An infinite family of Steiner triple systems without parallel classes

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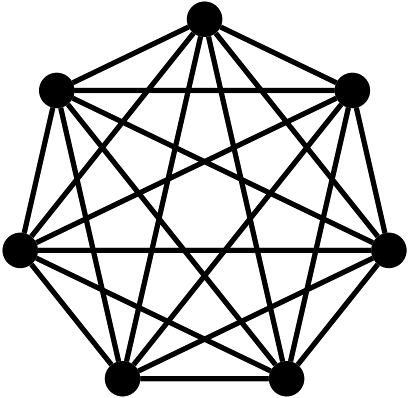
Joint work with Darryn Bryant (University of Queensland)

Part 1:

Steiner triple systems and parallel classes

Steiner triple systems

Steiner triple systems



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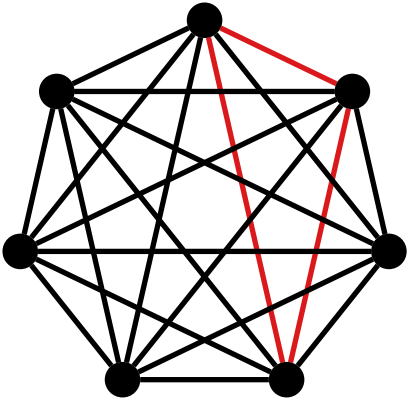

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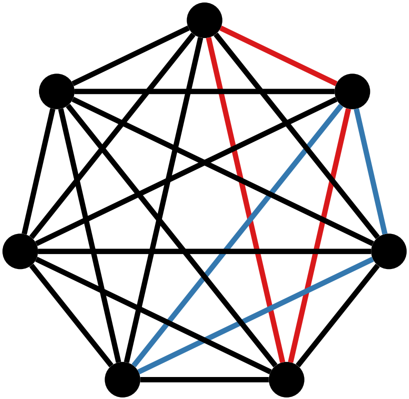

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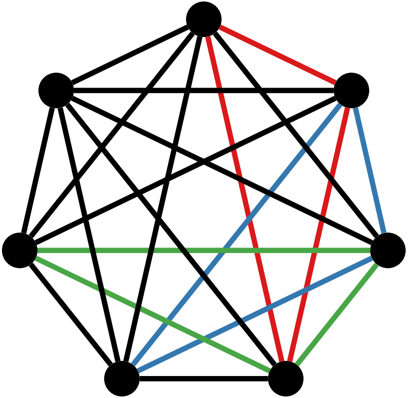

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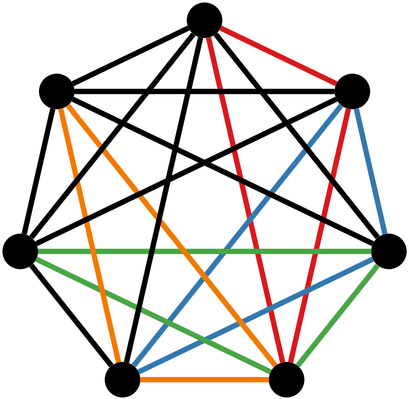

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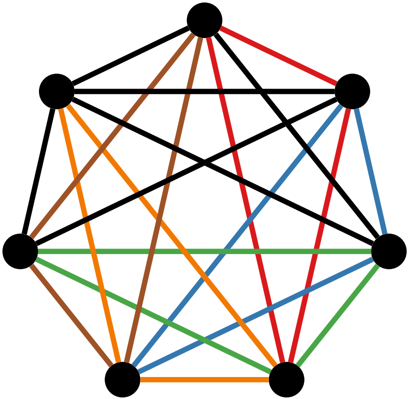

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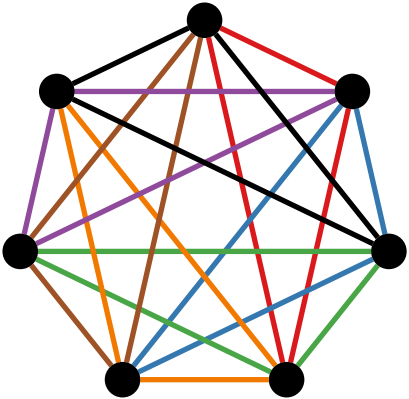

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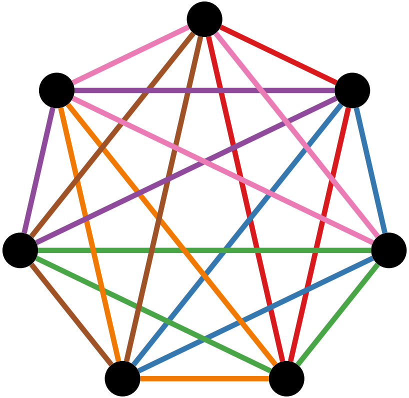

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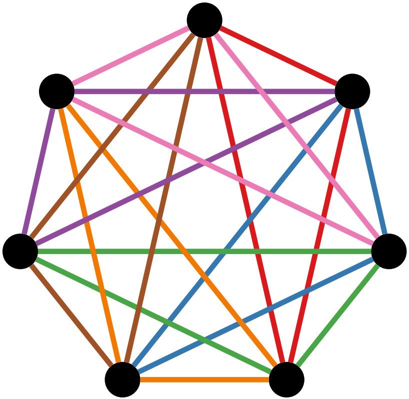
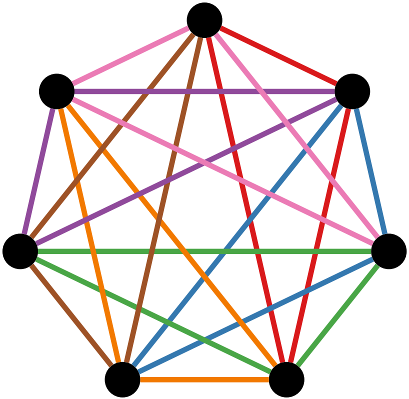


An STS(7)

Steiner triple systems



An STS(7)

Theorem (Kirkman 1847) An STS(v) exists if and only if $v \geq 1$ and $v \equiv 1$ or $3 \pmod 6$.

Parallel classes

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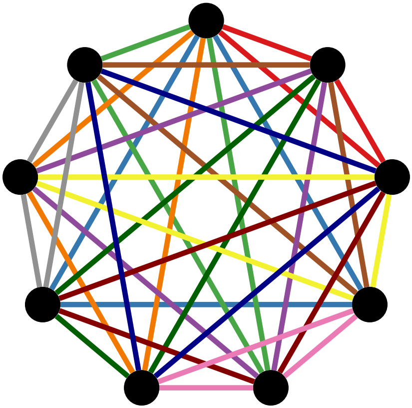

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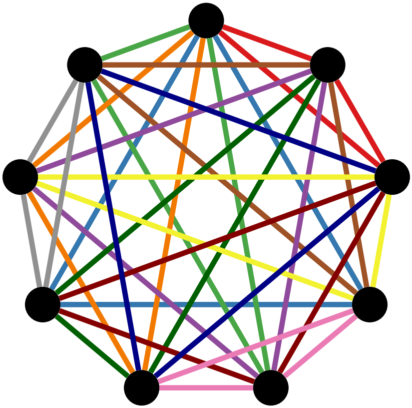

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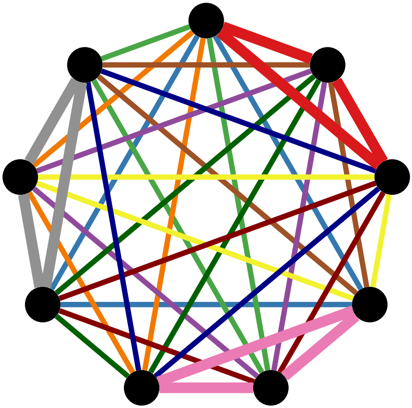

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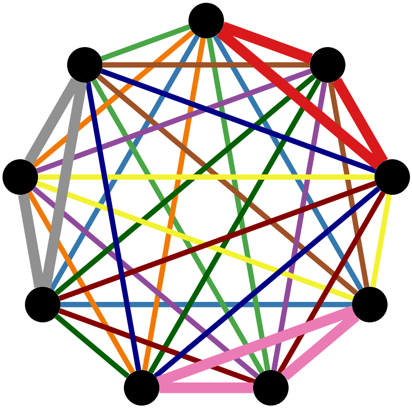

An STS(9) with a PC

Almost parallel classes

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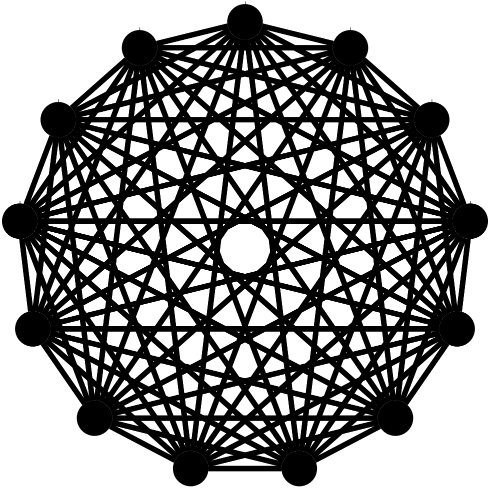
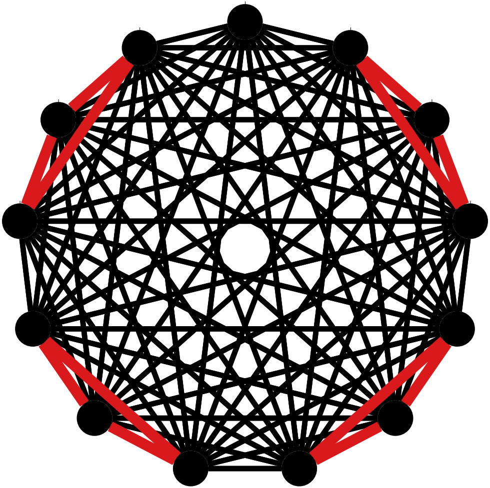


An STS(13)

Almost parallel classes



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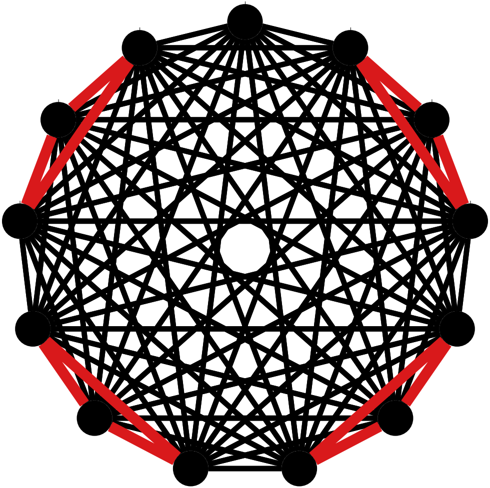

An STS(13) with an APC

A question

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Question What can we say about when an STS(v) has a PC/APC?

If $v \equiv 3 \pmod 6$, the STS(v) might have a PC.
If $v \equiv 1 \pmod 6$, the STS(v) might have an APC.

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STSs without PCs/APCs seem rare.

Conjectures

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Conjecture (Mathon, Rosa) There is an STS(v) with no PC for all $v \equiv 3 \pmod 6$ except $v = 3, 9$.

Conjecture (Rosa, Colbourn) There is an STS(v) with no APC for all $v \equiv 1 \pmod 6$ except $v = 13$.

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Theorem (Bryant, Horsley, 201?) For each $v \equiv 27 \pmod{30}$ such that $\mathrm{ord}_p(-2) \equiv 0 \pmod{4}$ for every prime divisor p of $v - 2$, there is an STS(v) with no PC. There are infinitely many such values of v.

Part 2:

Our result

Construction

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- Let $v = 5n + 2$ and $G = \mathbb{Z}_5 \times \mathbb{Z}_n$ (remember $v \equiv 27 \pmod{30}$). Note $n \equiv 5 \pmod{6}$.

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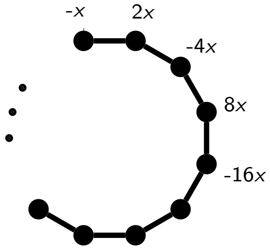

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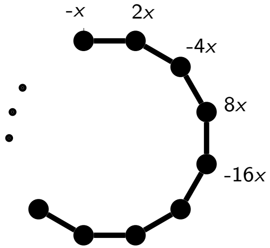

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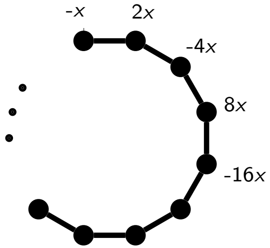

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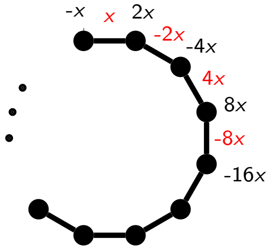

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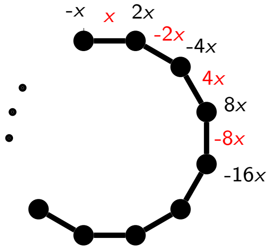

- Every point in $G \setminus \{(0, 0)\}$ is in one such cycle.
- Consider the weights of these edges.
- For each $g \in G \setminus \{(0, 0)\}$ there is exactly one unused edge of weight g.

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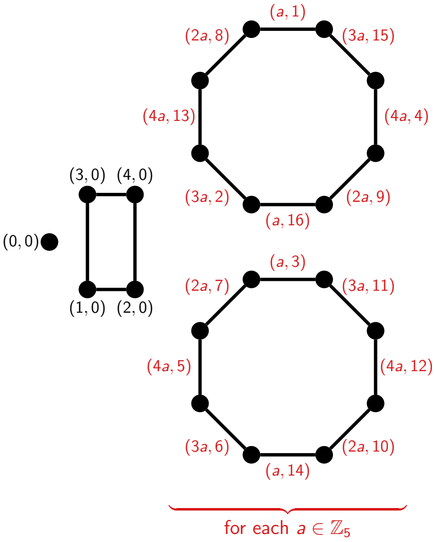

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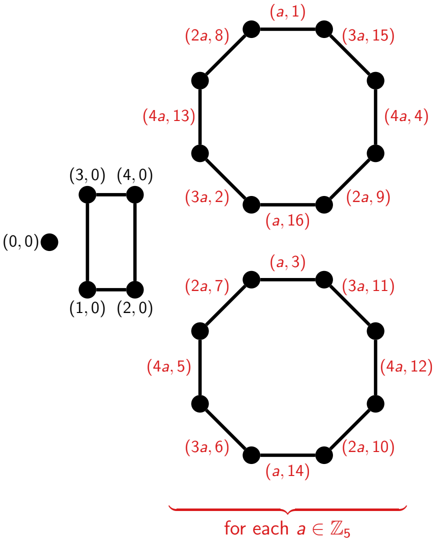

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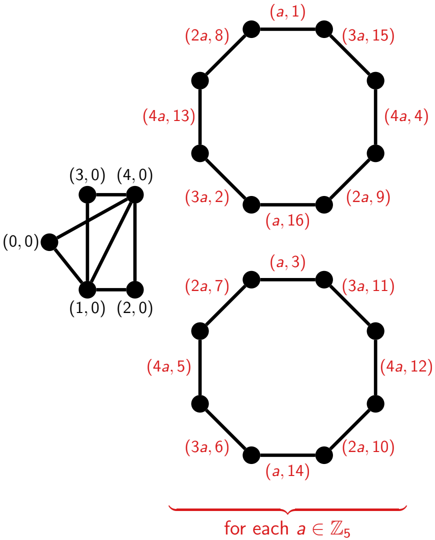

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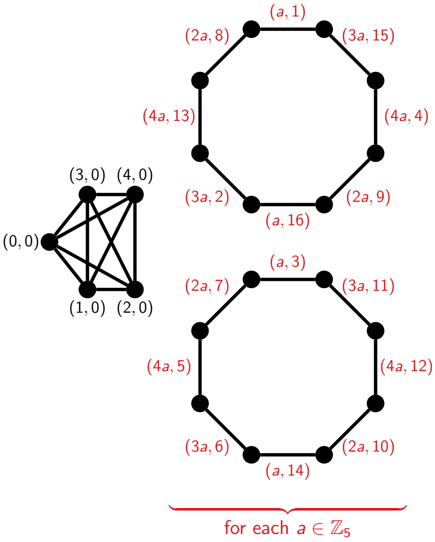

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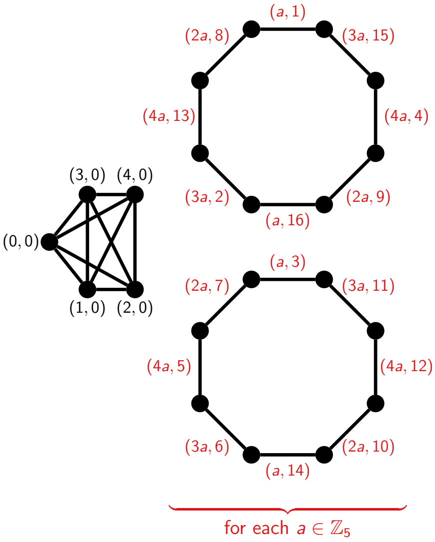

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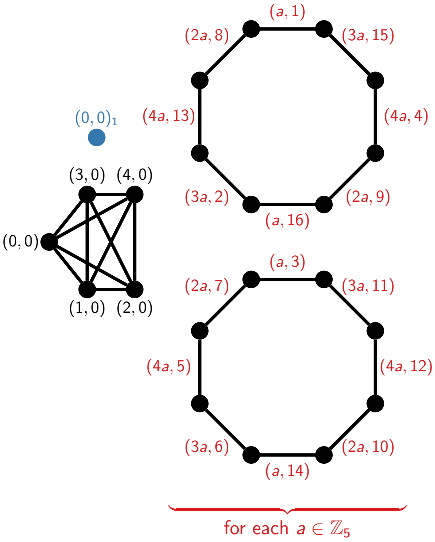

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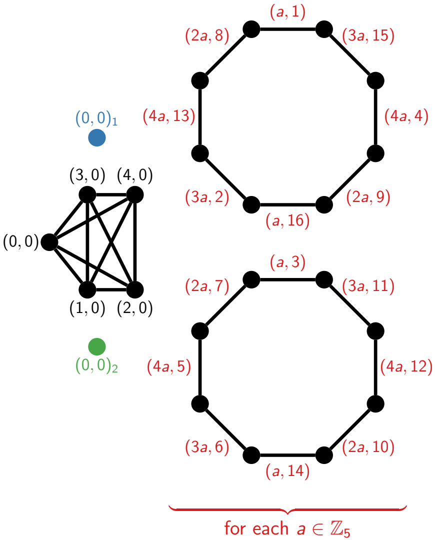

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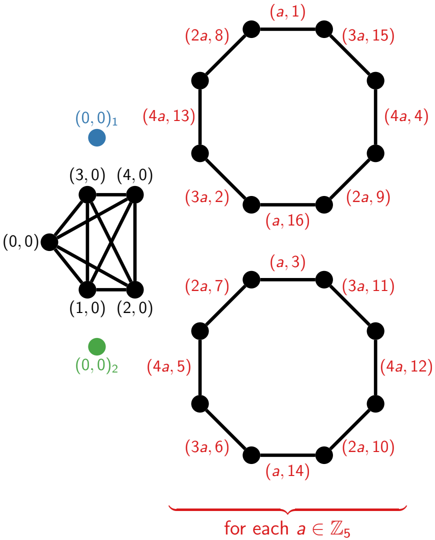

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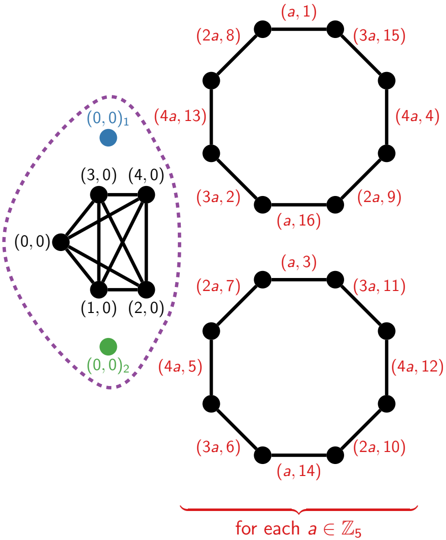

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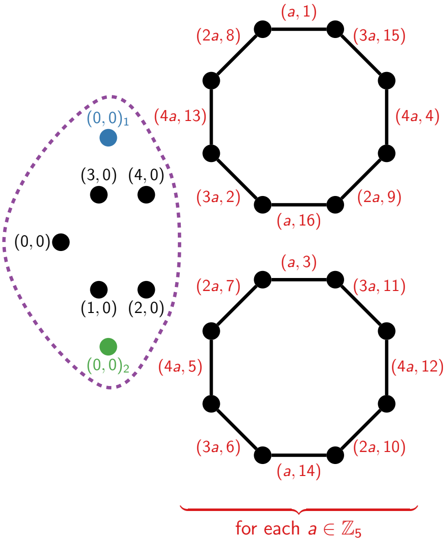

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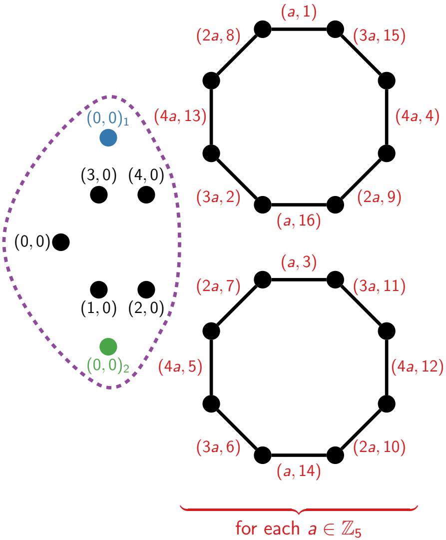

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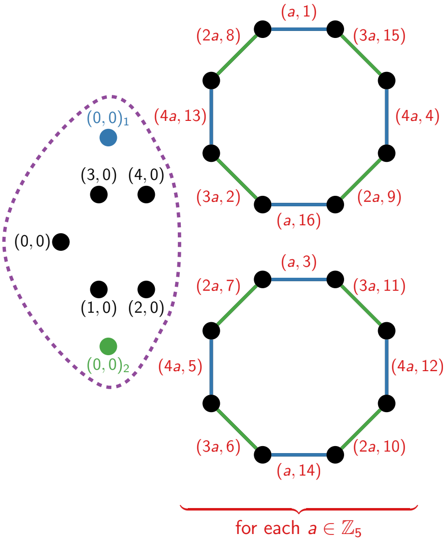

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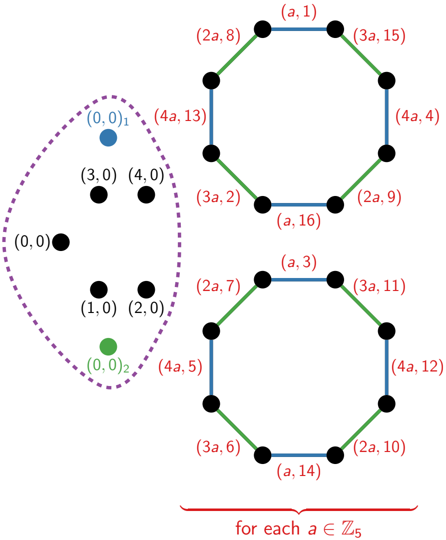

- Remove $\{(0,0),(2,0),(3,0)\}$ and $\{(0,0),(1,0),(4,0)\}$.
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- Add an STS(7) not containing $\{(0,0),(0,0)_1,(0,0)_2\}$ on the specified vertices.
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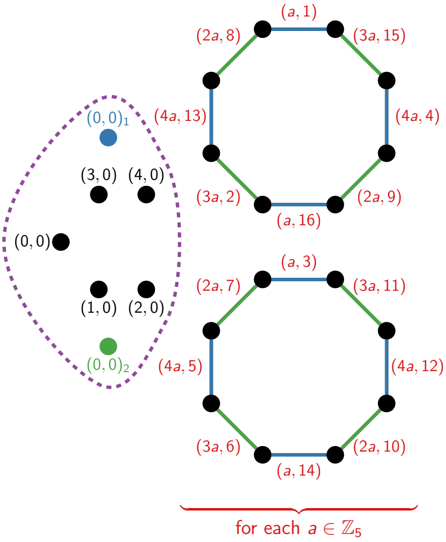

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- The result is an STS(87) which I claim has no PC.

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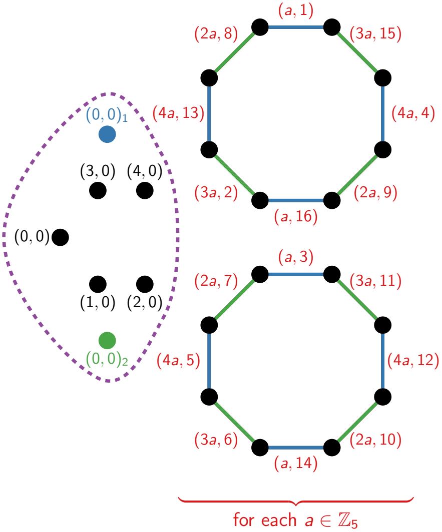

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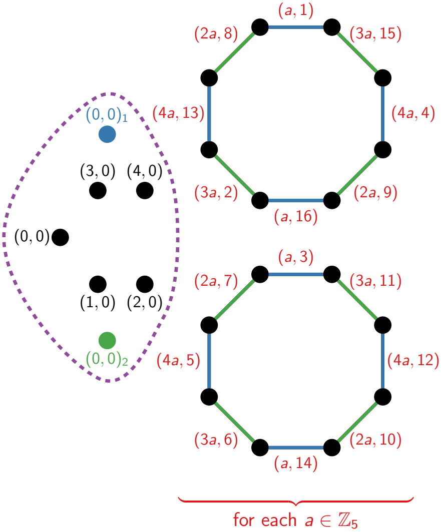

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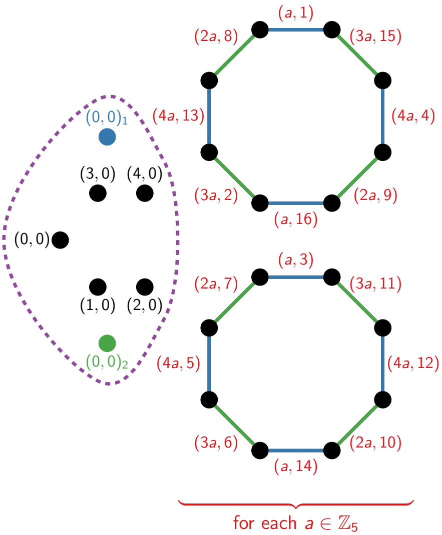

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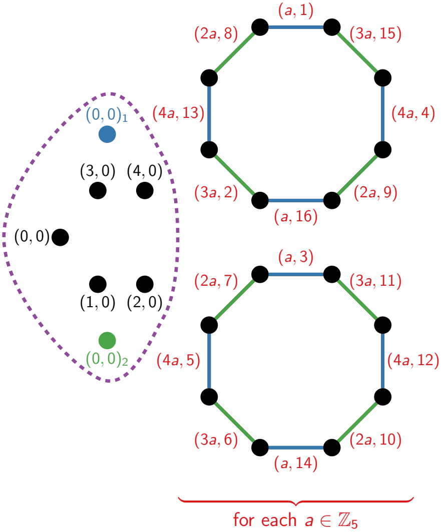

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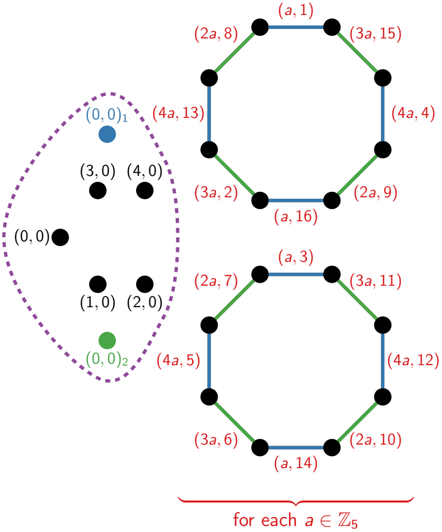

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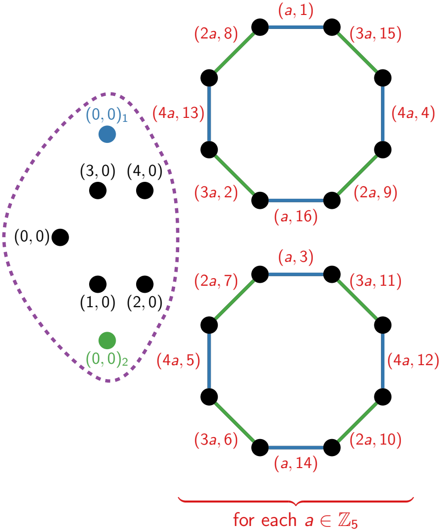

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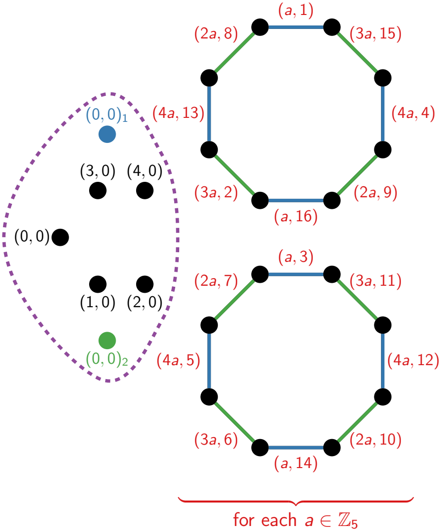

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- If T_1 is in the STS(7), then the triples except T_2 have weight $(*, 0)$. Contradiction.
- So T_1 and T_2 are not in the STS(7). By the properties of the edge colouring, their weights cannot add to $(*, 0)$. But the rest have weight $(*, 0)$. Contradiction.

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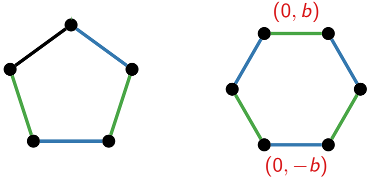

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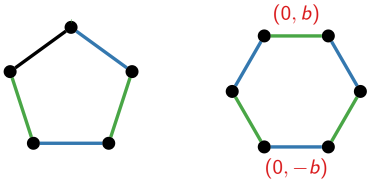

Theorem (Bryant, Horsley, 201?) For each $v \equiv 27 \pmod{30}$ such that $\text{ord}_p(-2) \equiv 0 \pmod 4$ for every prime divisor p of $v-2$, there is an STS(v) with no PC.

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- $\mathcal{P} = \{p : \mathrm{ord}_p(-2) \equiv 0 \pmod 4\}$ contains all primes congruent to $5 \pmod 8$, some primes congruent to $1 \pmod 8$, and no others.
- So we can apply the theorem for any $v = 5p_1 \cdots p_t + 2$ where $p_1, \ldots, p_t$ is a list of primes from $\mathcal{P}$ containing an odd number of primes congruent to $5 \pmod 8$.

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We think we can adapt our argument to find STS of many more orders with chromatic index at least $3\lfloor \frac{v}{6} \rfloor + 3$.

The End