.

Theorem (Vizing 1964)

Every graph $G$ has a proper $(\Delta(G) + 1)$-edge colouring.
Switching in proper edge colorings

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Feels kind of switchy

Is reminiscent of the argument on the last slide.
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This talk is about applying these kinds of switching techniques to edge decompositions of graphs *other than* edge colourings.
Part 1:
Embedding partial Steiner triple systems
Steiner triple systems

**STS(v):** A decomposition of $K_v$ into triangles.

**PSTS(u):** A packing of $K_u$ with triangles.

Theorem (Kirkman 1847)

An STS($v$) exists if and only if $v \equiv 1, 3 \pmod{6}$.

Call such orders admissible.
Steiner triple systems

**STS(v):** A decomposition of $K_v$ into triangles.

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Steiner triple systems

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**PSTS(μ):** A packing of $K_μ$ with triangles.

An STS(7)

An STS(7)
Steiner triple systems

**STS(\(v\)):** A decomposition of \(K_v\) into triangles.

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An STS($v$) exists if and only if $v \equiv 1, 3 \pmod{6}$.

Call such orders *admissible*. 
Embedding PSTSs

Problem
Given a PSTS, find the smallest $v$ for which there is an STS $(v)$ containing the PSTS.
Embedding PSTSs

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An embedding of the PSTS(7) of order 9
History

Treash (1971): Every PSTS has an embedding (of order at most $2^u$).

Lindner (1975): Every PSTS has an embedding of order $6^u + 3$.

Conjecture (Lindner 1977): Every PSTS has an embedding of order $v$ for each admissible $v \geq 2^u + 1$.

Andersen, Hilton, Mendelsohn (1980): Every PSTS has an embedding of order $v$ for each admissible $v \geq 4^u + 1$.

Bryant (2004): Every PSTS has an embedding of order $v$ for each admissible $v \geq 3^u - 2$.


Work on embeddings of PTS $(v,\lambda)$s and quasigroup variants by Andersen, Colbourn, Hamm, Hao, Hoffman, Lindner, Mendelsohn, Raines, Rodger, Rosa, Stubbs, Wallis in the 1970s, 80s, 90s and 00s.
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Lemma (Andersen, Hilton, Mendelsohn 1980)

If there is a PSTS \((u)\) with \(t\) triangles, then there is a PSTS \((u)\) with \(t\) triangles such that the numbers of triangles on any two vertices differ by at most one.
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Lemma (Andersen, Hilton, Mendelsohn 1980)

If there is a PSTS \( (u) \) with \( t \) triangles, then there is a PSTS \( (u) \) with \( t \) triangles such that the numbers of triangles on any two vertices differ by at most one.
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Lemma (Andersen, Hilton, Mendelsohn 1980)
If there is a PSTS \((u)\) with \(t\) triangles, then there is a PSTS \((u')\) with \(t\) triangles such that the numbers of triangles on any two vertices differ by at most one.
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Lemma (Andersen, Hilton, Mendelsohn 1980)

If there is a PSTS($u$) with $t$ triangles, then there is a PSTS($u$) with $t$ triangles such that the numbers of triangles on any two vertices differ by at most one.
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Adding a triangle to a packing

Say we wish to add another triangle to a partial embedding. The leave of our packing has no triangles, but does contain a "lasso" on the new vertices.
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Say we wish to add another triangle to a partial embedding. The leave of our packing has no triangles, but does contain a “lasso” on the new vertices.
Theorem (Bryant, H. 2009)

Every PSTS \((u)\) has an embedding of order \(v\) for each admissible \(v \geq 2u + 1\).
Theorem (Bryant, H. 2009)

Every PSTS($u$) has an embedding of order $v$ for each admissible $v \geq 2u + 1$. 
More switching-assisted results on embeddings

Bryant, Buchanan (2007): Every partial totally symmetric quasigroup of order $u$ has an embedding of order $v$ for each even $v \geq 2u + 4$.

Bryant, Martin (2012): For $u \geq 28$, every PTS($u, \lambda$) has an embedding triple of order $v$ for each admissible $v \geq 2u + 1$.

Martin, McCourt (2012): Any partial 5-cycle system of order $u \geq 255$ has an embedding of order at most $\frac{1}{4}(9u + 146)$.

H. (2014): “Half” of the possible embeddings of order less than $2u + 1$ for PSTS($u$)s with $\Delta(L) \leq \frac{1}{4}(u - 9)$ and $|E(L)| < \frac{1}{32}(u - 5)(u - 11) + 2$ exist.

H. (2014): Any PSTS($u$) with at most $\frac{1}{50}u^2 + o(u)$ triples has an embedding for each admissible order $v \geq \frac{1}{5}(8u + 17)$. 
Part 2:
Cycle decompositions
Cycle decompositions of complete graphs

There is a decomposition of $K_n$ into cycles of lengths $m_1, \ldots, m_t$. 

$K_7 \Rightarrow 7, 6, 4, 4$
Cycle decompositions of complete graphs

\[ K_n \leadsto m_1, \ldots, m_t \]

“There is a decomposition of \( K_n \) into cycles of lengths \( m_1, \ldots, m_t \).”
Cycle decompositions of complete graphs

\[ K_n \sim m_1, \ldots, m_t \]

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“There is a decomposition of $K_n$ into cycles of lengths $m_1, \ldots, m_t$.”

$K_7 \leadsto 7, 6, 4, 4$
If \( K_n \Rightarrow m_1, \ldots, m_t \), then
\begin{enumerate}
\item \( n \) is odd;
\item \( n \geq m_1, \ldots, m_t \geq 3 \); and
\item \( m_1 + \cdots + m_t = \left( \frac{n^2}{2} \right) \).
\end{enumerate}

Alspach's cycle decomposition problem (1981)

Prove (1), (2) and (3) are sufficient for \( K_n \Rightarrow m_1, \ldots, m_t \).
If $K_n \sim m_1, \ldots, m_t$ then

(1) $n$ is odd;

(2) $n \geq m_1, \ldots, m_t \geq 3$; and

(3) $m_1 + \cdots + m_t = \binom{n}{2}$. 
If $K_n \rightarrow m_1, \ldots, m_t$ then

(1) $n$ is odd;

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Alspach’s cycle decomposition problem (1981)

Prove (1), (2) and (3) are sufficient for $K_n \rightarrow m_1, \ldots, m_t$. 
History (highlights)

When does $K_n \Rightarrow m, \ldots, m$?

▶ Kirkman and Walecki solved special cases in the 1800s.
▶ Results from Kotzig, Rosa, Huang in the 1960s.
▶ Reductions of the problem from Bermond, Huang, Sotteau and from Hoffman, Lindner, Rodger in the 1980s and 90s.

When does $K_n \Rightarrow m_1, \ldots, m_t$?

▶ Work on limited sets of cycle lengths from Adams, Bryant, Heinrich, Horák, Khodkar, Maehaut, Rosa in the 1980s, 90s and 00s.
▶ A more general result from Balister in 2001.
▶ Solved by Bryant, H., Pettersson in 2014.
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**Theorem (Raines, Szaniszló 1999)**

For $m \in \{4, 5\}$, if there is a packing of $K_n$ with $t$ $m$-cycles, then there is a packing of $K_n$ with $t$ $m$-cycles such that the numbers of cycles on any two vertices differ by at most one.
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- This works for packings with cycles of arbitrary lengths.
- They must be twin vertices in the underlying graph. (For a packing of $K_n$, this is trivial.)
Switching in cycle packings

this works for packings with cycles of arbitrary lengths
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(for a packing of $K_n$ this is trivial)
Using switching in cycle packings

Equalising lemma (Bryant, H.)

\[ K^n \Rightarrow (m_1, m_2, \ldots, m_t, x, y) = K^n \Rightarrow (m_1, m_2, \ldots, m_t, x + 1, y - 1) \]
when \( x < y \) and \( x + y \geq n + 2 \).

Merging lemma (Bryant, H.)

\[ K^n \Rightarrow (m_1, m_2, \ldots, m_t, c, x, y) = K^n \Rightarrow (m_1, m_2, \ldots, m_t, c, x + y) \]
when \( c \geq \frac{1}{2}(x + y) \) and \( x + y + c \leq n + 1 \).

Reduction (Bryant, H.)
To solve Alspach's problem for \( K^n \) it suffices to solve it for lists of the form
\[ 3, 3, \ldots, 3, 4, 4, \ldots, 4, 5, 5, \ldots, 5, k, n, n, \ldots, n. \]
Using switching in cycle packings

**Equalising lemma** (Bryant, H.)

\[ K_n \sim (m_1, m_2, \ldots, m_t, x, y) \implies K_n \sim (m_1, m_2, \ldots, m_t, x + 1, y - 1) \]

when \( x < y \) and \( x + y \geq n + 2 \).

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**Using switching in cycle packings**

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\[ K_n \rightsquigarrow (m_1, m_2, \ldots, m_t, c, x, y) \implies K_n \rightsquigarrow (m_1, m_2, \ldots, m_t, c, x + y) \]

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**Reduction** (Bryant, H.)

To solve Alspach’s problem for \( K_n \) it suffices to solve it for lists of the form

\[ 3,3,\ldots,3,4,4,\ldots,4,5,5,\ldots,5,k,n,n,\ldots,n. \]
Theorem (Bryant, H., Pettersson 2014)

There is an \((m_1, \ldots, m_t)\)-decomposition of \(K_n\) if and only if

1. \(n\) is odd;
2. \(n \geq m_1, \ldots, m_t \geq 3\); and
3. \(m_1 + \cdots + m_t = \binom{n}{2}\).
Switching-assisted cycle decomposition results

- Bryant (2010): Characterisation of when $\lambda K_n$ has a decomposition into paths of lengths $m_1, \ldots, m_t$.
- H. (2012): Partial results on when a complete multipartite graph has a decomposition into cycles of length $m$.
- H., Hoyte (2016, 2017): Partial results on when $K_n - K_h$ has a decomposition into cycles of lengths $m_1, \ldots, m_t$.
- Asplund, Chaffee, Hammer (2017+): Partial results on when $\lambda K_a, b$ has a decomposition into cycles of lengths $m_1, \ldots, m_t$.
- Bryant, H., Maenhaut, Smith (2017+): Characterisation of when $\lambda K_n$ has a decomposition into cycles of lengths $m_1, \ldots, m_t$.
- Hoyte (2017+): Characterisation of when $\lambda K_n$ has a packing with cycles of lengths $m_1, \ldots, m_t$.

Note the underlying graphs in these results have large sets of pairwise twin vertices.
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**Hoyte** (2017+): Characterisation of when $\lambda K_n$ has a packing with cycles of lengths $m_1, \ldots, m_t$.

Note the underlying graphs in these results have large sets of pairwise twin vertices.
Part 3:
Almost regular decompositions
Regularising improper edge colourings
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In an improper edge colouring of $K_n$, we want to make the colour classes “as regular as possible” (without changing their sizes).
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Regularising improper edge colourings

In an improper edge colouring of $K_n$, we want to make the colour classes “as regular as possible” (without changing their sizes).

\[ \text{Diagram of } K_n \]
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Our previous switching techniques can also be viewed in this framework.
Applications

4 edges of each colour, arbitrary

This argument can be extended to give a neat proof of Cruse (1974) and Andersen and Hilton (1980) that characterise when an (improper) edge colouring of $K_u$ can be extended to a $k$-factorisation of $K_v$.

Bryant recently extended these arguments to hypergraphs, where they give elegant proofs of many generalisations of Baranyai's theorem.
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Future directions

Can hypergraph switching be used in a more sophisticated way?
Keevash and Barber, Csaba, Glock, Kuhn, Lo, Osthus, Treglown have recently obtained very strong results on edge decomposition of dense graphs. Can switching be usefully applied in this setting?

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That’s all