

### Computing with finitely presented groups

Lecture series to be held at *Computational Aspects of Thin Groups* NUS IMS, June 3-7, 2024.

<b>Title:</b>	Computation with finitely presented groups.
<b>Teaching Team:</b>	Heiko Dietrich (lecturer) and Khánh Lê (TA, Rice University)
<b>Duration:</b>	3 x ~60 minutes, plus some assisted homework sessions.
<b>Audience:</b>	mainly graduate students and post-docs.
<b>Assumed knowledge:</b>	basic group theory knowledge (such as group actions, free groups, etc), but not so much about computational aspects.
<b>Pace:</b>	between a graduate lecture and a normal talk; pdf slides, but with some detailed examples and some GAP output).
<b>Material:</b>	pdf slides, a problem sheet, a list of open review questions, references.
<b>Technology:</b>	pdf slides and projector.
<b>Website:</b>	<a href="https://users.monash.edu/~heikod/imsnus2024">https://users.monash.edu/~heikod/imsnus2024</a>
<b>References:</b>	provided in a separate document (see website)

**Abstract.** Groups are mathematical objects that capture the concept of symmetry, and therefore have rich applications in mathematics and other sciences. Computational Group Theory fuses fundamental research in group theory with algorithmic methods. How well one can compute with groups depends significantly on how the group is presented. Common ways to describe groups are permutation groups, matrix groups, and group presentations. While there exist efficient algorithms for computing with permutation and matrix groups, dealing with groups given by a finite presentations can be challenging because many computational problems for these groups are known to be undecidable in general. The aim of this lecture series is to explain what is possible when computing with these groups.

**Anticipated outline.** Depending on time, the following topics might be covered.

**Lecture 1:** free groups; group presentations; Dehn problems; Tietze transformations; von Dyck's Theorem; Todd-Coxeter coset enumeration; low index subgroups; Reidemeister-Schreier.

**Lecture 2:** polycyclic presentations, collection, consistency; abelian quotients; ANUPQ,  $p$ -quotients,  $p$ -cover, Burnside problems; nilpotent and solvable quotients.

**Lecture 3:** some comments on non-solvable quotients; rewriting systems; Knuth-Bendix procedure; Dehn's rewriting system, Dehn presentation; automata; automatic groups; KBMAG.