Macroscopic Fundamental Diagrams of arterial road networks governed by adaptive traffic signal systems

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A comparative study of Macroscopic Fundamental Diagrams of arterial road networks governed by adaptive traffic signal systems

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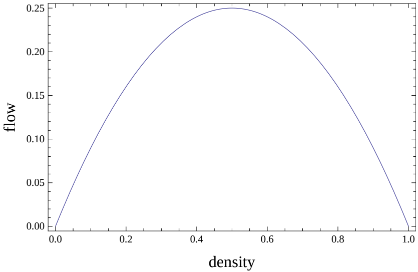
Fundamental Diagram

- Consider a one-dimensional flow (vehicles along a freeway)
- The functional relationship between flow and density is the **fundamental diagram** (Greenshields, 1935)



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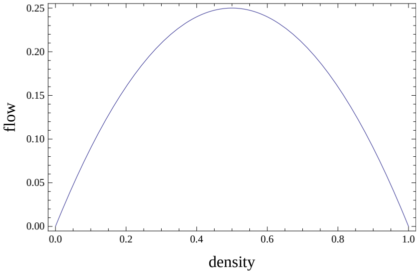

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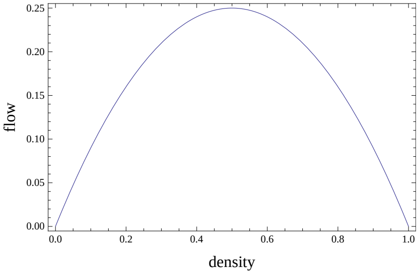

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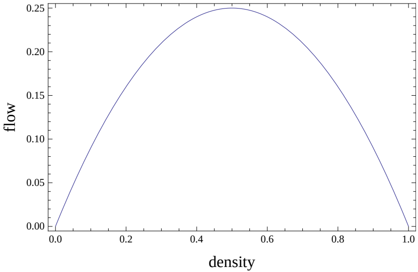

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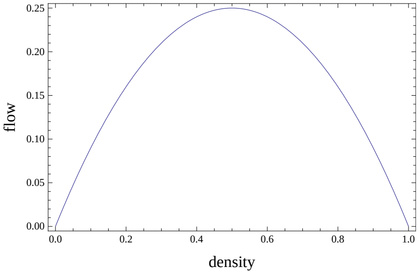

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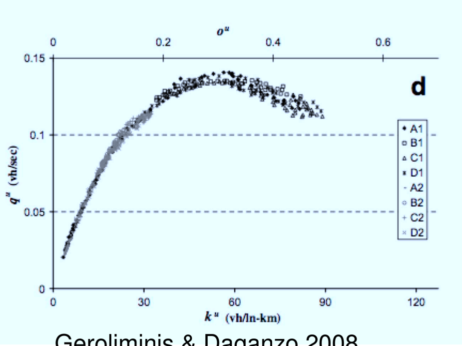
- What should happen in a network?
- How should one even **define** network flow?
(No prescribed direction)

Macroscopic Fundamental Diagrams

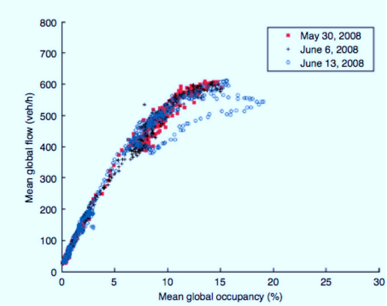
- Simplest idea: relate arithmetic means of link density and flow
- If network has link set Λ:

$$\rho = \frac{1}{|\Lambda|}\sum_{\lambda\in\Lambda}\rho_\lambda, \qquad J = \frac{1}{|\Lambda|}\sum_{\lambda\in\Lambda}J_\lambda$$

- ρ_λ is density of link λ and J_λ is its flow



Geroliminis & Daganzo 2008
Empirical data from Yokohama



Buisson & Ladier 2009
Empirical data from Toulouse

Two Extreme Cases

- Existence of MFDs is trivial:
  - If all links have the same FD
  - and if the distribution of congestion is always perfectly uniform
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  - If one has a network and is free to vary the demand on each link in any way imaginable, then no MFD can exist
  - e.g. half the links have $\rho_\lambda = 1$ and other half have $\rho_\lambda = 0$, then $\rho = 1/2$ and $J = 0$
  - e.g. all links have $\rho_\lambda = 1/2$, then $\rho = 1/2$ but $J > 0$ (could even have $J = J_{\text{max}}$)
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  - Existence of MFDs clearly not independent of demand
- MFDs are interesting because there is something in between
- In practice, on many networks the demand will rise and fall in a fairly constrained way during a typical day

What are MFDs?

Consider a fixed network with link set Λ

First of all, one needs to agree on what ρ and J mean.

- $\rho_\lambda(t)$ and $J_\lambda(t)$ are stochastic processes
- Aggregate variables

$$\rho(t) = \frac{1}{|\Lambda|}\sum_{\lambda\in\Lambda}\rho_\lambda(t) \qquad J(t) = \frac{1}{|\Lambda|}\sum_{\lambda\in\Lambda}J_\lambda(t)$$

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- Can be interested in instantaneous or stationary MFDs
- "Heterogeneity" is also important

$$h(t) = \sqrt{\frac{1}{|\Lambda|}\sum_{\lambda\in\Lambda}[\rho_\lambda(t) - \rho(t)]^2}$$

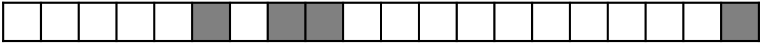
  Helbing 2009; Mazloumian, Geroliminis & Helbing 2010; Geroliminis & Sun 2011; de Gier, G & Zhang 2013
- J, ρ, h all stochastic processes
- In time dependent context, heterogeneity can explain hysteresis

Asymmetric Simple Exclusion Process (ASEP)

"*Everything should be made as simple as possible, but not simpler*"

(Albert Einstein)

- Want an Ising model of traffic flow
- One-dimensional stochastic cellular automata very popular in statistical mechanics starting in 1990s
  - Such models do a reasonable job of explaining qualitative behaviour of freeway traffic
  - "Phantom" jams emerge as consequence of collective behaviour
- Cellular automata are discrete dynamical systems
- Space, time, and state variables are discrete



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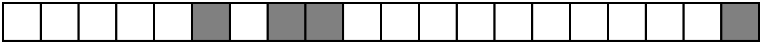

- ASEP with open boundaries:
  - If $x_1(t) = 0$, then with probability α, $x_1(t+1) = 1$
  - For each cell $i = 1, \ldots, L$ with $x_i(t) = 1$
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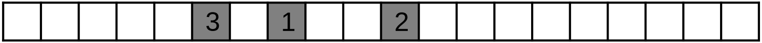
  - Vehicles can have different speeds $0, 1, \ldots, v_{\max}$



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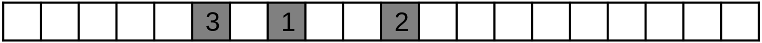

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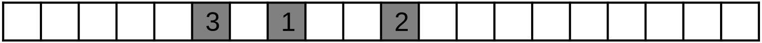

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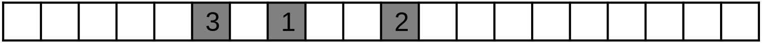

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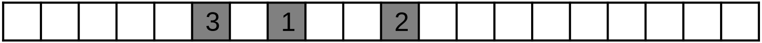

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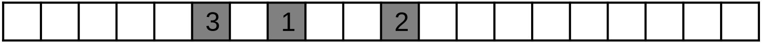

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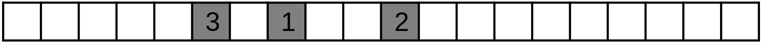

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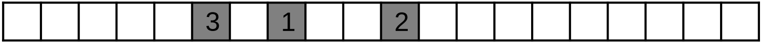

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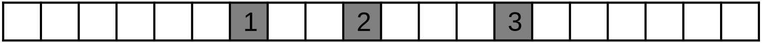

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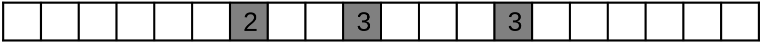

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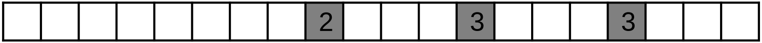

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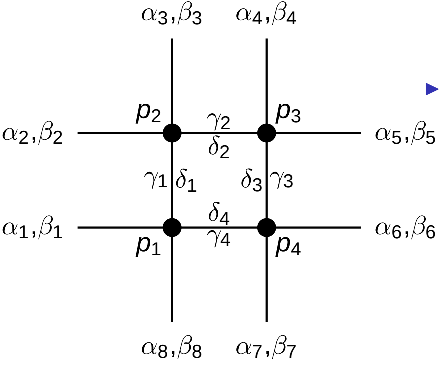
NetNaSch model

Goal: Minimal stat-mech model that can mimic realistic traffic signals

- Take multiple NaSch models and glue them together

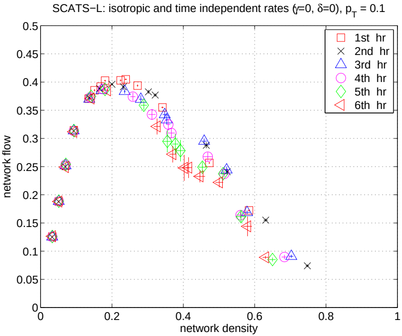


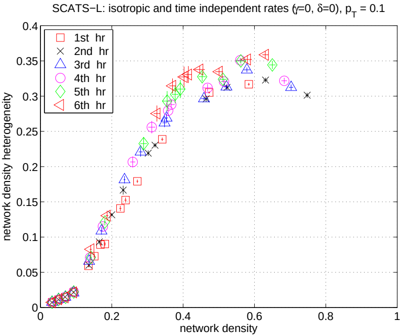
- Need to include:
  - Multiple lanes with lane changing
  - Turning decisions (random)
  - Input and output (endogenous/exogenous)
  - Appropriate rules for how vehicles traverse intersections

Varying all the $\alpha_\lambda, \beta_\lambda, \gamma_\lambda, \delta_\lambda, p_n$... cannot give an MFD
Varying a lower-dimensional space of parameters can

Static demand – Approach to Stationarity

Generate MFD by setting $\alpha_\lambda = \alpha$, $\beta_\lambda = \beta$, $\gamma_\lambda = \delta_\lambda = 0$ for all $\lambda \in \Lambda$

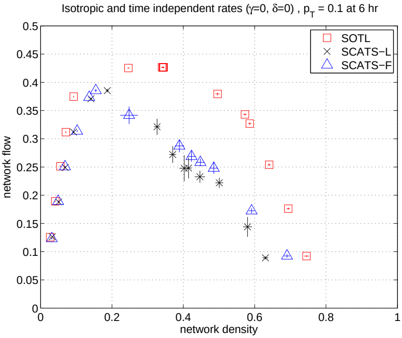




- Intersections governed by model of SCATS with adaptive linking
- Instantaneous MFD converges to stationary curve
- Although there is uniform boundary demand, the density distribution in the network is not homogeneous

Static demand – Stationary MFDs

- Use MFDs to quantify performance of signal systems



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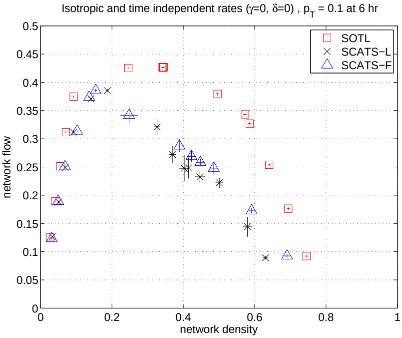
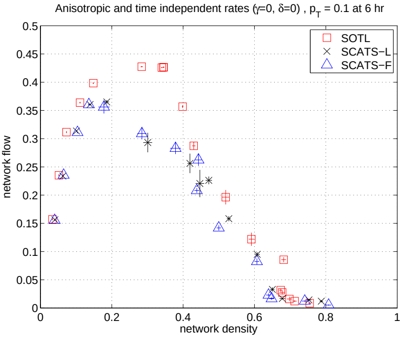


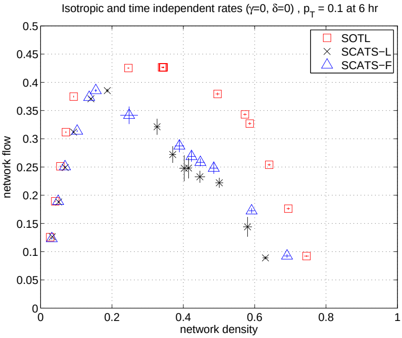
Isotropic boundary demand

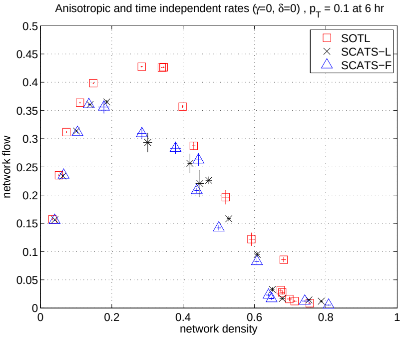


Higher demand on west side

Static demand – Stationary MFDs

- Use MFDs to quantify performance of signal systems



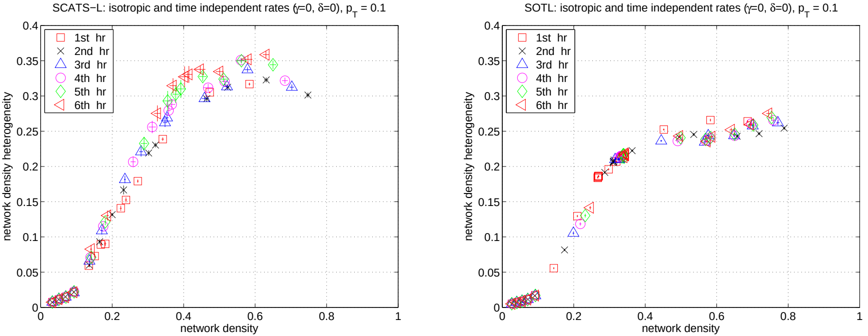


Isotropic boundary demand Higher demand on west side

- Anisotropic demand can still produce well-defined MFD

Self-organizing traffic lights

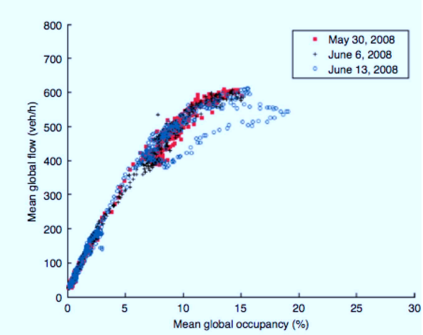
- SOTL is a toy model of a highly adaptive acyclic signal system
- Always gives green to phase with the highest demand



- SOTL has lower heterogeneity than SCATS
- Accounts for its better MFD

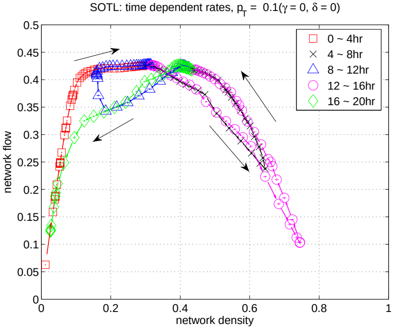
Time-dependent demand

- Vary α, β over 24 hours to mimic am/pm peaks
- Hysteresis observed - clockwise and anticlockwise



Buisson & Ladier 2009

Empirical data from Toulouse

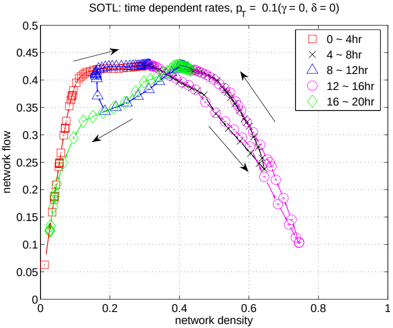


Zhang, G & de Gier 2013

Simulated data

Time-dependent demand

- Hysteresis in MFD consequence of heterogeneity



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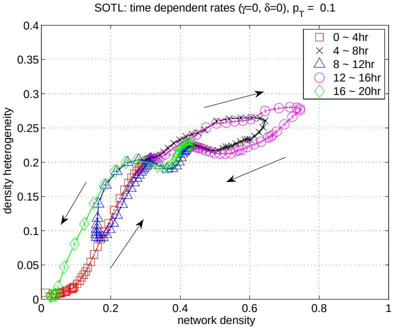

Zhang, G & de Gier 2013
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Two-bin model

- Consider two adjacent networks (bins) exchanging vehicles
- Each bin has same well-defined MFD $J(\rho)$

$$\frac{\mathrm{d}\rho_1}{\mathrm{d}t} = \frac{a_1 - b_1 J(\rho_1) + p_2 J(\rho_2) - p_1 J(\rho_1)}{L_1}$$

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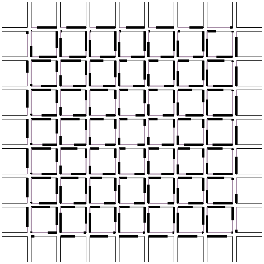
- Let bin 1 be boundary layer, bin 2 the interior



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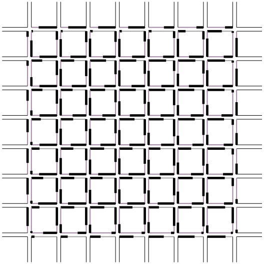

Recovery

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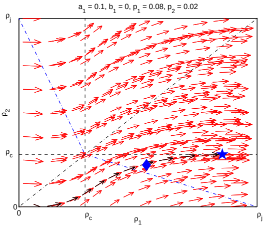

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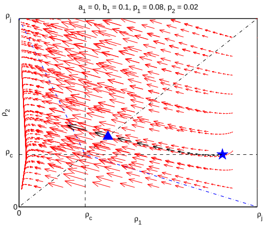

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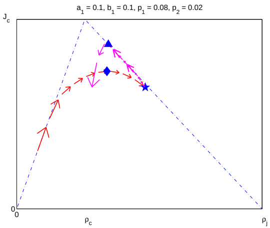

Instantaneous MFD

Open Problems

- Can we observe anticlockwise hysteresis empirically?
- Can we understand cross-correlations between flow, density and density heterogeneity?
- How does driver adaptivity affect the shape of MFDs?

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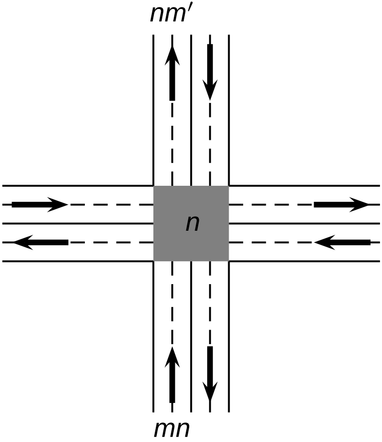
- Can we observe anticlockwise hysteresis empirically?
- Can we understand cross-correlations between flow, density and density heterogeneity?
- How does driver adaptivity affect the shape of MFDs?
- How should one partition networks in order to produce well-defined MFDs?
- Several groups are attempting to use MFDs as a basis for perimeter control?

Paths

Consider a particular node n in a traffic network



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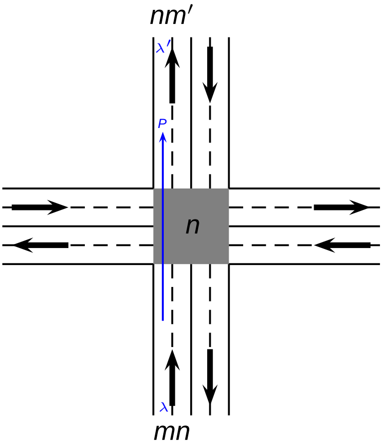

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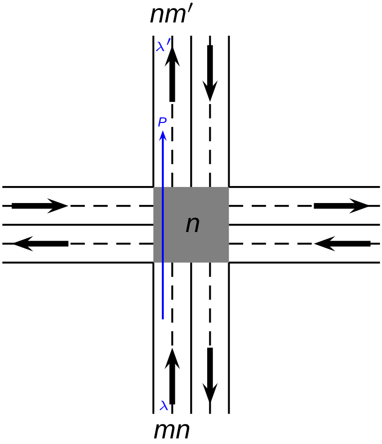

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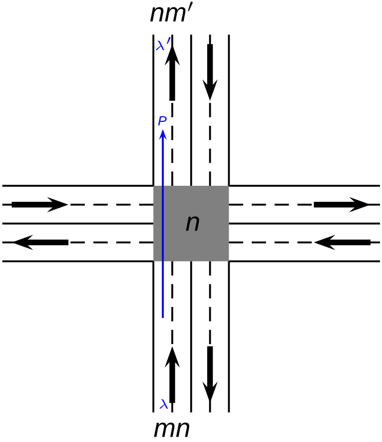

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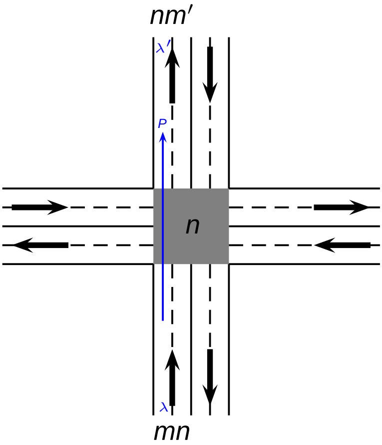

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- No cells in the intersection – we use paths to glue the CA on adjacent links together

Phases

We can't simply let all paths be traversed at once – vehicles would crash inside the intersection

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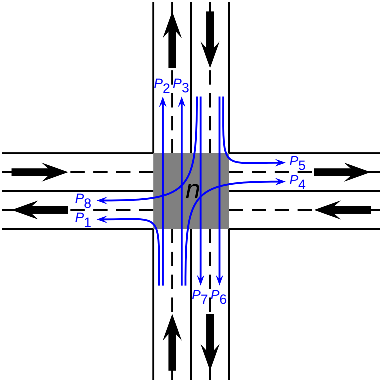

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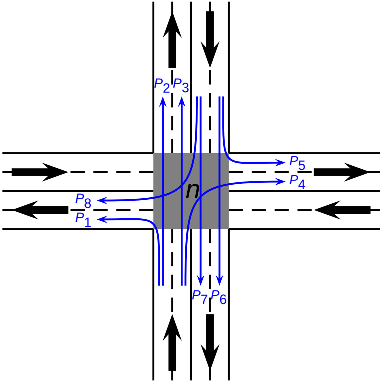

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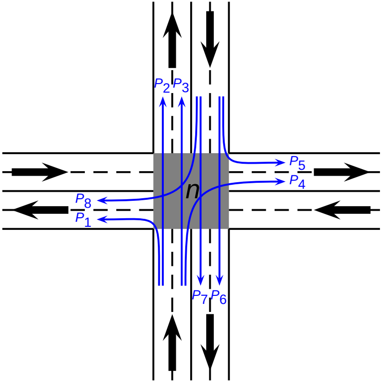

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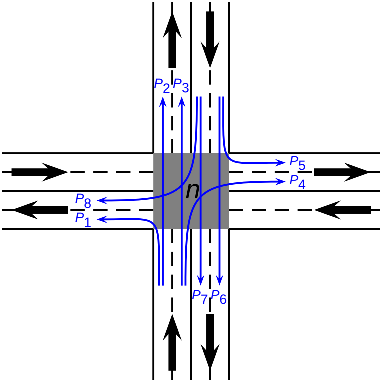

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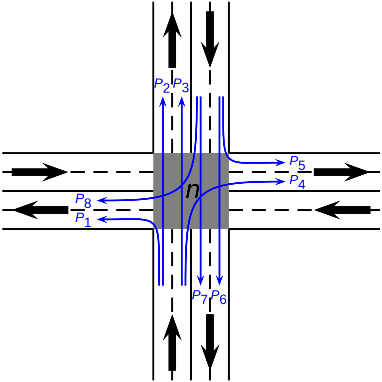

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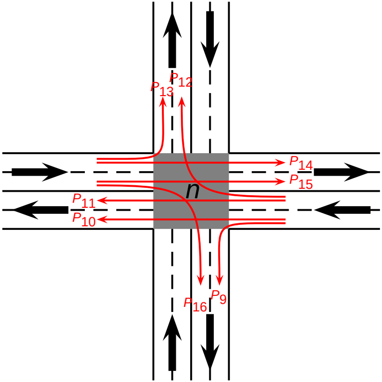

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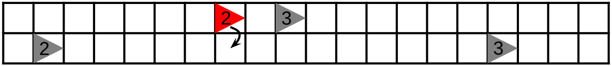
- Time $t + \Delta t$: $\mathcal{P}_{\text{current}} = \mathcal{P}_2 = \{P_9, \ldots, P_{16}\}$

Lane changing (dynamic)

In order to model freeways or urban networks we need multiple lanes and lane changing



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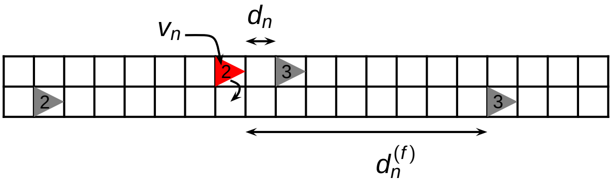

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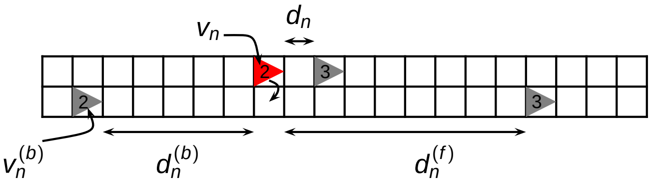

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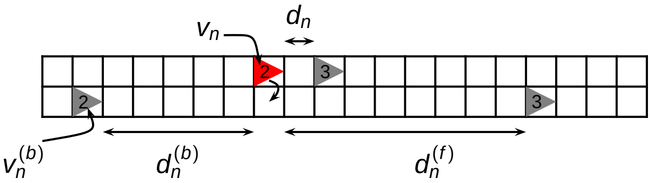

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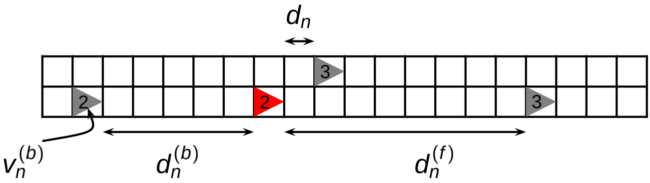

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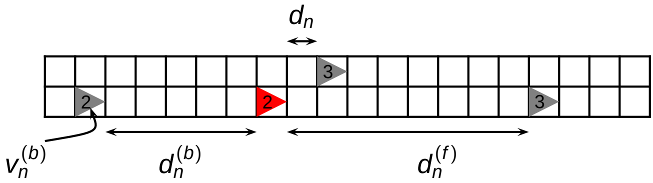

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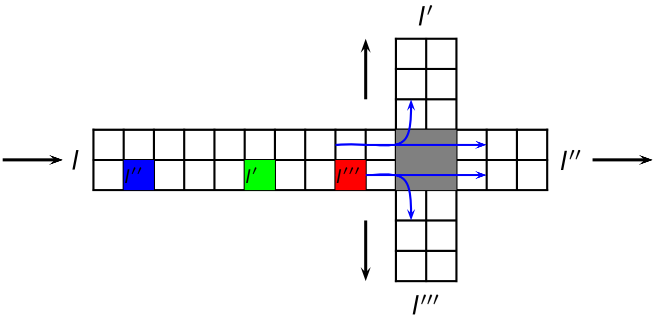
- Allow only left→right (right→left) at odd (even) time steps

Lane changing (topological)



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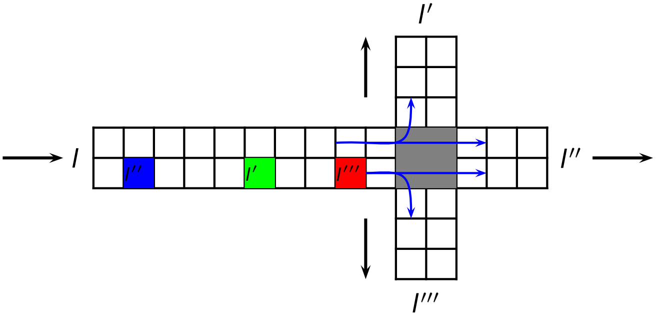

- Each vehicle wants to be in a lane for which there exists a path consistent with its desired turn

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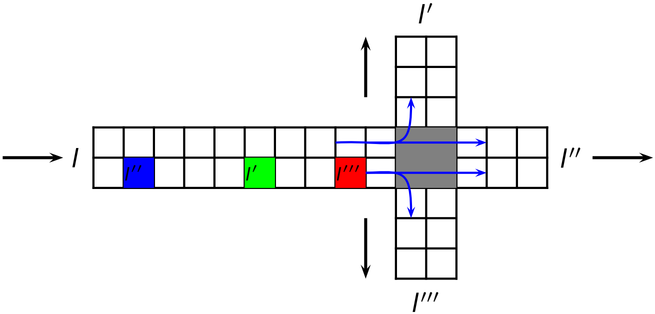

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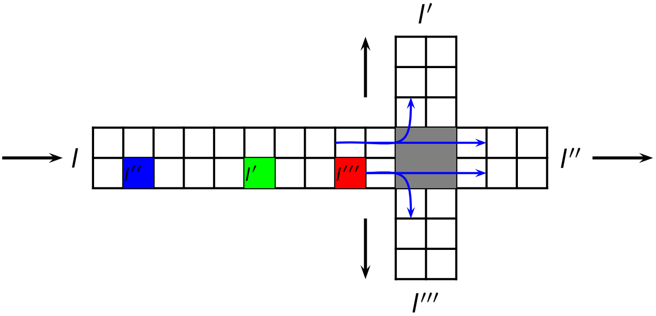

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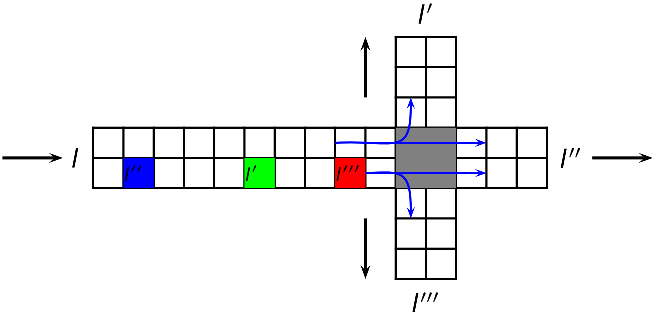

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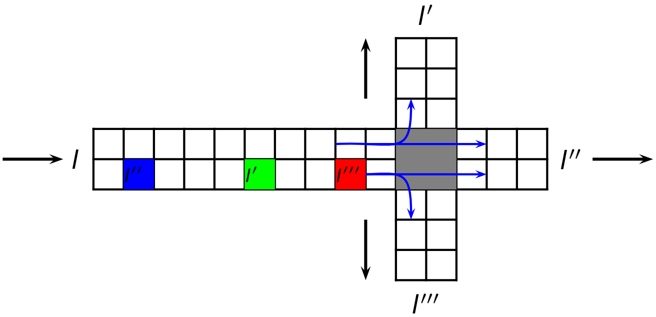

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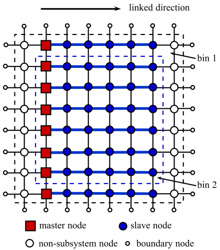
- Only allow dynamical lane changing if it doesn't contradict topological lane changing – only blue car can

Boundaries

- We must consider open systems
- So some links only have one endpoint in the network



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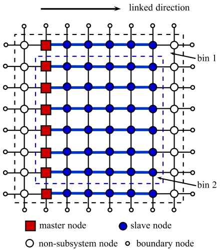

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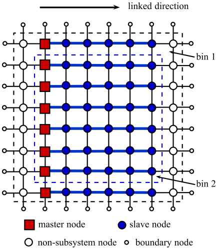

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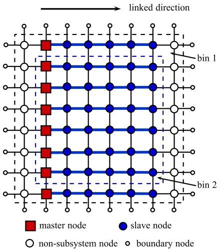

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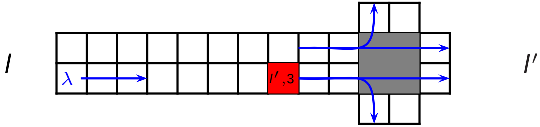
- Turning decisions affect lane changing dynamics

Mark paths



- Consider each lane λ of each link l

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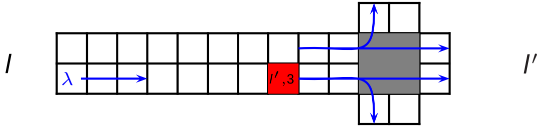

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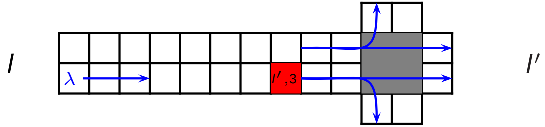

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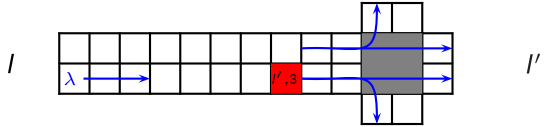

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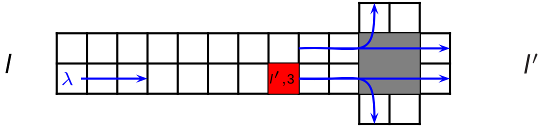
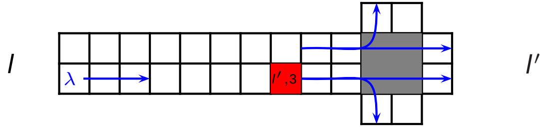
- If there exists $P \in \mathcal{P}_{\text{current}}$ with:

Mark paths



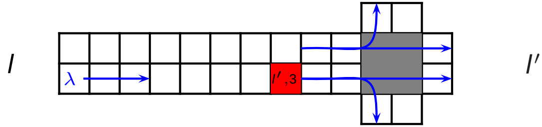
- Consider each lane λ of each link l
- Let **v** be the last vehicle on λ
- Suppose $x(\mathbf{v}) + v(\mathbf{v}) > length(\lambda)$
- If there exists $P \in \mathcal{P}_{\text{current}}$ with:
  - $inlane(P) = \lambda$

Mark paths



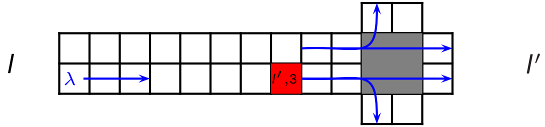
- Consider each lane λ of each link l
- Let $\mathbf{v}$ be the last vehicle on λ
- Suppose $x(\mathbf{v}) + v(\mathbf{v}) > length(\lambda)$
- If there exists $P \in \mathcal{P}_{\text{current}}$ with:
  - $inlane(P) = \lambda$
  - $outlane(P)$ has unoccupied first cell

Mark paths



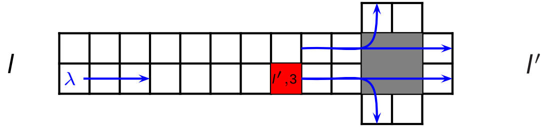
- Consider each lane λ of each link l
- Let $\mathbf{v}$ be the last vehicle on λ
- Suppose $x(\mathbf{v}) + v(\mathbf{v}) > \mathit{length}(\lambda)$
- If there exists $P \in \mathcal{P}_{\text{current}}$ with:
  - $\mathit{inlane}(P) = \lambda$
  - $\mathit{outlane}(P)$ has unoccupied first cell
  - $\mathit{outlink}(P) = \mathit{turn}(\mathbf{v})$

Mark paths



- Consider each lane λ of each link l
- Let $\mathbf{v}$ be the last vehicle on λ
- Suppose $x(\mathbf{v}) + v(\mathbf{v}) > length(\lambda)$
- If there exists $P \in \mathcal{P}_{\mathrm{current}}$ with:
  - $inlane(P) = \lambda$
  - $outlane(P)$ has unoccupied first cell
  - $outlink(P) = turn(\mathbf{v})$
- Then associate $\mathbf{v} \leftrightarrow P$ (in this case we say P is marked)

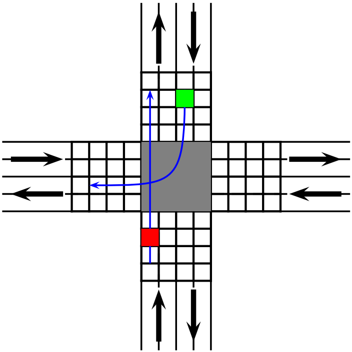
Mark paths



- Consider each lane λ of each link l
- Let $\mathbf{v}$ be the last vehicle on λ
- Suppose $x(\mathbf{v}) + v(\mathbf{v}) > \mathit{length}(\lambda)$
- If there exists $P \in \mathcal{P}_{\text{current}}$ with:
  - $\mathit{inlane}(P) = \lambda$
  - $\mathit{outlane}(P)$ has unoccupied first cell
  - $\mathit{outlink}(P) = \mathit{turn}(\mathbf{v})$
- Then associate $\mathbf{v} \leftrightarrow P$ (in this case we say P is marked)
- Else stop $\mathbf{v}$ at the end of λ

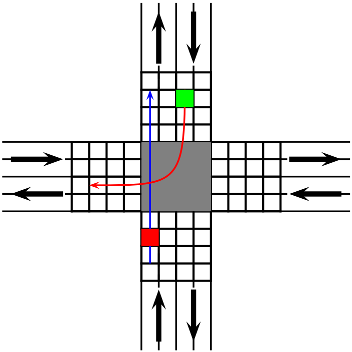
Clear paths

Consider each marked path P of each node n



Clear paths

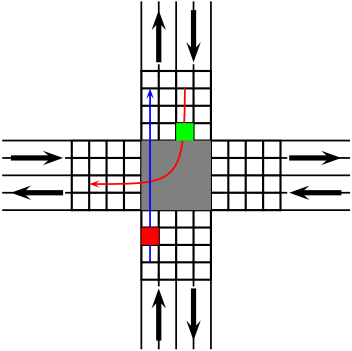
Consider each marked path P of each node n



- If P must give way to another marked path P' of n

Clear paths

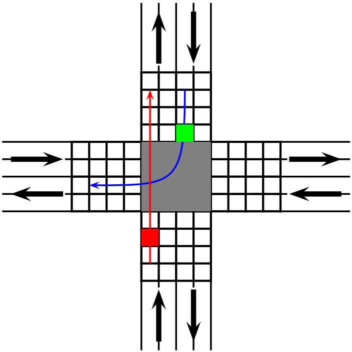
Consider each marked path P of each node n



- If P must give way to another marked path P' of n
  - Stop the vehicle $\mathbf{v} \leftrightarrow P$ on the last cell of *inlane*(P)

Clear paths

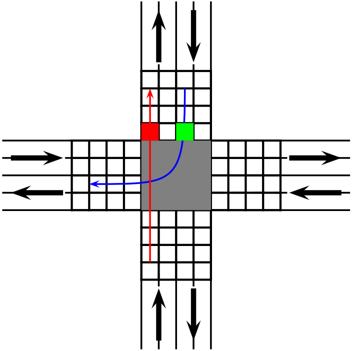
Consider each marked path P of each node n



- If P must give way to another marked path P' of n
  - Stop the vehicle $\mathbf{v} \leftrightarrow P$ on the last cell of *inlane*(P)
- Else move the vehicle $\mathbf{v} \leftrightarrow P$ to the first cell of *outlane*(P)

Clear paths

Consider each marked path P of each node n



- If P must give way to another marked path P' of n
  - Stop the vehicle $\mathbf{v} \leftrightarrow P$ on the last cell of $inlane(P)$
- Else move the vehicle $\mathbf{v} \leftrightarrow P$ to the first cell of $outlane(P)$