Reception Range Estimation in CDMA Multihop Cellular Networks

A. A. N. Ananda Kusuma    Lachlan L. H. Andrew    Stephen V. Hanly
ARC Special Research Centre for Ultra-Broadband Information Networks (CUBIN)
Department of Electrical and Electronic Engineering
The University of Melbourne, Victoria 3010, Australia
{a.kusuma,l.andrew,s.hanly}@ee.mu.oz.au

Abstract—In multihop networks, there is a trade-off between the transmission errors and hop size. In a system with base stations overlay, traffic is aggregated towards base stations and there is a need to investigate the impact of relaying to the user’s own traffic. The optimal hop size and transmission strategy to maximise minimum user's throughput is studied in the context of a spread-spectrum CDMA multihop cellular networks. A heuristic and receiver-based routing algorithm is proposed to achieve the target hop size. To estimate user's throughput, an approximate model is introduced, in addition to simulation. We observed that the improvement in minimum user's throughput can be made by allowing target radius for base station and the other nodes to be different.

I. INTRODUCTION

There has been a lot interest in deploying ad-hoc networks (packet radio networks) where a set of nodes can be self-organised and help other nodes’ data to their destination. Recently, ad-hoc networks have also been used in the context of cellular or multihop cellular networks (MCN), to enhance performance in terms of power requirements, self-organisation, coverage and capacity [1–7].

In this work, the uplink of MCN is analysed where all traffic goes to the base stations. This causes traffic aggregation towards the base stations, where nodes near the base station carry more traffic than those further away. This raises the issue of fairness which we address by seeking to maximise the minimum throughput over all nodes in the networks (excluding traffic relayed on behalf of other nodes). In particular, our work is to investigate whether the transmission/reception range should be a function of distance from the base station. While there have been many studies into optimal range in ad-hoc networks with uniform traffic distribution [8–11], limited results are available for those with aggregated traffic [4,5].

II. RELATED WORK

Early work on optimal transmission ranges in ad-hoc networks focused on TDMA system [8,9], and the concept of range is coupled with the notion of average number of terminals in a particular area of a random networks.

In the context of spread-spectrum, a model which includes the total contribution of received interference powers (affected by propagation model) was introduced in [10]. Assuming that the terminals can be positioned exactly at the chosen distance from a receiver, an optimal range can be determined. The model ignored thermal noise since it was assumed to be insignificant compared to the multi-user interference, and the optimal range is shown to be a function of processing gain. The concept for considering the total received powers [10] is adopted in [11] for the model from transmitting side which includes forward progress routing towards destination and transmit power adjustment. In [12], the work of [10] is extended by including fading. Nevertheless, in all of above, the transmit probabilities are assumed to be spatially uniform.

In our work, transmit probabilities are spatially non-uniform, which is influenced by the routing and transmission policy. A routing scheme is developed where the target range can be adjusted by varying the routing’s radius.

III. PROBLEM FORMULATION

The general problem in multihop networks is complex as it involves topology management, routing, MAC, power control, etc. We use the following simplified model.

A single cell with unit radius \( r_{\text{max}} = 1 \) is assumed as shown in Fig. 1, with no influence from the neighbouring cells. The nodes are spatially distributed as a two-dimensional Poisson process. This follows [8–11], but it is contrast to [4,5] where the nodes are assumed to form a continuum and relays always found in the straight line to destination. Node density is given by \( \lambda \) per cell or \( \lambda/\pi \) per unit area. All the nodes transmit with constant (equal) transmit powers and no fading is assumed; thus, the power received from a unit-power transmitter at distance \( d \) is simply \( d^{-n} \) where \( n \) is the path loss exponent. This offers simplicity and potential implementation of networks with distributed and bursty nodes. Without fading and power control, the effect of interference is more localised, which justifies the single-cell assumption. Relevant studies with ideal power allocation are in [6,7].

Transmission is time-slotted, and a node can not transmit and receive simultaneously. Traffic sources are greedy and nodes always have data to forward. At each time slot, with probability \( p_0 \), a node \( i \) will transmit its own data (new or retransmitted), and with a time-invariant probability \( p_1(i) = p_0 \) it will forward another node’s data. Note that as \( p_0 \) is fixed, the throughput is unchanged by the node retransmitting its own data. In contrast, \( p_1(i) \) is set such that packets which are not receiving correctly are retransmitted.
Spread-spectrum multiple access with fixed carrier to interference ratio (CIR) requirement \( \alpha \) is assumed. The effect of capture is ignored, and any packet received will be successfully decoded if its CIRs is greater than \( \alpha \). Capture in spread-spectrum ad-hoc networks is dependent on the employed spreading-code protocol. The difficulty typically comes due to the lack of central controller. In this work, there is central controller (base station), and all links are assumed to have independent spreading codes. Furthermore, multiple packet reception is a potential feature for future ad-hoc networks [13]. The Probability that CIR > \( \alpha \) is denoted by \( p_s \), and by [14,10,15],

\[
CIR = \frac{Y_0}{2L \sum_{i} Y_i + N_o},
\]

where \( Y_0 \) is the target received power, \( Y_i \) the received power from the \( i \)th interferer, \( L \) is the processing gain, and \( N_o \) is the thermal noise. Our power unit is in the unit of \( N_o \).

Nodes are stationary and the paths from all nodes to the base station form a tree \( T \). \( T \) is parameterised by a target range (average hop size) \( R(r) \) at distance \( r \) from the base station. A user’s throughput \( u \) is defined as the probability of successfully transmitting its own data in a given slot. Recall that relays will retransmit the data to guarantee its arrival. Successful own’s transmission requires three independent events: the node transmits (probability \( p_o \)), the receiver is not transmitting with probability \( (1-p_r) \), the interference at the receiver is sufficiently low that, if node \( j \) transmits to \( i \), it has \( CIR > \alpha \) (probability \( p_e(i) \)). For a given \( p_o \) and \( T \), user \( j \)’s throughput is thus \( u(j) = p_o(1-p_r(i))p_e(i) \). Averaging over realisation of the Poisson process, the throughput of a user is determined by its receiver’s distance to the base station, \( r \), by circular symmetry. On average a user is at distance \( R(r) \) (the reception range) of the receiver. Using similar notation for average value, we write

\[
u(r) = p_o \cdot (1 - p_r(i)) \cdot p_e(r) \]

The aim is to find \( p_o \) and a means to generate \( T \) which will maximise the minimum expected transmission rate. Given \( R(r) \), \( T \) depends on the routing algorithm. When transmit powers are constant, but traffic is spatially non-uniform, the success of transmission depends on the interference at the receiver, but no property of the transmitter. Thus, the optimal hop size is a property of the receiver, and we use the term “reception range” instead of the more familiar term “transmission range”. The fundamental question is whether or not the reception range should be a function of distance from base station \( r \), due to the aggregation of traffic. The performance objective is formulated as

\[
\max_{p_o,R} \{ \min_r u(r) \}. \tag{3}
\]

To make this concrete, the following section introduces a specific routing algorithm, for which we will determine the optimal reception range \( R(r) \). However, we are not advocating this as the optimal algorithm; it is hoped that the function \( R(r) \) that we find will provide a tool for finding better algorithm.

### IV. HEURISTIC ROUTING ALGORITHM

Traditional routing assigns a fixed cost to each link. However, for wireless networks, the appropriate costs depend on the route eventually chosen, and so other methods must be used. Many algorithms [16] seek a path from transmitter to receiver to achieve its target transmission range. Because our target hop size is (potentially) dependent on the receiver’s location, we propose a receiver-oriented algorithm.

The proposed routing algorithm grows a tree starting from the base station. Nodes are added in increasing order of their distance to the base station. A node is added by selecting the node (relay or BS) to which it will transmit. The choice of relay seeks to optimise multiple objectives, in decreasing order of priority: a receiver, \( j \), should not receive from a node further than \( D_j \) away; loads should be balanced; the maximum hop length should be minimised; the hop count should be minimised.

The algorithm uses the following quantities: \( d(i,j) \) is the distance between nodes \( i \) and \( j \); the path \( P(j) = (P_{j,N_j},\ldots,P_{j,0}) = (j,\ldots,b) \) is the ordered list, of length \( N_j \), of nodes which carry a packet from node \( j \) to the base station; \( m_j = \max_n d(P_{j,n},P_{j,n-1}) \) is the maximum hop length on path \( P(j) \); \( m_{i,j} = \max(d(i,j),m_j) \) is the maximum hop length which would result from relaying packets from \( i \) via \( j \); \( f(j) \) is the number of flows relayed by the most heavily loaded node on path \( P(j) \), that is, the number of nodes \( n \) such that \( P_{j,1} = P_{n,1} \); \( A \) is the set of allocated nodes. Note that \( f(j) \) increases as more nodes are added to the tree, while the other quantities are constant.

Two greedy passes are used; nodes which cannot be connected in the first pass without violating the receive range constraint are pushed onto a queue to be processed in the second pass. In each pass, a subroutine AdOneOf() is called. The algorithm can now be stated as follows:

\[
P(b) \leftarrow (b), A \leftarrow b, f(j) \leftarrow 0 \text{ for all } j
\]

**foreach** node \( i \), in increasing \( d(i,b) \)

\[
J_1 \leftarrow \{ j \in A : d(i,j) < D_j \}
\]

if \( J_1 = \emptyset \)

push \( i \) into the queue

else

AddOneOf(\( J_1 \))

endif

Fig. 1. Single-Cell Multihop Model
endfor
while the queue is not empty
    pop i
    AddOneOf(A)
endwhile

Subroutine AddOneOf(S)
    J_2 \leftarrow \{ j \in S : f(j) = \min_{k \in S} f(k) \}
    J_3 \leftarrow \{ j \in J_2 : m_{i,j} = \min_{k \in J_3} m_{i,k} \}
    J_4 \leftarrow \{ j \in J_3 : N_j = \min_{k \in J_4} N_k \}
    j = \arg\min_{k \in J_4} d(i,k)
    P(i) \leftarrow (i,P(j))
    N_i \leftarrow N_j + 1
    m_{i,j} \leftarrow m_{i,j}
    A \leftarrow A \cup i
endforeach k\text{ such that } P_{k,1} = P_{j,1}

f(k) \leftarrow f(k) + 1
endfor

The algorithm is centralised and it is still appropriate for table driven and base-centric routing protocol, e.g. [17], where the complete topology is available. Designing the distributed version of the algorithm is our future work.

The algorithm maps the specified radius D to the target range (average hop size) R. Fig. 2 shows the linear relationship between R and D. Due to the nature of Poisson process, the position of the terminals are uniform, and for single-hop case it can be shown that \( R = 2/3 \cdot D \). In ad-hoc region, R can be larger than 2/3 \cdot D since the routing policy allows a link to hop over a particular terminal. The algorithm has an edge-effect when no terminal is found within its target radius, it then connects to the next nearest terminal. Thus, R is clipped at \( D_{min} \approx 1/\sqrt{\lambda} \) in single-hop region and \( D_{min} \approx 2/\sqrt{\lambda} \) in ad-hoc region, where \( \lambda \) is the terminal density per cell of \( \pi \) unit area. We consider the deviation from \( R = 2/3 \cdot D \) not too much, and for simplicity we assume \( R = 2/3 \cdot D \) in both region as long as \( D > D_{min} \).

V. BENEFITS OF SPATIALLY VARYING RECEPTION RANGES

For computationally tractability, we limit ourselves to having one reception range, \( R_{bs} \) for the base station and another \( R_{ms} \) for all other nodes. Both \( R_{bs} \) and \( R_{ms} \), together with \( p_o \) are optimised by exhaustive simulation (as described in VII).

Table I and Fig. 3 demonstrate that under optimal condition there is a benefit from allowing \( R \) to vary spatially. However, exhaustive search by means of simulation is prohibitively slow. The rest of this paper will investigate a model which is simple enough to allow \( R_{bs} \) and \( R_{ms} \) to be determined more efficiently.

VI. ANALYTICAL MODELLING

A. Two-hop model

From Fig. 3, it can be seen that the nodes whose data being relayed by the receiver at \( r \approx D_{bs} \) are the worst. The intuitive reason is because all nodes at \( r < D_{bs} \) will share all the load from traffic generated in ad-hoc region, and the relay nodes at \( \approx D_{bs} \) have the worst wireless channel to the base station. Thus, the task reduces to modelling the throughput of two-hop path as shown in Fig. 4. A proportion of traffic generated in ad-hoc region is pumped from \( A \), forwarded by \( O \) to \( bs \). The throughput of \( A \), \( u_A = p_o \cdot (1 - p_b(D_{bs})) \cdot P_{O}^{A} \), and \( p_b(D_{bs}) = (p_s^A \cdot p_f^O)/P_s^O + p_o \) where \( P_f^O \) reflects the amount of traffic from \( A \) to be forwarded by \( O \) to \( bs \). The parameters used in this computation are affected by the choice of \( p_o \) and the reception ranges. The following section explains how those parameters are estimated.

B. Transmit Probability

To directly model \( p_o(r) \) by including transmission policy is complicated. An easier way is to estimate based only on routing configuration. Assuming \( p_o \) is very small, then \( p_s \approx 1 \) and \( p_o(r) \) is basically found by counting the number of flows at \( r \).
By referring to Fig. 1, we make an assumption that all packets originating outside a circle of radius $r$ must cross the circle; thus, the number of forwarded flows at $r$ is proportional to the area outside the circle. The number of available relays also decreases as $r$ decreases, which may result in an increase of forwarding load. Transmit probability is then $p_t(r) = p_f(r) + p_o$, where

$$p_f(r) = \frac{\kappa(r_{\text{max}}^2 - r^2)}{2r},$$

(4)

$k$ is the constant of proportionality (influenced by $R$), which needs to be found empirically, see Appendix.

All traffic generated in ad-hoc region must be forwarded to the base station by the relays inside base station coverage. Due to the flow load-balanced property of the routing algorithm, it can be seen that the number of forwarded flows tend to be constant at inside base station coverage ($r < D_{bs}$) and gradually drops as $r$ decreases due to the drop on the number of potential relays. We take piecewise approximation of $p_f$ and the width of relay region inside base station coverage is approximately $R$. The peak of $p_f$ at $r < D_{bs}$ can be determined by distributing all traffic from ad-hoc region to the last relays to base station, i.e.

$$p_f(D_{bs}^2 - (D_{bs} - R)^2) = p_o(r_{\text{max}}^2 - D_{bs}^2).$$

Thus,

$$p_f(r) = \frac{p_o(r_{\text{max}}^2 - D_{bs}^2)}{R(2D_{bs} - R)},$$

(5)

We refer $p_f(r)$ in 4 as $p_f^1$ and in 5 as $p_f^2$. From Fig. 5, the piecewise approximation results are close with simulation results. It can be observed that the shape of $p_o(r)$ can vary by varying $R$ and $D_{bs}$.

C. Statistics of Interference

To compute $p_o$, the statistics of interference at the receivers need to be computed. Since the nodes are governed by Poisson random process on the plane $A$, then transmitting nodes are also Poisson random process $X_t$ with intensity $\lambda_t(r) = p_t(r)\lambda$. With each sample function of $X_t$, at each receiving point, we can associate the random variable of the received interference power $Y = \sum g(x_i) g(x_i) = d_i^{-n}$, where $d_i$ is the distance of the $i^{th}$ point in the sample function to the receiving point. The characteristic function of $Y$ (Campbell’s theorem [18]),

$$E(\exp(zY)) = \exp\left(\int_A \exp(zg(dA)) - 1\right)\lambda_t(dA)$$

(6)

When $\lambda_t$ is constant as in spatially uniform traffic, the results are presented in [10], and for path loss exponent $n = 4$, a closed form for cdf of $Y$ is given by

$$F_Y(y) = \text{erfc}\left(\frac{n^{1/2}\lambda_t}{2\sqrt{y}}\right).$$

(7)

In our work, $\lambda_t(r)$ is based on $p_t(r)$ derived in previous subsection, and $F_Y(y)$ is estimated at $r = 0$ and $r = D_{bs}$. We numerically computed the characteristic function of 6 by standard standard integration routine for oscillatory integrand [19]. We found that integration is stable for estimation at $r = 0$, but not at $r = D_{bs}$. We’ll further investigate this. $F_Y(y)$ is then obtained by numerically inverting the characteristic function by means of fast numerical inversion of [20]. Numerical inversion was found to be stable. For the receiver at $r = D_{bs}$, we adopt locally uniform assumption and compute the average $\lambda_t$ on the circle with radius $R$ centred at $r = D_{bs}$, and use the equation 7.

$p_o$ can be found from $F_Y(y)$ by setting the threshold $y_t$ for successful reception which depends on target received power and CIR requirement.

Fig. 6 shows the comparison of the estimate $F_Y(y)$ with the one obtained from simulation with the same input parameters. The simulation data is described in Section VII. The estimate is more optimistic than simulation result, and it is due to the estimate of $p_t(r)$ does not look like Fig 5 when $p_o$ is not small. Fig. 7 shows $p_t(r)$ using parameters estimated in Section VII. Generally, it is no longer flat at $r < D_{bs}$, but tend to peaked at $r = D_{bs}$ (more retransmission due to worse wireless link to the base station compared to other transmitters inside $D_{bs}$). The second peak at $r > D_{bs}$ is also due to more retransmission to the busiest node at $r = D_{bs}$. Further investigation will be carried out to accurately predict $p_t(r)$.

VII. NUMERICAL RESULTS

Simulation was done by taking average performance over realisations of Poisson process. On any realisation, a tree was

---

Fig. 5. Forwarding flows, $D_{bs} = D_{bs}$, with $\lambda = 1000$ per cell

Fig. 6. Cumulative probability distribution of the received interference power $y$
Simulation parameters. Even though the throughput values predicted will continue to improve the approximate model, and develop a guidance for determining the appropriate ranges. Our work is close, we haven’t showed the true optimal parameters for large feasible optimal simulators for large

feasible optimal simulators for large


throughput. The wireless parameters used are

faster than direct computation using recursive algorithm of [15].

In simulation, exhaustive search was done by bracketing

Fig 6 shows the average of

throughput can be made by allowing different target range for the base station and other relays. A receiver-based routing algorithm which utilises the concept of reception range is proposed. Furthermore, an approximate model is introduced for fast estimation of the optimal parameters.

ACKNOWLEDGMENT

We thank Taka Sakurai for his assistance. This work was supported by the Australian Research Council.

APPENDIX I

FITTING PARAMETERS FOR THE ESTIMATE TRANSMIT PROBABILITY

\( \kappa \) measures expected number of times the traffic outside \( r \) to be forwarded by relays in annulus at \( r \). Fig 8 shows the relationship between \( \kappa \) and \( R \). It can be seen that \( \kappa \) decreases (hence \( p_f(r) \)) as \( \lambda \) increases and vice versa. Thus, \( \kappa \) is fitted as \( \kappa = c/R^\theta \). Fig 9 shows the fitting parameters for several values of \( \lambda \). Asymptotic property was observed for dense networks, and we defer the theoretical study for future work. For the rest of the experiment, we take \( c \approx 0.427 \) and \( \theta \approx 1.424 \).

REFERENCES


Parameters in $\kappa = \frac{1}{\pi}$

Fig. 9. Parameters in $\kappa = \frac{1}{\pi}$


