

2–3 October 2014 Monash University, Clayton

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information

Welcome to the 32nd Victorian Algebra Conference. This year's conference features 22 talks, including 7 by students.

venue

All talks are in Bld 29, theatre S13; see the map on Page 8. The theatre has blackboards, as well as a computer and data projector. Registration and morning & afternoon teas will be next to the conference theatre. Restrooms and a water dispenser can be found on the ground floor of Bld 28: if you enter the building, turn right, and you'll find them behind the lifts.

There is free-parking available in the blue permit areas in the N1 multi-level car park and the SE4 multi-level car park, see the map on Page 8. (You must display a permit for all other blue parking areas at all times.)

dinner

The conference dinner is on Thursday, 7.00pm, at *Taste Baguette*, on Clayton Campus (Bld 11, see the map on Page 8). The location will be open from 6:30pm, serving drinks.

lunch

There are many options for lunch in the Campus Centre (Bld 10); additional coffee shops are located in the S.T.R.I.P. (Bld 75) or New Horizon (Bld 82).

coffee/tea

During the morning and afternoon breaks, the conference will provide coffee, tea, and a selection of sweets in front of the conference theatre. The morning break on Friday will also feature some additional snacks. If you want plain water, there is a water dispenser on the ground floor of Bld 28.

internet

The University-wide wireless network is available via 'eduroam' (for those with accounts at participating institutions).

sponsors

Thanks to the Australian Mathematical Society for its support of the Victorian Algebra Group over the past 32 years. Thanks also to the School of Mathematical Sciences for its support of this year's conference.

questions?

Feel free to ask us, or one of the other locals, if you need assistance during the conference.

Enrico Carlini, Heiko Dietrich, Ian Wanless – Conference Organisers

participants

as of 29 September 2014

Dr Iain Aitchison University of Melbourne Miss Gair Asha La Trobe University Mr Darcy Best Monash University Dr José Burillo Universitat Politecnica de Catalunya Dr Graham Clarke **RMIT** University Dr Michael Coons The University of Newcastle Dr Zajj Daugherty University of Melbourne Prof Brian Davey La Trobe University Dr Heiko Dietrich Monash University Dr Norman Do Monash University Dr James East University of Western Sydney Dr Murrav Elder The University of Newcastle Dr Enrico Carlini Monash University Dr Graham Farr Monash University Dr Barry Gardner University of Tasmania Dr Alex Ghitza Melbourne University Dr Michael Giudici The University Of Western Australia Miss Amelia Gontar University of Sydney Mr Josh Grant Monash University Mrs Charles Grav La Trobe University Mr Cheng Guo University of Technology, Sydney Ms Lucy Ham La Trobe University Mr Nick Ham University of Tasmania Miss Sophie Ham Monash University Mr John Harrison The University of Newcastle Dr Roozbeh Hazrat University of Western Sydney Dr Nhan Bao Ho La Trobe University Mr Joshua Howie University of Melbourne Dr Marcel Jackson La Trobe University Dr Tomasz Kowalski La Trobe University Mrs Jayama Mahamendige Monash University Dr Daniel Mathews Monash University Mrs Stacey Mendan La Trobe University Prof Chuck Miller University of Melbourne Mr Pol Naranjo Barnet Universitat Politecnica de Catalunya Dr Padraig Ó Catháin Monash University Dr Jane Pitkethly La Trobe University Dr Arun Ram University of Melbourne Dr Lawrence Reeves University of Melbourne Dr Amin Sakzad Monash University Miss Tian Sang University of Melbourne Mr Murray Smith La Trobe University Mr Christopher Taylor La Trobe University Ms Na Wang Monash University Dr Ian Wanless Monash University Mr Timothy Wilson Monash University Dr David Wood Monash University Dr Wenting Zhang La Trobe University

abstracts

1. Artin Conjectures: links with geometry, and an elliptic curve

Iain Aitchison (with Lawrence Reeves, Ryuji Abe)

A simple graph derived from the parity of integers leads to a well-studied class of knots and links. The simple geometry of these in turn raises an interesting connection with Artin's Conjecture related to generators of finite fields. In turn, this leads to problems associated with another of Artin's Conjectures, that which led to the Riemann Hypothesis over finite fields. In our context this relates to counting points on a specific and simple elliptic curve. This talk presents these relationships, culminating in a tantalizing elementary conjecture concerning quadratic residues.

2. Parity of transversals in latin squares

Darcy Best

A transversal of a latin square of order n is a set of n entries which has exactly one representative from each row, column and symbol. A long standing conjecture of Ryser states that the number of transversals in a latin square is congruent to the order of the latin square modulo 2. In 1990, Balasubramanian confirmed this conjecture to be true in the even orders. In this talk, we extend this proof to show that in squares of order 2 mod 4, the number of transversals is necessarily a multiple of 4. Moreover, the set of transversals may be further broken down into smaller classes which also contain a special property modulo 2.

3. Metric Properties of Baumslag-Solitar groups

Jose Burillo (with Murray Elder)

We use Britton's normal forms to find estimates for the word metric in the Baumslag-Solitar groups. With these estimates, we find lower bounds for the growth rate for the groups.

4. From rational to regular

Michael Coons

I will provide an introduction to regular sequences by surveying some classical examples and results as well as presenting a few new ones.

5. Topological recursion and the quantum curve for monotone Hurwitz numbers

Norman Do (with Alastair Dyer, Daniel Mathews)

Take a permutation and count the number of ways to express it as a product of a fixed number of transpositions — you have calculated a Hurwitz number. By adding a mild constraint on such factorisations, one obtains the notion of a monotone Hurwitz number. We have recently shown that the monotone Hurwitz problem fits into the socalled topological recursion/quantum curve paradigm. This talk will attempt to give the flavour of what exactly the previous sentence means.

6. There are 132 069 776 distinct transformation semigroups on 4 points

James East

(with Attila Egri-Nagy, James Mitchell)

We enumerated and classified transformation semigroups acting on up to 4 points. The degree 4 case requires extensive computation and in this talk we describe how the ideal structure of the full transformation semigroups allows us to parallelize the enumeration. We also give a short summary of the classification results and the new open problems.

7. Equations in free groups and free monoids

Murray Elder (with Laura Ciobanu, Volker Diekert)

An equation in a free group/monoid is an expression like $aX^2 = XY$, where a is an element of the group/monoid and X, Y are variables. A solution is an assignment of elements to X and Y so that the equation is true in the group/monoid.

Using some clever new results by Diekert, Jez and Plandowski, we are able to describe the set of all solutions to an equation in a free group or free monoid with involution, as a formal language of reasonably low complexity. In my talk I will describe the problem, the formal language class, and briefly describe how the result works.

8. Minors and Tutte invariants for alternating dimaps

Graham Farr

A graph H is a minor of a graph G if it can be obtained from G by a sequence of deletions and/or contractions of edges. Minors play a central role in graph theory, in characterising graph properties and in counting various structures associated with graphs. The theory of minors is intimately related to duality in graphs and matroids.

This talk will describe a theory of minors for alternating dimaps (i.e., orientably embedded digraphs in which the edges incident at a vertex are directed alternately into, and out of, the vertex). There are now three fundamental reductions, rather than two (as for deletion and contraction), and they are related by a transform due to Tutte (1948) called triality or trinity, which extends duality.

The theory has many analogies with standard minor theory. However, these minor operations are non-commutative. This makes their theory more difficult, but we are still able to establish some of the kinds of results one would hope for in a theory of minors, such as excluded minor characterisations and enumerative invariants that obey linear recurrence relations analogous to the deletioncontraction relation for the Tutte polynomial.

9. On Skew Polynomial Rings and Some Related Rings

Barry Gardner (with Elena Cojuhari)

Skew monoid rings (which include skew polynomial rings)form a rather restricted subclass of a certain class of modified monoid rings. We shall examine their relationship with the larger class and their connection with finite normalizing extensions. We shall also present a characterization of rings with a grading by the cyclic group of order 2.

10. Commuting graphs

Michael Giudici

Given a group G, the commuting graph of G is the graph with vertices the noncentral elements of G, and two vertices are adjacent if and only if they commute. Commuting graphs of other algebraic structures can be defined similarly. Commuting graphs of groups have received a lot of attention in recent years. I will discuss some recent results about the structure of such graphs, and in particular their diameter and which graphs can be the commuting graphs of groups and other algebraic structures.

11. Polynomial Rank and Transformation Between Symmetric Pure States

Cheng Guo

Multipartite entanglement has been widely studied since it is a proven asset to information processing and computational tasks. In order to make quantitative comparison between different types of quantum information resources via Stochastic Local Operations and Classical Communication (SLOCC), we want to find the structure of these protocol mathematically. We notice that the tensor rank of any state cannot increase by SLOCC. When these pure states are symmetric, polynomial rank show the upper bound of the tensor rank and the lower bound of the transformation radio. We will also introduce some recent result.

12. Preservation Theorems in Finite Structures

Lucy Ham

It is widely known that most classical results in model theory fail or become meaningless when restricted to the realm of finite structures. A famous example is the Los-Tarski Preservation Theorem, which says that a first-order sentence is preserved by taking extensions if and only if it is equivalent to an existential sentence: this fails at the finite level for both relational and algebraic signatures.

The one notable exception is Rossman's Finite Homomorphism-Preservation Theorem: a firstorder sentence is preserved under homomorphisms on finite relational structures if and only if it is equivalent, on the class of finite relational structures, to an existential positive sentence.

Relativising such theorems can change their truth: the Los-Tarski Theorem for example is recovered at the finite level when restricted to classes of acyclic graphs closed under substructures and disjoint unions. We show that some broad cases of Rossmans result continue to hold under relativisations. But in contrast, we show the corresponding relativised theorem fails in algebraic signatures, giving perhaps the first case of a classical preservation theorem holding for relational structures at the finite level, but not for algebraic structures at the finite level. It remains unknown if Rossmans result (unrelativised) holds in the algebraic setting.

13. The Fauser Monoid

Nick Ham

The Jones monoid \mathcal{J}_n is generated by hook diagrams, by including generators of the symmetric group we get the Brauer monoid \mathcal{B}_n . We will be looking at the monoids generated when we replace the hooks in the generators of \mathcal{J}_n and \mathcal{B}_n with triapses, the latter of which we refer to as the Fauser monoid \mathcal{F}_n . I will begin by discussing various bits of information about these monoids, such as: characterising and counting the elements; difficulties with terminology; Green's relations; and complications with my so far hopeless search for a presentation. Then end by discussing what we hope to achieve with the semigroup algebra of \mathcal{F}_n .

14. Graded Algebras, classifications via Graded K-theory

Roozbeh Hazrat

From a graph (e.g., cities and flights between them) one can generate an algebra which captures the movements along the graph.

This talk is about one type of such correspondences, i.e., Leavitt path algebras.

Despite being introduced only 10 years ago, Leavitt path algebras have arisen in a variety of different contexts as diverse as analysis, symbolic dynamics, noncommutative geometry and representation theory. In fact, Leavitt path algebras are algebraic counterpart to graph C*-algebras, a theory originated and nourished in Australian universities which has become an area of intensive research globally. There are strikingly parallel similarities between these two theories. Even more surprisingly, one cannot (yet) obtain the results in one theory as a consequence of the other; the statements look the same, however the techniques to prove them are quite different (as the names suggest, one uses Algebra and other Analysis). These all suggest that there might be a bridge between Algebra and Analysis yet to be uncovered.

In this talk, we introduce Leavitt path algebras and then try to understand the behaviour and to classify them by means of (graded) K-theory. We will ask nice questions!

15. On the closureness of impartial combinatorial games

Nhan Bao Ho

An impartial combinatorial game is a two-player game in which the players move alternately, following some given set of rules of moves. We assume the game has only a finite number of positions each of which can be visited at most once and so the game must terminate after some number of moves. The game has no hidden information and no element of luck. The player who makes the last move wins. In sum of two games, a player can choose any game and then makes a legal move in that game. The next player can choose either of game to move. We discuss the closureness of two families of impartial games called "tame" and "miserable" under the sum operator.

16. The relative 1-line property for knot exteriors

Joshua Howie

The relative 1-line property is a condition on a family of intersections of surface subgroups of a 3-manifold group constructed from the knot group.

The relative 1-line property allows us to give an algebraic characterisation of what it means for a knot to be alternating, answering an old question of Ralph Fox. We can also use this to characterise a class of knots which have alternating projections onto higher genus surfaces.

We will also consider the problem of deciding whether two surface subgroups have the relative 1-line property.

17. Ghosts on a plane. And nilpotence.

Marcel Jackson

A Natural Duality is a dual equivalence between a class of algebraic objects and a class of topological structures, arising over some generating object by way a particular canonical construction. Many classical dualities fall under this umbrella, including vector space duality (over the field of scalars) and Stone's duality for Boolean algebras (over the two element Boolean algebra).

Not every algebraic object can carry a natural duality (as a generating object), and so it emerges as of obvious interest to understand which can and which cannot.

In this talk we return to the currently incomplete classification of finite dualisable semigroups. We introduce a geometric argument for showing nondualisability of semigroups (and other algebras), and find it sufficiently powerful to show that almost all finite semigroups are not dualisable. Amongst other broad applications of the technique is a fresh proof that a finite group is nondualisable when it has a nonabelian Sylow subgroup.

18. Quasivarieties of directed paths

Tomasz Kowalski

(with Marcel Jackson, Michal Stronkowski)

A quasivariety is a class of relational structures defined by a set of quasi-identities, that is, universally quantified formulas $\varphi_1 \& \ldots \& \varphi_n \to \psi$, where φ_i and ψ are atomic. In the language of directed graphs atomic formulas are: x = y, and E(x, y) (stating that there is a directed edge between x and y). A quasivariety Q is finitely based, if Q is defined by a finite set of quasi-identities. A quasivariety Q is finitely generated, if it is precisely the class of isomorphic copies of substructures of direct products of a finite set S of structures.

It is well known that in the class of (undirected) graphs there are only 4 finitely based quasivarieties, but for directed graphs there are infinitely many. We investigate quasivarieties generated by directed paths and show that: (1) If Q is a quasivariety generated by "mostly upward" directed paths, then Q is finitely based if and only if it is finitely generated. (2) There are continuum many distinct quasivarieties generated by "mostly upward" paths.

Time permitting, we mention some results aiming at a complete characterisation of finitely based quasivarieties of directed paths.

19. Near-autoparatopisms of Latin squares

Jayama Mahamendige (with Ian Wanless)

A Latin square L of order n is an $n \times n$ array containing n symbols from $[n] = \{1, 2, ..., n\}$ such that each element of [n] appears exactly once in each row and each column of L. Rows and columns of L are indexed by elements of [n]. The element in the i^{th} row and j^{th} column is denoted by L(i, j).

The set of n^2 ordered triples,

$$O(L) = \{(i, j, L(i, j)); i, j \in [n]\}$$

is called the orthogonal array representation of L.

The Hamming distance between two Latin squares L and L' is defined by

$$dist(L,L') = \#\{d \in O(L); d \notin O(L')\}.$$

The elements of the form $\sigma = (\alpha, \beta, \gamma; \lambda) \in S_n \wr S_3$ are known as paratopisms. When $\lambda = \varepsilon$, σ is said to be an isotopism. In other words, $\theta =$ $(\alpha, \beta, \gamma) \in S_n^3$ is said to be an isotopism. When $\theta = (\alpha, \alpha, \alpha) \in S_n^3$, then θ is known as an isomorphism.

If $dist(L, L^{\sigma}) = 0$, that is, $L^{\sigma} = L$, then σ is known as an autoparatopism. If $dist(L, L^{\sigma}) =$ 4, then σ is called a near-autoparatopism of L. If $dist(L, L^{\theta}) = 0$, that is, $L^{\theta} = L$, then θ is known as autotopism. If $dist(L, L^{\theta}) = 4$, then θ is called a near-autotopism of L. The similar definitions for isomorphisms are automorphisms and near-automorphisms.

If a submatrix M of a Latin square L is also a Latin square then M is called a subsquare of L. A subsquare of order 2 is an intercalate.

Cavenagh and Stones gave a proof to the following result.

For all $n \geq 2$ except $n \in [3, 4]$ there exists a Latin square L of order n that admits a nearautomorphism. In this process, they found a necessary and sufficient condition for α to be a

near-automorphism when α has the cycle structure (2, n-2), where $n \ge 0$ and $n \equiv 2 \mod 4$.

We determine a family of Latin squares with unique subsquare of order two(intercalate) which admits a near-autoparatopism. In this process, we find a necessary and sufficient condition for $\sigma = (\varepsilon, \beta, \gamma : (12)) \in S_n \wr S_3$ to be a nearautoparatopism when the cycle structure of β and γ are $(n-2)^{1}1^2$ and $(n-2)^{1}2^1$ respectively.

20. Unitary Precoding for Integer-Forcing MIMO Linear Receivers

Amin Sakzad

(with Emanuele Viterbo)

A flat fading point-to-point multiple-antenna channel is considered where the channel state information is known at both transmitter and receiver. At the transmitter side, we use a lattice encoder to map information symbols to lattice codewords. The lattice coded layers are then precoded using unitary matrices satisfying nonvanishing minimum product distance. At the receiver side, an integer-forcing linear receiver is employed. This scheme is called "unitary precoded integer-forcing". We show that by applying the proposed precoding technique full-diversity can be achieved. We then verify this result by conducting computer simulations in a 2×2 and 4×4 multipleinput multiple-output (MIMO) channel using fulldiversity algebraic rotation precoder matrices.

21. The Boundary Between Decidability and Undecidability in the Tarski Signature

Murray Smith

Algebras of relations can be viewed as a natural extension of Boole's algebra of logic: as well as the usual set theoretic operations, one has composition, converse and the identity relation. Tarski proposed a succinct equational axiomatisation and showed it covered much of the basic theory. But these axioms turned out to be incomplete, and moreover it is now known that no finite system of laws will suffice. Worse still: recognising when a finite abstract algebra is isomorphic to an algebra of relations is undecidable!

The purely compositional fragment of relation algebra is easily captured: it is semigroup theory. So where in the signature do these complications arise? We investigate identify reducts of the Tarski signature lying at the boundary between decidability and undecidability.

22. Discriminator Varieties of Double-Heyting Algebras

Christopher Taylor

Just as Boolean algebras provide an algebraic counterpart to classical logic, Heyting algebras are the algebraic counterpart of intuitionistic logic, where the law of the excluded middle (i.e. "p or $\neg p$ " is always true) is rejected. The rejection of this law means that implication $p \rightarrow q$ cannot be defined as $\neg p$ or q in an intuitionistic setting.

A Heyting algebra is a bounded distributive lattice endowed with an additional binary operation \rightarrow (corresponding to intuitionistic implication). Including the dual of implication as a further binary operation gives us the class of *double-Heyting algebras*.

A discriminator variety is an equational class \mathcal{V} that has a term t(x, y, z) such that t is a discriminator term on every subdirectly irreducible member of \mathcal{V} , i.e. t(x, y, z) = z if x = y and x otherwise. Discriminator algebras are perhaps the most successful generalisation of Boolean algebras. It is well-known that any discriminator variety \mathcal{V} is semisimple, that is, every subdirectly irreducible member of \mathcal{V} is simple. However the converse does not hold in general. We show that in the case of double-Heyting algebras, the converse is true.



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SE4 Multi-level Car Park E4 Senior Chemistry D2

Sport and Recreation D4

University Staff Club D3

Yarrawonga Building E2

Lecture Theatres

Campus Centre G149 D3

Humanities H1-H10 D3

Education G23, 245 E3

Engineering E1-E5 C3

Engineering E7, G03 C2

Engineering EH1-EH4, E6 C3

Law Moot Court L1 - L5 E2

Central One C3

Short Courses Centre D3 Sir Alexander Stewart Theatre C2

Sir Robert Blackwood Concert Hall D4

Zoology Environmental Laboratories B5

Teaching Facilities Support Unit E2

Senior Zoology D2

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Science S5-S6 D2

Zoology S7-S8 D2

Hargrave-Andrew C2 John Medley D3

Briggs C4 Central Building B5

Normanby House A4

South Eastern Flats E5

Halls of Residence

South One E2

Libraries

Matheson E3

Deakin **B5**

Farrer A5

Howitt A5

Jackomos C5

Richardson A4

Roberts B4

Law E2

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85b

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- Alexander Theatre E3 Australian Pulp and Paper C2 Bicycle Arrival Station C2 Biochemistry Laboratories D2 Biology 17 Biomedical Imaging Laboratory D6 Botany B5 Campus Centre D3 Central Science D2 Centre for Electron Microscopy C2 Childcare Centre A4 Computer Science D3 Engineering Building 31, 33, 35, 36, 36a, 37, 60, 69 & 72 C3 Facilities and Services B3 Faculty of Education E3 Faculty of Information Technology C3 Faculty of Law E2 Faculty of Medicine E2 First Year Biology D2 First Year Chemistry D2
- Japanese Study Centre E2 John Monash Science School E1 59 Krongold Centre E3 80A Mannix College F3 Mathematics and Information 16 17 Technology Services c2 220 Medical and Health Science D2 42 Melbourne Centre for Nanofabrication E6 10 19 Menzies Building D3 81 Microbiology D2 Monash Children's Centre E1 83 26 Monash College F2 Monash Connect D3 Monash House E2 31 Monash Oakleigh Legal Service E1 40 Monash S.T.R.I.P. D1 75, 76 & 77 6 Monash Science Centre A4 Monash University Business Park 680, 700, 710 and 738 63 12
- ▶ All talks are in Bld 29, S13. ▶ Registration, tea, and coffee are next to S13.

Blackburn Road A6

North 1 Multi-level Car Park B2

New Horizons B2

64

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▶ Conference dinner is at *Taste Baguette*, at the back of Bld 11.

Thursday 2 October

9.00-9.20 9.20-9.30	Registration Conference opening
9.30-10.25	Michael Giudici: Commuting graphs.
10.30-10.55	Coffee break
11.00-11.25	Darcy Best: Parity of transversals
-	in latin squares.

12.00-12.25 Nick Ham: The Fauser monoid.

12.30-1.55	Lunch
2.00-2.25	Joshua Howie: The relative 1-line property for knot exteriors.
2.30-2.55	Jayama Mahamendige: Near- autoparatopisms of latin squares.
3.00-3.25	Murray Smith: The boundary between decidability and undecidability in the Tarski signature.
3.30-3.55	Coffee break
4.00-4.25	Christopher Taylor: Discriminator varieties of double- Heyting algebras.
4.30-4.55	Norman Do: <i>Topological</i> recursion and the quantum curve for monotone Hurwitz numbers.
5.00-5.25	James East: There are 132069776 distinct transformation semigroups on 4 points.
5.30-5.55	Iain Aitchison: <i>Artin conjectures: links with geometry, and an elliptic curve.</i>
7.00	Dinner at "Taste Baguette"
	(drinks from 6.30)

Friday 3 October

9.00-9.55	Cheng Guo: Polynomial rank and transformation between symmetric pure states.
10.00-10.25	Graham Farr: <i>Minors and Tutte invariants for alternating dimaps.</i>
10.30-10.55	Coffee break
11.00-11.25	Murray Elder: Equations in free groups and free monoids.
11.30-11.55	Barry Gardner: On skew polynomial rings and some related rings.
12.00-12.25	Roozbeh Hazrat: Graded algebras, classifications via graded K-theory.
12.30-1.55	Lunch & VAG AGM
2.00-2.25	Nhan Bao Ho: On the closureness of impartial combinatorial games.
2.30-2.55	Marcel Jackson: Ghosts on a plane. And nilpotence.
3.00-3.25	Tomasz Kowalski: Quasivarieties of directed paths.
3.30-3.55	Coffee break
4.00-4.25	Amin Sakzad: Unitary precoding for integer-forcing MIMO linear receivers.
4.30-4.55	José Burillo: Metric properties of Baumslag-solitar groups.
5.00-5.25	Michael Coons: From rational to regular.

All talks are in S13 (Bld 29).