Alspach's problem

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Cycle Decompositions of Complete Graphs

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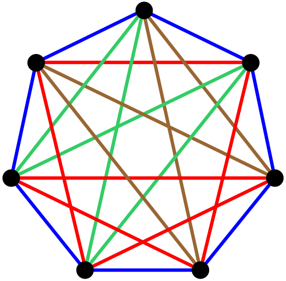

$(4, 4, 6, 7)$-decomposition of K_7

If there exists an $(m_1, m_2, \ldots, m_t)$-decomposition of K_n then

(1) n is odd;

(2) $3 \leq m_1, m_2, \ldots, m_t \leq n$; and

(3) $m_1 + m_2 + \cdots + m_t = |E(K_n)|$.

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Alspach's Cycle Decomposition Problem (1981): Prove (1), (2) and (3) are also sufficient for an $(m_1, m_2, \ldots, m_t)$-decomposition of K_n.

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Alspach also posed the equivalent problem for $K_n - I$ when n is even.

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If a list satisfies (2) and (3) for some odd n we call it *n-admissible*.

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- The cycles can have as many as n vertices.
- We have to find a very large number of decompositions for each value of n

Existing Results

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There exists a $(m_1, m_2, \ldots, m_t)$-decomposition of K_n when the obvious necessary conditions and any of the following hold.

- $m_1 = m_2 = \cdots = m_t$ — Alspach, Gavlas, Šajna
- each $m_i \in \{3, 4, 5\}$ — Balister
- each $m_i \in \{3, n\}$ — Bryant, Maenhaut
- each $m_i \in \{2^k, 2^{k+1}\}$ for some $k \geq 2$ — Heinrich, Horák, Rosa
- n is large and each $m_i \leq \lfloor \frac{n-112}{20} \rfloor$ — Balister
- etc.

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Packings and Leaves

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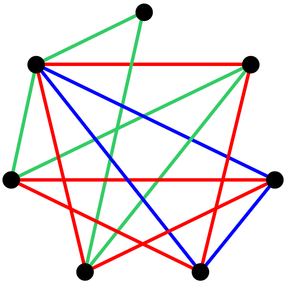

$(6, 5, 3)-$packing of K_7

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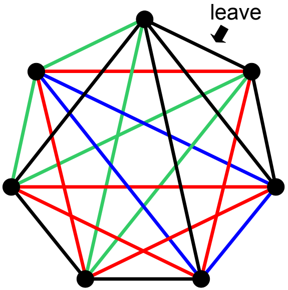

$(6, 5, 3)-$packing of K_7

Equalise Cycles Lemma

Equalise Cycles Lemma

Suppose there exists an $(m_1, m_2, \ldots, m_t, x, y)$-decomposition of K_n where $x < y$ and $x + y \geq n + 2$. Then there exists an $(m_1, m_2, \ldots, m_t, x + 1, y - 1)$-decomposition of K_n.

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Example:

$$
\begin{aligned}
& (m_1, m_2, \ldots, m_t, 10, 15)\text{-decomposition of } K_{23} \\
\Rightarrow \quad & (m_1, m_2, \ldots, m_t, 11, 14)\text{-decomposition of } K_{23} \\
\Rightarrow \quad & (m_1, m_2, \ldots, m_t, 12, 13)\text{-decomposition of } K_{23}
\end{aligned}
$$

Join Cycles Lemma

Join Cycles Lemma

Suppose there exists an $(m_1, m_2, \ldots, m_t, c, x, y)$-decomposition of K_n where

(1) $c \geq \frac{1}{2}(x + y)$; and

(2) $x + y + c \leq n + 1$.

Then there exists an $(m_1, m_2, \ldots, m_t, c, x + y)$-decomposition of K_n.

Join Cycles Lemma

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Then there exists an $(m_1, m_2, \ldots, m_t, c, x + y)$-decomposition of K_n.

Example: $(m_1, m_2, \ldots, m_t, 10, 7, 6)$-decomposition of K_{23}

$\Rightarrow$ $(m_1, m_2, \ldots, m_t, 10, 13)$-decomposition of K_{23}

A Reduction of Alspach's Problem

$m_1, m_2, \ldots, m_t, x, y$

$3 < x \leq y < n, \;\; x + y \geq n + 2$

A Reduction of Alspach's Problem

$$m_1, m_2, \ldots, m_t, x, y$$
$$\Uparrow$$
$$m_1, m_2, \ldots, m_t, x-1, y+1$$

$3 < x \leq y < n, \;\; x + y \geq n + 2$

A Reduction of Alspach's Problem

$$m_1, m_2, \ldots, m_t, x, y \qquad 3 < x \leq y < n, \;\; x + y \geq n + 2$$

$$\Uparrow$$

$$m_1, m_2, \ldots, m_t, x - 1, y + 1$$

$$\Uparrow$$

$$\vdots$$

$$\Uparrow$$

$$m_1, m_2, \ldots, m_{s-1}, m_s, c, n, n, \ldots, n \qquad m_i \leq c, \;\; m_i + c \leq n + 1$$

A Reduction of Alspach's Problem

$$m_1, m_2, \ldots, m_t, x, y \qquad 3 < x \leq y < n, \ \ x + y \geq n + 2$$

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$$\vdots$$

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$$m_1, m_2, \ldots, m_{s-1}, m_s, c, n, n, \ldots, n \qquad m_i \leq c, \ \ m_i + c \leq n + 1$$

$$\Uparrow$$

$$m_1, m_2, \ldots, m_{s-1}, 3, m_s - 3, c, n, n, \ldots, n \qquad (\text{if } m_s \notin \{3, 4, 5\})$$

A Reduction of Alspach's Problem

$$m_1, m_2, \ldots, m_t, x, y \qquad 3 < x \leq y < n, \;\; x + y \geq n + 2$$

$$\Uparrow$$

$$m_1, m_2, \ldots, m_t, x - 1, y + 1$$

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$$\vdots$$

$$\Uparrow$$

$$m_1, m_2, \ldots, m_{s-1}, m_s, c, n, n, \ldots, n \qquad m_i \leq c, \;\; m_i + c \leq n + 1$$

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$$m_1, m_2, \ldots, m_{s-1}, 3, m_s - 3, c, n, n, \ldots, n \qquad (\text{if } m_s \notin \{3, 4, 5\})$$

$$\Uparrow$$

$$\vdots$$

$$\Uparrow$$

$$3, 3, \ldots, 3, 4, 4, \ldots, 4, 5, 5, \ldots, 5, c, n, n, \ldots, n$$

A Reduction of Alspach's Problem

n-ancestor lists: n-admissible lists of the form

$$3, 3, \dots, 3, 4, 4, \dots, 4, 5, 5, \dots, 5, k, n, n, \dots, n$$

Theorem: If an $(m'_1, m'_2, \dots, m'_t)$-decomposition of K_n exists for each n-ancestor list $m'_1, m'_2, \dots, m'_t$, then an $(m_1, m_2, \dots, m_t)$-decomposition of K_n exists for each n-admissible list $m_1, m_2, \dots, m_t$.

More Reduction

Lemma: Suppose there exists an $(m_1, m_2, \ldots, m_t, 3, 3, 4)$-decomposition of K_n where $m_1, m_2, \ldots, m_t$ contains at least $\frac{n-9}{3}$ 3s. Then there exists an $(m_1, m_2, \ldots, m_t, 5, 5)$-decomposition of K_n.

Lemma: Suppose there exists an $(m_1, m_2, \ldots, m_t, 3, 4, 4, 4)$-decomposition of K_n where $m_1, m_2, \ldots, m_t$ contains at least $\frac{n-5}{2}$ 4s. Then there exists an $(m_1, m_2, \ldots, m_t, 5, 5, 5)$-decomposition of K_n.

More Reduction

Lemma: Suppose there exists an $(m_1, m_2, \ldots, m_t, 3, 3, 4)$-decomposition of K_n where $m_1, m_2, \ldots, m_t$ contains at least $\frac{n-9}{3}$ 3s. Then there exists an $(m_1, m_2, \ldots, m_t, 5, 5)$-decomposition of K_n.

Lemma: Suppose there exists an $(m_1, m_2, \ldots, m_t, 3, 4, 4, 4)$-decomposition of K_n where $m_1, m_2, \ldots, m_t$ contains at least $\frac{n-5}{2}$ 4s. Then there exists an $(m_1, m_2, \ldots, m_t, 5, 5, 5)$-decomposition of K_n.

n-ancestor lists: n-admissible lists of the form

$$3, 3, \ldots, 3, 4, 4, \ldots, 4, 5, 5, \ldots, 5, k, n, n, \ldots, n$$

- if there are at least three 5s, then there are fewer than $\frac{n-9}{3}$ 3s and fewer than $\frac{n-5}{2}$ 4s.

Our Previous Results

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There exists a $(m_1, m_2, \ldots, m_t)$-decomposition of K_n or $K_n - I$ when the obvious necessary conditions and any of the following hold.

- each $m_i \geq \frac{n+5}{2}$ Bryant, Horsley
- each $m_i \leq \frac{n-1}{2}$ and $m_t \leq 2m_{t-1}$ Bryant, Horsley
- n is odd and sufficiently large Bryant, Horsley

An example

An example

$K_{55} \rightsquigarrow 220 \times C_3, 110 \times C_4, 7 \times C_{55}$

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$K_{55} \rightsquigarrow 220 \times C_3, 110 \times C_4, 7 \times C_{55}$
$K_{55} \rightsquigarrow (4 \times 55) \times C_3, (2 \times 55) \times C_4, 7 \times C_{55}$

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$K_{55} \rightsquigarrow 220 \times C_3, 110 \times C_4, 7 \times C_{55}$
$K_{55} \rightsquigarrow (4 \times 55) \times C_3, (2 \times 55) \times C_4, 7 \times C_{55}$

$\{1, \ldots, 12\} \rightsquigarrow \{1, 9, 10\}, \{2, 4, 6\}, \{3, 8, 11\}, \{5, 7, 12\}$
$\langle \{1, \ldots, 12\} \rangle_{55} \rightsquigarrow (4 \times 55) \times C_3$

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$\{13, \ldots, 20\} \rightsquigarrow \{13, 14, 15, 16\}, \{17, 18, 19, 20\}$
$\langle\{13, \ldots, 20\}\rangle_{55} \rightsquigarrow (2 \times 55) \times C_4$

An example

Lemma: $\langle\{a, b\}\rangle_n \rightsquigarrow 2 \times C_n$ if and only if $\gcd(a, b, n) = 1$.

Lemma: $\langle\{a, b, c\}\rangle_n \rightsquigarrow 3 \times C_n$ if $\gcd(a, n) = 1$.

$K_{55} \rightsquigarrow 220 \times C_3, 110 \times C_4, 7 \times C_{55}$
$K_{55} \rightsquigarrow (4 \times 55) \times C_3, (2 \times 55) \times C_4, 7 \times C_{55}$

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$\{13, \ldots, 20\} \rightsquigarrow \{13, 14, 15, 16\}, \{17, 18, 19, 20\}$
$\langle\{13, \ldots, 20\}\rangle_{55} \rightsquigarrow (2 \times 55) \times C_4$

$\{21, \ldots, 27\} \rightsquigarrow \{21, 22\}, \{23, 24\}, \{25, 26, 27\}$
$\langle\{21, \ldots, 27\}\rangle_{55} \rightsquigarrow 7 \times C_{55}$

Another example

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$$K_{123} \rightsquigarrow 30 \times C_3, 39 \times C_4, 1235 \times C_5, 1 \times C_{98}, 8 \times C_{123}$$

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$$K_{123} \rightsquigarrow 30 \times C_3, 39 \times C_4, (10 \times 123 + 5) \times C_5, 1 \times C_{98}, 8 \times C_{123}$$

Another example

Lemma: $\langle\{1,2,3\}\rangle_n \rightsquigarrow m_1, m_2, \ldots, m_t, k$ if each $m_i \in \{3,4,5\}$, $3 \leq k \leq n$ and $m_1 + m_2 + \cdots + m_t + k = 3n$.

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$\langle\{1,2,3\}\rangle_{123} \rightsquigarrow 30 \times C_3, 39 \times C_4, 5 \times C_5, 1 \times C_{98}$

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$\langle\{1,2,3\}\rangle_{123} \rightsquigarrow 30 \times C_3, 39 \times C_4, 5 \times C_5, 1 \times C_{98}$

$\{4, \ldots, 53\}$ has an odd sum
$\langle\{4, \ldots, 52\} \cup \{54\}\rangle_{123} \rightsquigarrow (10 \times 123) \times C_5$

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$\{4,\ldots,53\}$ has an odd sum
$\langle\{4,\ldots,52\} \cup \{54\}\rangle_{123} \rightsquigarrow (10 \times 123) \times C_5$

$\{53\} \cup \{55,\ldots,61\} \rightsquigarrow \{53,61\}, \{55,56\}, \{57,58\}, \{59,60\}$
$\langle\{53\} \cup \{55,\ldots,61\}\rangle_{123} \rightsquigarrow 8 \times C_{123}$

Another example

Lemma: $\langle\{1,2,3\}\rangle_n \rightsquigarrow m_1, m_2, \dots, m_t, k$ if each $m_i \in \{3,4,5\}$, $3 \le k \le n$ and $m_1 + m_2 + \cdots + m_t + k = 3n$.

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$\{53\} \cup \{55, \dots, 61\} \rightsquigarrow \{53, 61\}, \{55, 56\}, \{57, 58\}, \{59, 60\}$
$\langle\{53\} \cup \{55, \dots, 61\}\rangle_{123} \rightsquigarrow 8 \times C_{123}$

A non-example

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$K_{135} \rightsquigarrow 2517 \times C_3, 322 \times C_4, 1 \times C_{71}, 1 \times C_{135}$

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$K_{135} \rightsquigarrow 2517 \times C_3, 322 \times C_4, 1 \times C_{71}, 1 \times C_{135}$
$K_{55} \rightsquigarrow (18 \times 135 + 87) \times C_3, (2 \times 135 + 52) \times C_4, 1 \times C_{71}, 1 \times C_{135}$

A non-example

Lemma: $\langle\{1,2,3,4\}\rangle_n \rightsquigarrow m_1, m_2, \ldots, m_t, k$ if each $m_i \in \{3,4,5\}$, $3 \le k \le n$ and $m_1 + m_2 + \cdots + m_t + k = 4n$.

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$\langle\{1,2,3,4\}\rangle_{135} \rightsquigarrow 87 \times C_3, 52 \times C_4, 1 \times C_{71}$

A non-example

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$\{5, \ldots, 58\}$ has an odd sum
$\langle\{5, \ldots, 57\} \cup \{59\}\rangle_{135} \rightsquigarrow (18 \times 135) \times C_3$

A non-example

Lemma: $\langle\{1,2,3,4\}\rangle_n \rightsquigarrow m_1, m_2, \ldots, m_t, k$ if each $m_i \in \{3,4,5\}$, $3 \leq k \leq n$ and $m_1 + m_2 + \cdots + m_t + k = 4n$.

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$\{5, \ldots, 58\}$ has an odd sum
$\langle\{5, \ldots, 57\} \cup \{59\}\rangle_{135} \rightsquigarrow (18 \times 135) \times C_3$

$\{58\} \cup \{60, \ldots, 65\} \cup \{67\} \rightsquigarrow \{58, 60, 61, 63\}, \{62, 64, 65, 67\}$
$\langle\{58\} \cup \{60, \ldots, 65\} \cup \{67\}\rangle_{135} \rightsquigarrow (2 \times 135) \times C_4$

A non-example

Lemma: $\langle\{1,2,3,4\}\rangle_n \rightsquigarrow m_1, m_2, \ldots, m_t, k$ if each $m_i \in \{3,4,5\}$, $3 \leq k \leq n$ and $m_1 + m_2 + \cdots + m_t + k = 4n$.

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$\langle\{58\} \cup \{60, \ldots, 65\} \cup \{67\}\rangle_{135} \rightsquigarrow (2 \times 135) \times C_4$

$\langle\{66\}\rangle_{135}$ is not a hamilton cycle, $\gcd(66, 135) = 3$.

Main Result

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Theorem (we hope): There is a decomposition of K_n into cycles of lengths $m_1, m_2, \ldots, m_t$ if and only if n is odd, $3 \leq m_i \leq n$ for $i = 1, 2, \ldots, t$, and $m_1 + m_2 + \cdots m_t = \frac{n(n-1)}{2}$.

Theorem (we hope): There is a decomposition of $K_n - I$ into cycles of lengths $m_1, m_2, \ldots, m_t$ if and only if n is even, $3 \leq m_i \leq n$ for $i = 1, 2, \ldots, t$, and $m_1 + m_2 + \cdots m_t = \frac{n(n-2)}{2}$.

The End

Few hamilton cycles example

Few hamilton cycles example

Lemma: Let n be odd, and let $m_1, m_2, \ldots, m_t$ be an n-ancestor list with exactly one n at least $\frac{n-3}{2}$ 4s. If Alspach's conjecture holds for order $n-2$, then there is an $(m_1, m_2, \ldots, m_t)$-decomposition of K_n.