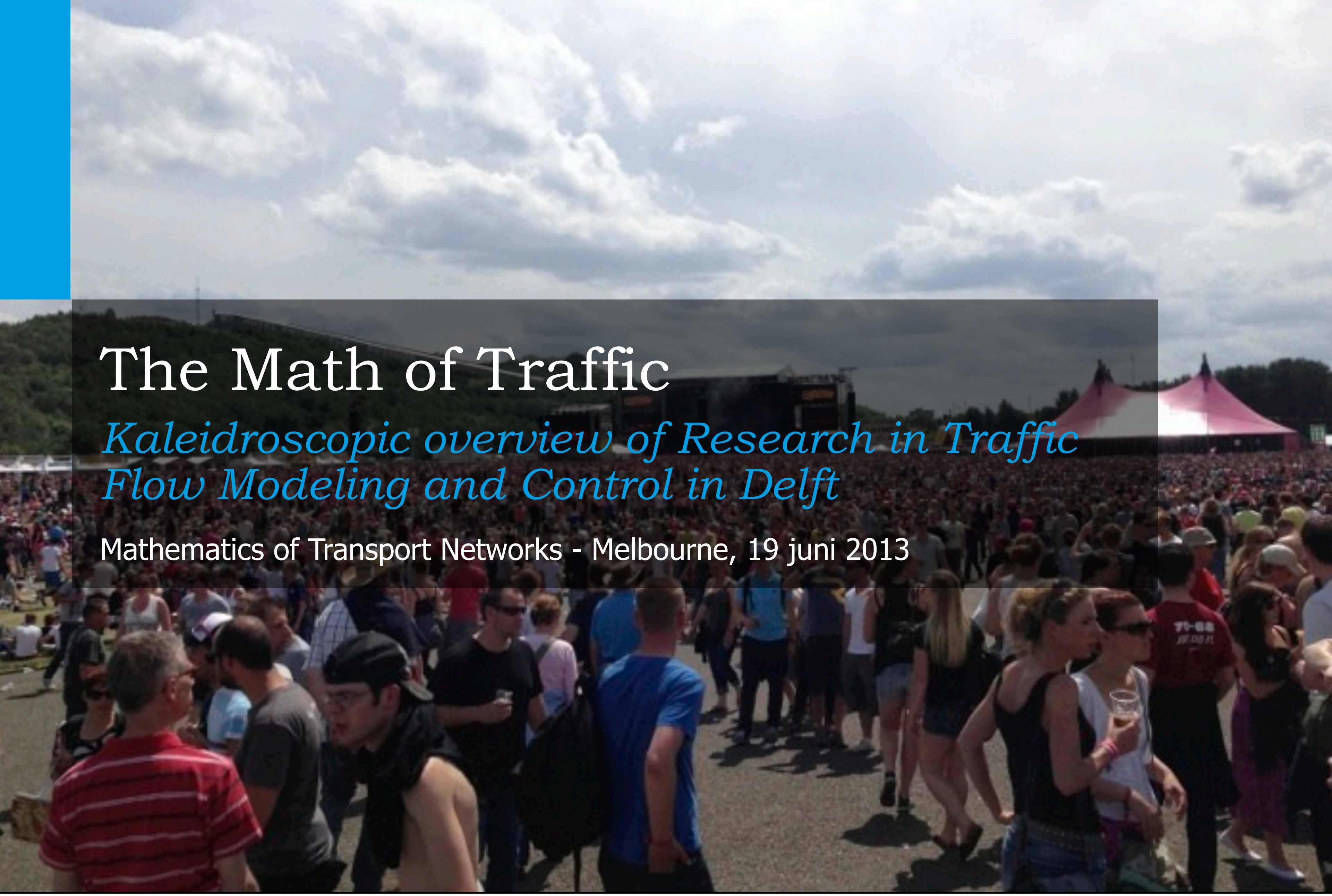


The Math of Traffic

Kaleidroscopic overview of Research in Traffic Flow Modeling and Control in Delft

Mathematics of Transport Networks - Melbourne, 19 juni 2013

Societal urgency: accessibility

Accessibility and Traffic Congestion

- History of traffic queues: from ‘unique sightseeing event’ to major and very common nuisance!

- Costs of traffic congestion in The Netherlands 4.6 billion Euros (2012), for Australia around 8.3 billion dollars (2005)

Societal urgency: accessibility

Reliability of Transport and Network Robustness

- In particular in peak-hours, travel times are hard to predict beforehand
- Trip planners have to take this uncertainty into consideration, resulting in extra cost (VOR = VOT!)

- Moreover, critically loaded networks are often not very robust (relatively small perturbations have very severe effects)
- Examples of robustness issues:
  - Extreme impact of weather (snow)
  - Impacts of incident on critical links


NRC HANDELSBLAD

Posthumeur van ouderen in verpleeghuizen Mens&

130 dagen gegijzeld in Noord-Mali Buitenland 9

Mannenmode is klassiek en veilig dit jaar Cultuur 15

• Langste spits ooit • Ruim 1.000 km file • Grote drukte op stations • Veel leerlingen vrij

36.250 leerlingen van school zonder diploma

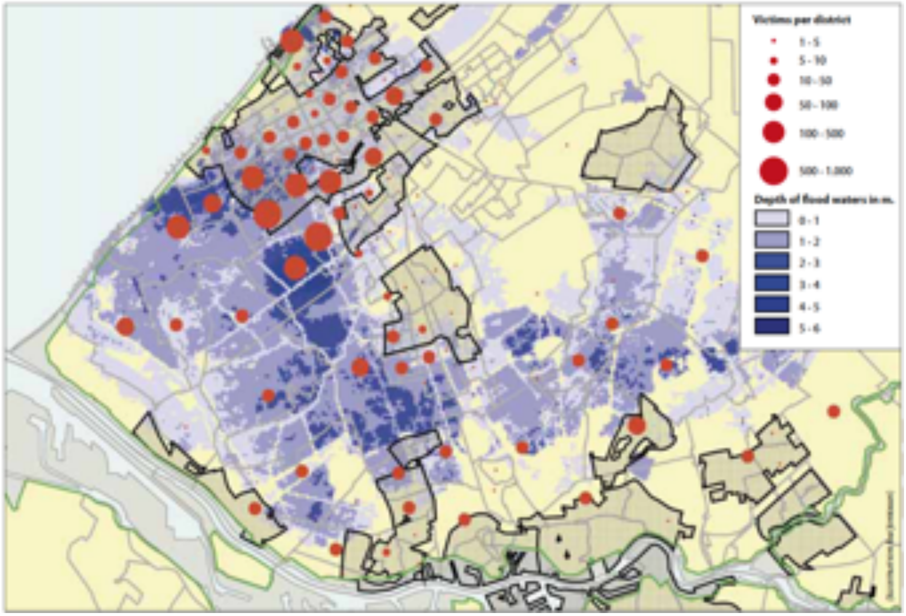
2,- EURO RETOUR

Societal urgency: Safety & Security

Emergencies and Evacuations

- Increasing risks of flooding of highly urbanized Randstad area
- Focus traditionally on prevention, but times are changing!

- Simple simulation
- Normal evacuation plans are inadequate and yield too long evacuation times (> 48 hours)
- How van we improve these plans or otherwise mitigate impacts of an emergency?

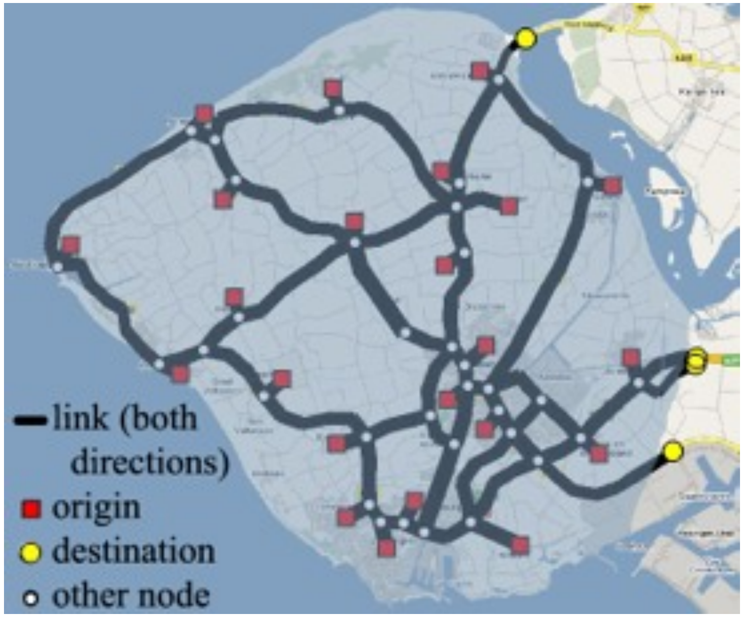


Example EVAQ application

Assessing and improving evacuation plans

- Flood strikes from West to East in six hours in which 120.000 residents / 48.000 cars need to be evacuated
- Capacity of outlinks = 8000 veh/h
- Spatio-temporal dynamics of hazard are known
- Evacuation instructions entail departure time, safe destination, and route to destination

  for specific groups of evacuees (e.g. per area code)
- Use shortest route to closest destination not overloading route



Evacuation of Walcheren

Assessing standard evacuation plan…

Number of evacuated people around 41000 (~34%)

Optimization objectives

Objective applied in this research

- Maximizing function of the number of arrived evacuees in each time period:

$$J(u) = \int_0^T q_u(t)\,dt$$

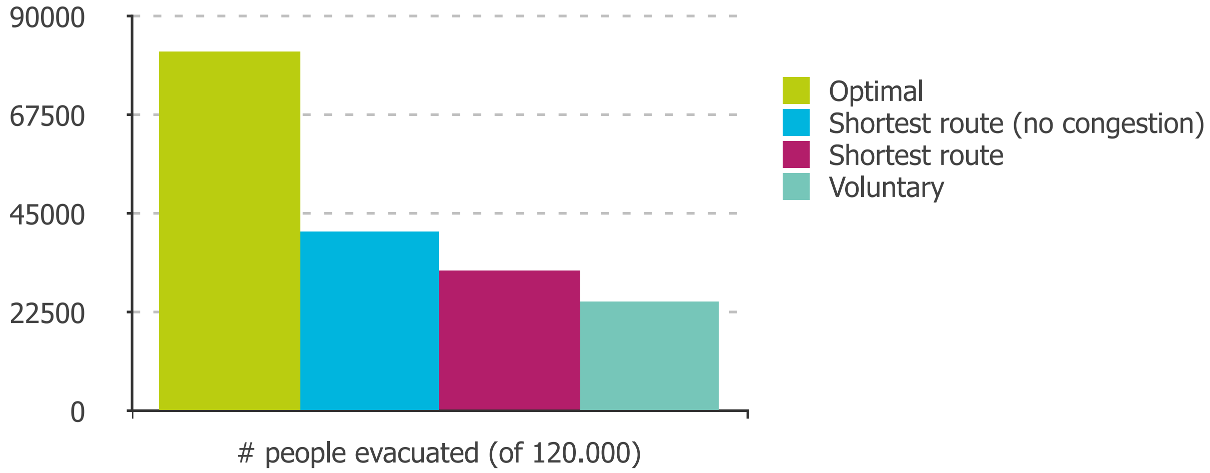
$q_u(t)$ number of arrived evacuees in time period t

u evacuation scheme

- Evacuate as many people as possible
- Use of evacuation simulation model EVAQ to compute J(u) as function of u
- NP hard problem: Ant Colony optimization

Example results
Strategy comparison

- Optimization of evacuation plan yields very significant improvement compared to other scenarios



- Computation times are large, even for small network (10 hrs)

Optimal pedestrian evacuation

Similar problem, different approaches

- Optimal departure time & routing:

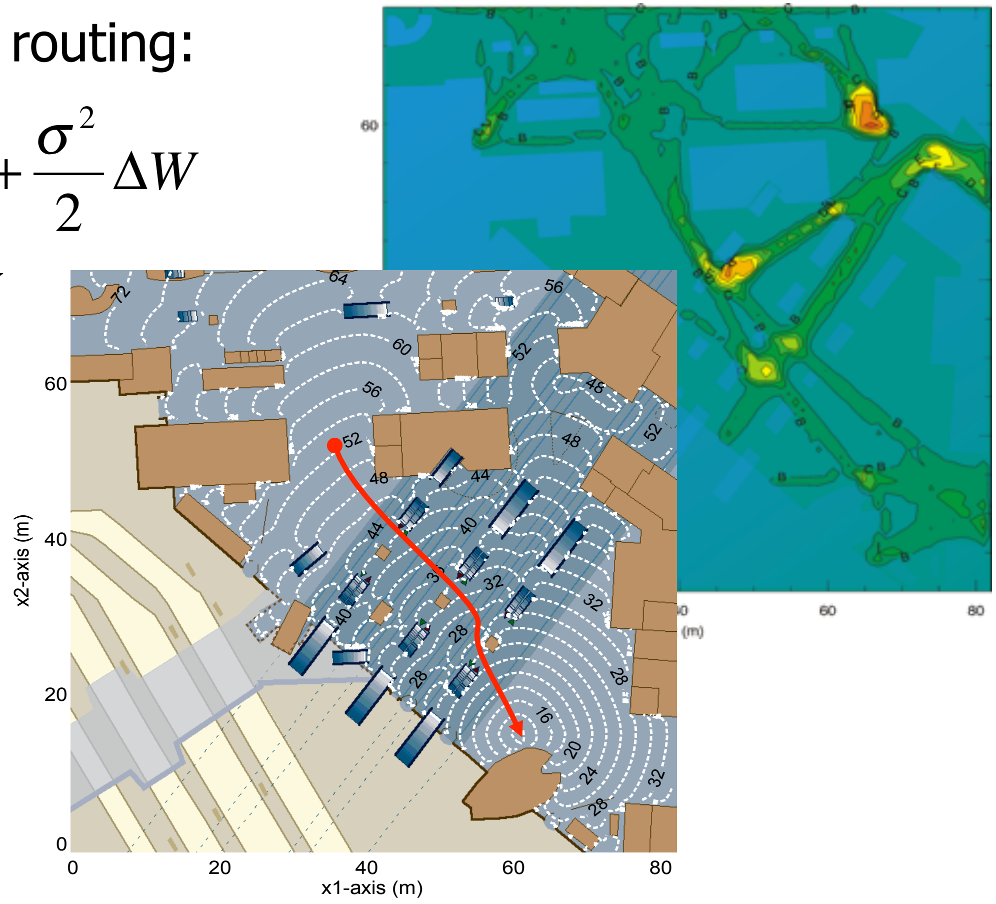
$$-\frac{\partial W}{\partial t} = L\left(t, \mathbf{x}, \mathbf{v}^*\right) + \mathbf{v}^* \nabla W + \frac{\sigma^2}{2} \Delta W$$

$$where\ \mathbf{v}^* = -c_0 \nabla W$$

- Network loading:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \mathbf{x}}\left(\rho \cdot \mathbf{v}\right) = 0$$

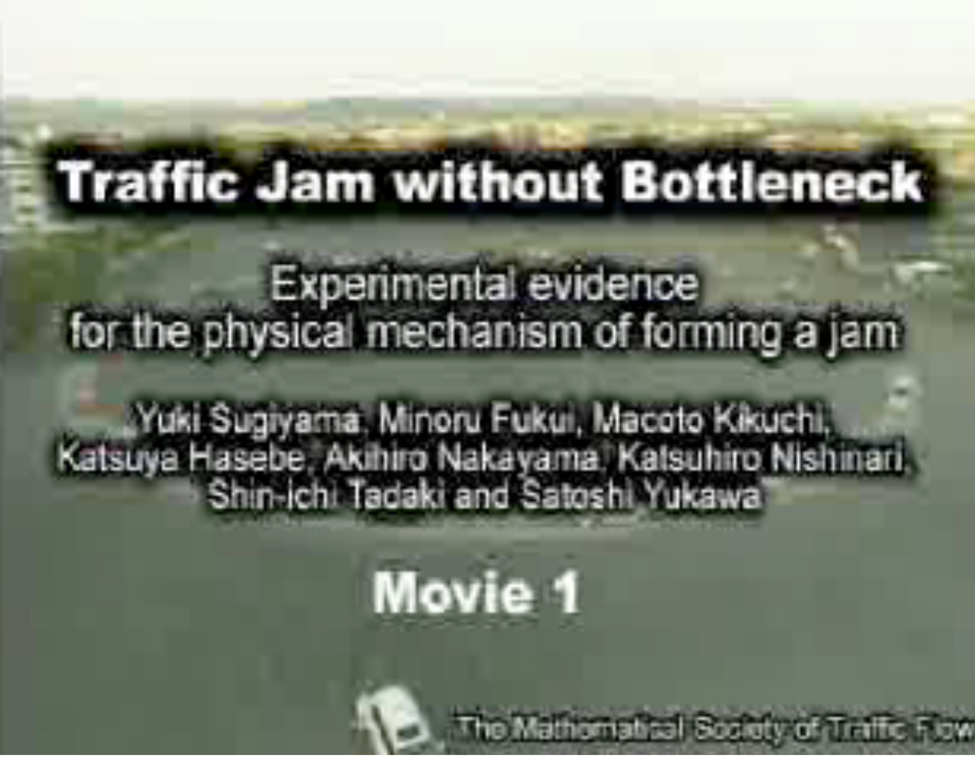
- Fixed point problem...

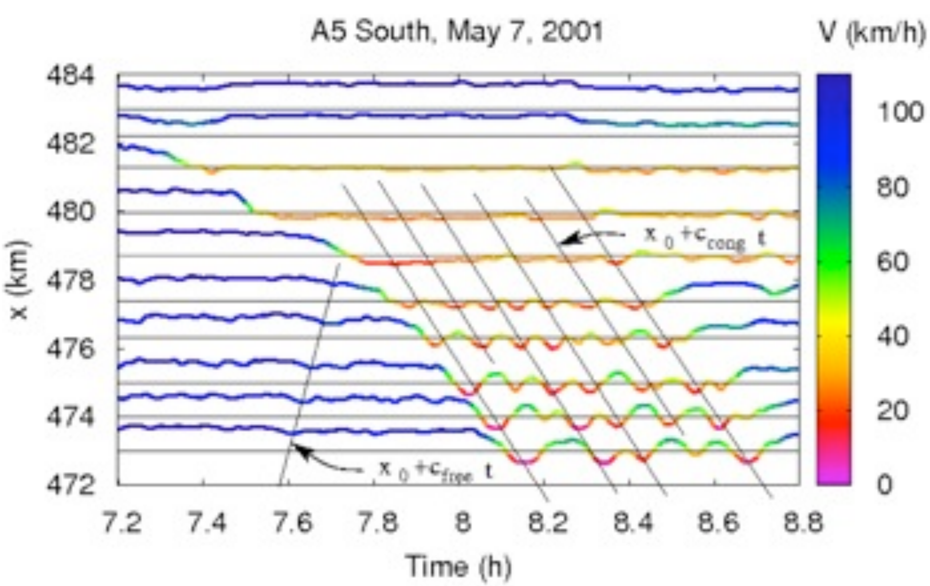


Math and traffic / transportation

Examples of using mathematical techniques

- Evacuation case is example of (off-line) model-based optimization (in this case: evacuation instructions; but also: design, planning)
- Example applications of mathematical techniques:
  - Model-based analysis of traffic and transportation phenomena, e.g. to understand key mechanisms or to determine key decision variables by fitting models
  - Mathematical modeling and simulation for off-line applications (scenario assessment, (network) designs, new ITS measures, etc.)
  - Improving data quality using data fusion by Kalman filtering
  - On-line traffic prediction and analysis of scenarios
  - On-line model-based optimization in for control purposes
- Let's take a look at some other examples...





Traffic instabilities

- Field data analysis (bottom figure) and physical experiments (top movie) show that in certain density regimes, traffic is unstable
- Small disturbances amplify as they travel from one vehicle to the next
- Eventually, disturbance grows into so-called wide moving jam, moving upstream in opposite direction of traffic at speed of 18 km/h
- Outflow of wide-moving jam is about 30% less than free flow capacity

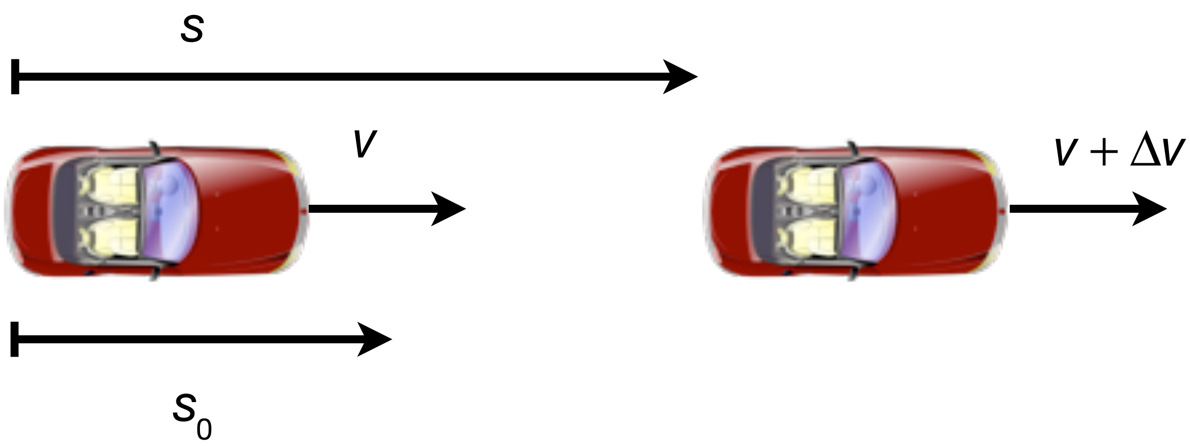
Understanding Traffic Instability

Using relatively simple models…

- CHM car-following model describes acceleration of vehicle in response to distance to predecessor, and speed:

$$\frac{d}{dt} v_i(t + T_r) = \kappa \cdot \Delta v_i(t)$$

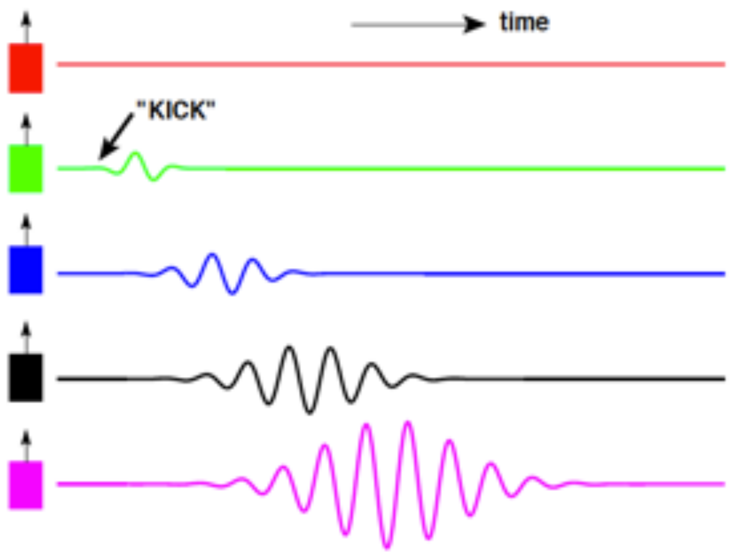
- Parameters are reaction time T_r and sensitivity κ

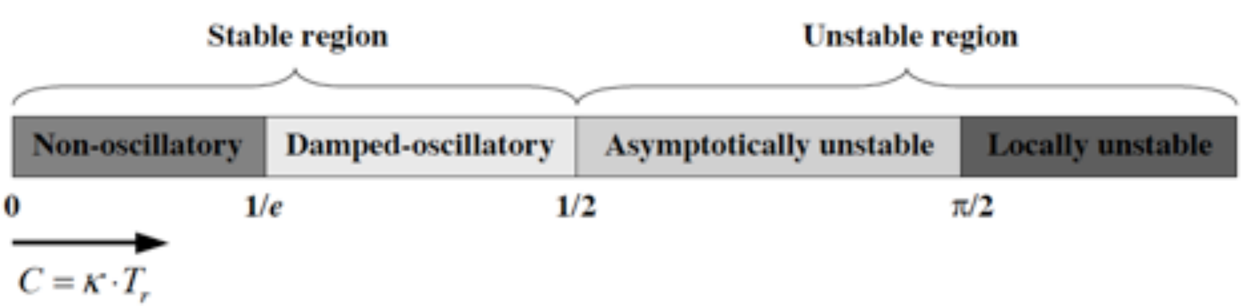


Understanding Traffic Instability

Using relatively simple models…

- Stability analysis of shows for which parameters we get asymptotic instability that is, disturbances grow as they traverse from one vehicle to the next
- It turns out that string stability is determined by:





Understanding Transit disturbances

Propagation of delays through transit networks

- Description of scheduled rail network as a Discrete Event System:

$$x_i(k) = \max\left(\max_j (a_{ij} + x_j(k - \mu_{ij})), d_i(k)\right)$$

k-departure time of train i — $x_i(k)$

travel time from i to j — a_{ij}

departures of previous trains on which i waits — $x_j(k - \mu_{ij})$

scheduled departure time — $d_i(k)$

- Max-plus algebra allows us to rewrite system as a linear system:

$$x_i(k) = \oplus_{j=1..n} (a_{ij} \otimes x_j(k - \mu_{ij})) \oplus d_i(k)$$

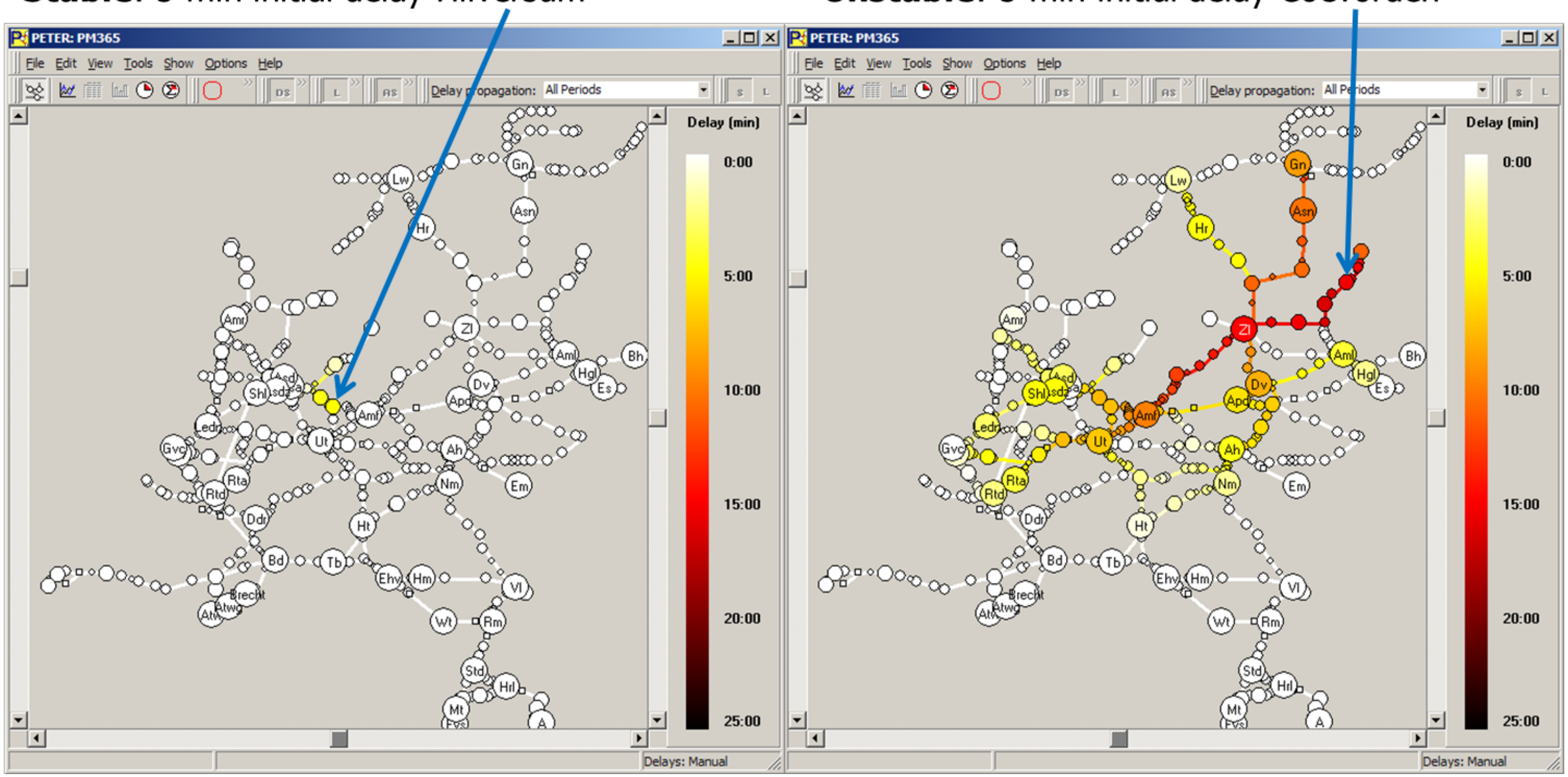
$$x(k) = A \otimes x(k) \oplus d$$

Understanding Transit disturbances

Propagation of delays through transit networks

Stable: 5 min initial delay Hilversum

Unstable: 5 min initial delay Coevorden



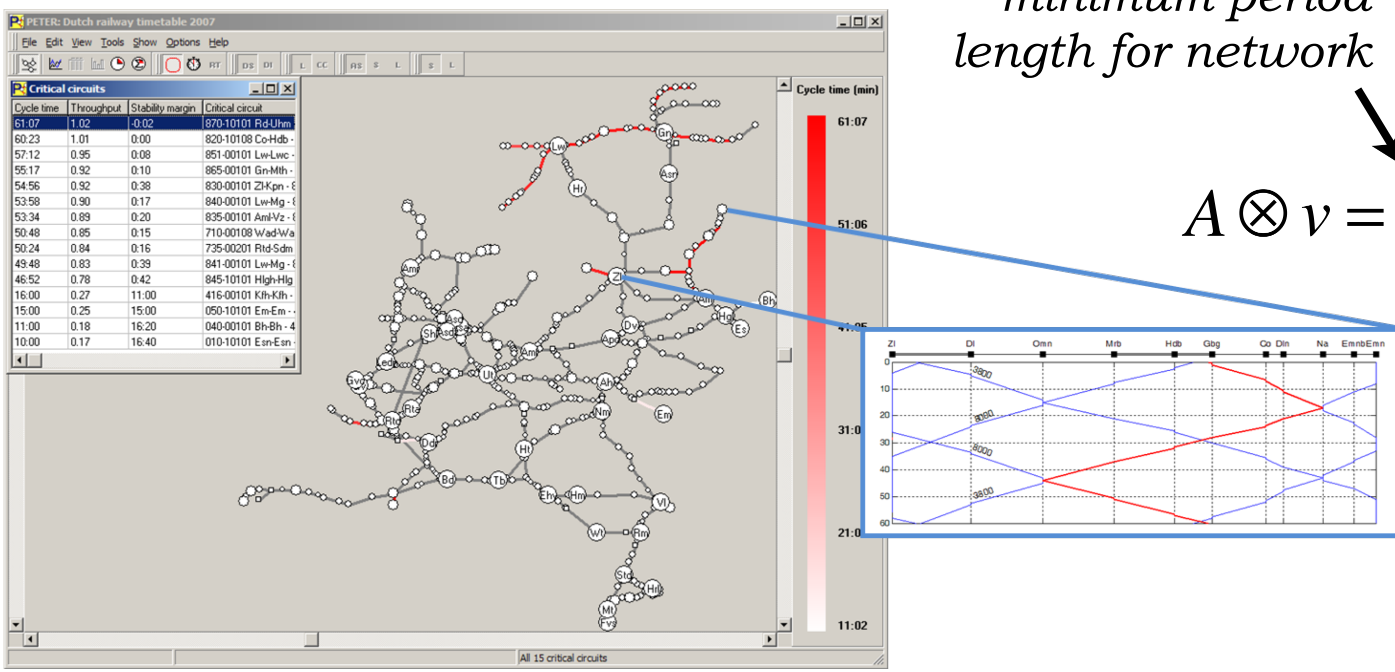
Understanding Transit disturbances

Propagation of delays through transit networks

- Stability of delay propagation can be analyzed by looking at eigenvalues of A

minimum period length for network

$$A \otimes v = \lambda \otimes v$$



periodic minimal time-table for all trains

State estimation

Making sense of real-time traffic data…

State estimation and data fusion

Estimate traffic state from different data sources

- Problems using Kalman filter approach using LWR model because of problematic linearization
- Use of Lagrangian formulation (change of coordinate system)

$$\frac{\partial \rho}{\partial t} + \frac{\partial q(\rho)}{\partial x} = 0 \longrightarrow \frac{\partial s}{\partial t} + \frac{\partial v(s)}{\partial n} = 0$$

Godunov Upwind

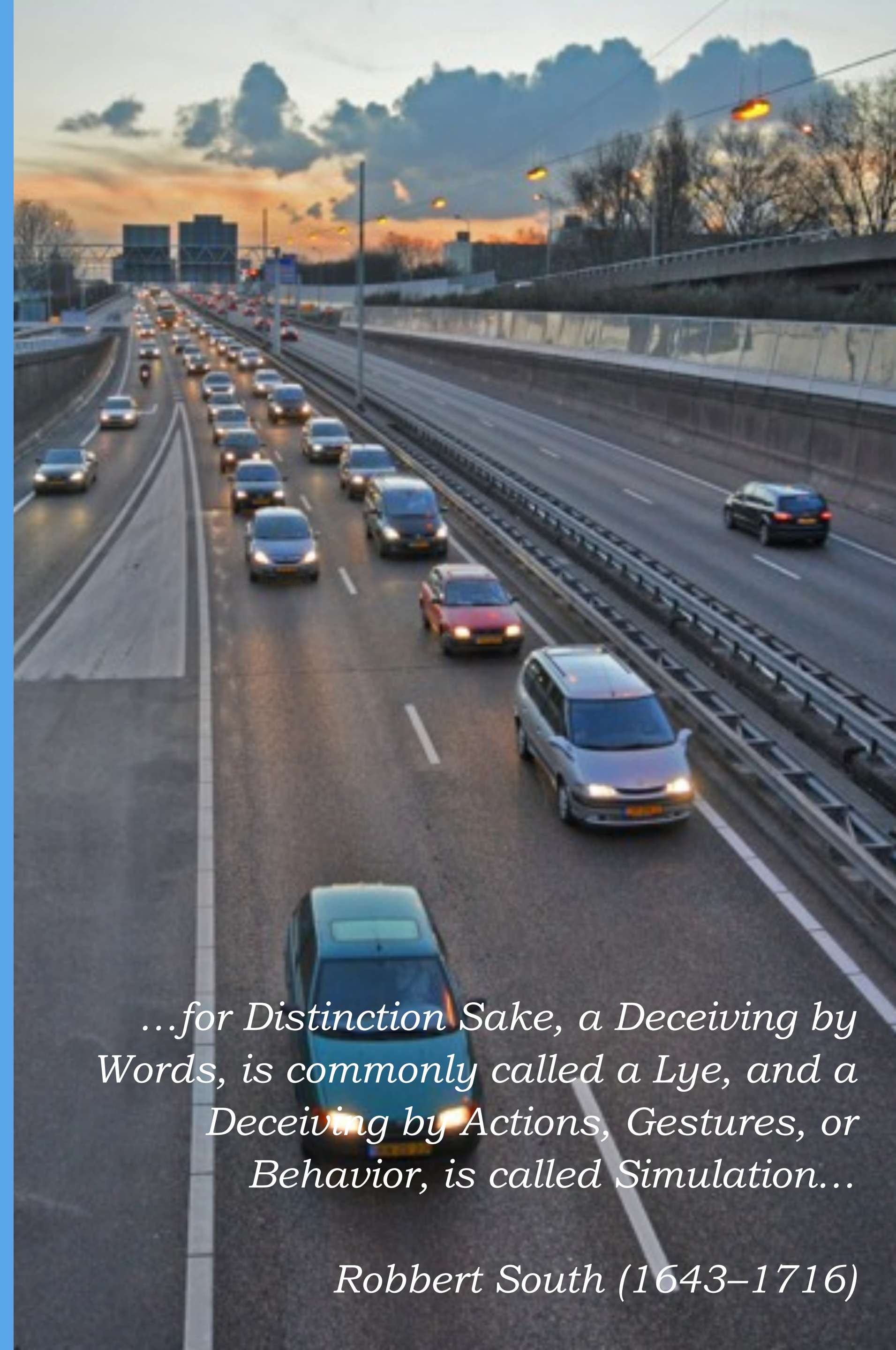
- Advantages of Lagrangian formulation:
  - Easy numerical discretization (upwind) with almost no num diffusion
  - A natural set of observation equations to deal with Lagrangian sensing data (probe vehicle, trajectory-based data)
  - Advantageous properties of application EKF (compared to Godunov)

Modeling

Not an exact science!

Traffic and Transport Models

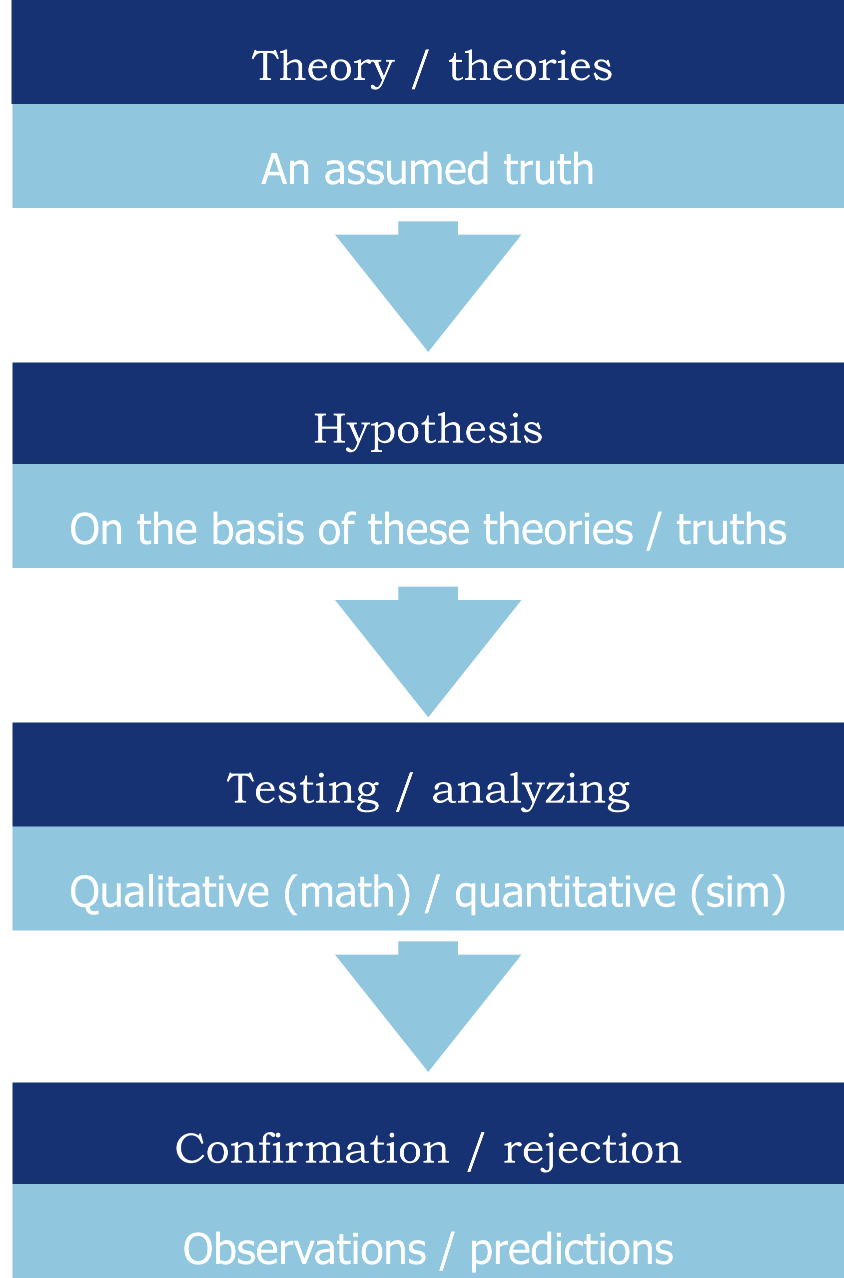
- Traffic operations result from human decision making and complex multi-actor interactions at different behavioral levels)
- Human behavior is ‘not easy to capture and predict’
- System is highly complex, non-linear, has chaotic features, etc.
- Challenge is to develop theories and models that represent and predict operations sufficiently accurate for application at hand
- But how is this achieved? Induction vs deduction...



Deduction

Modeling approaches

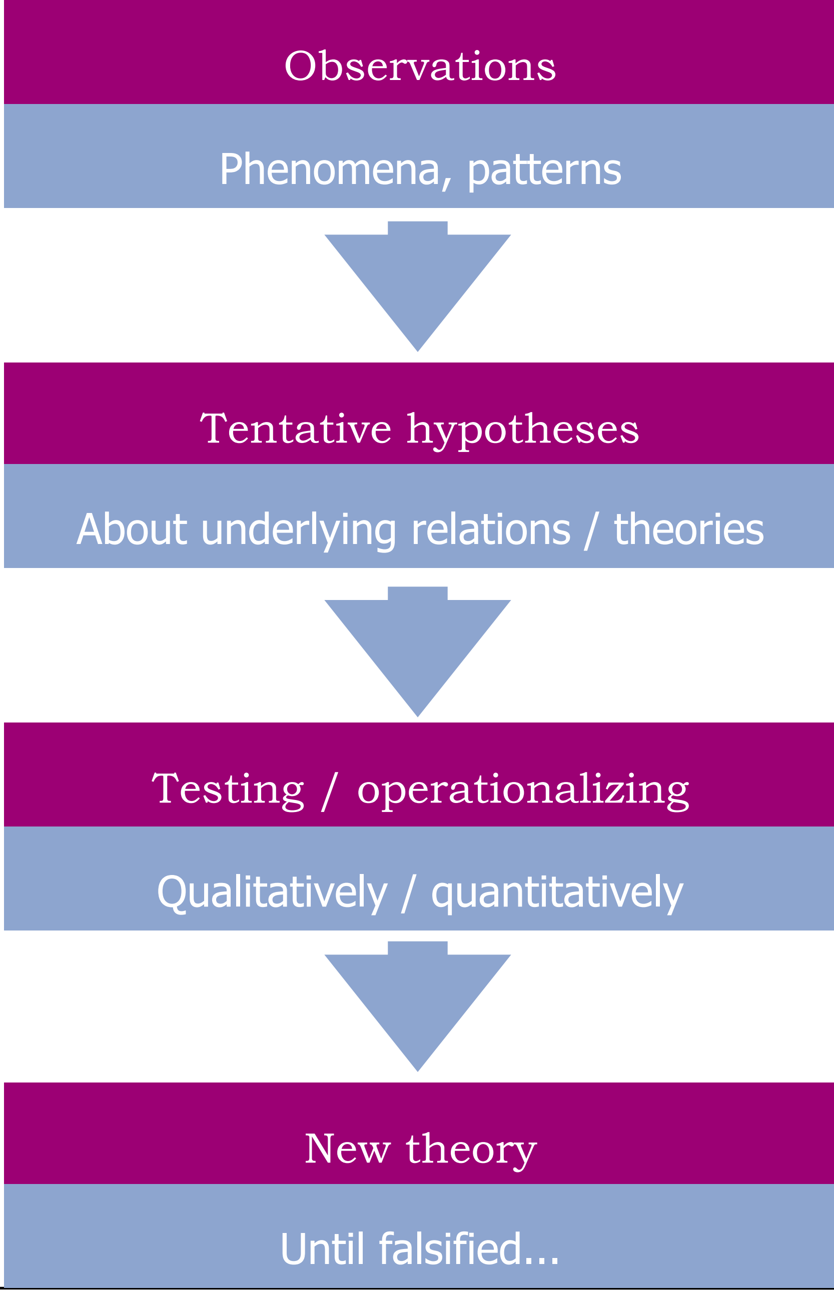
- Starts with an axiom, an assumed truth, a theory (which come from an observations, logic, other theories)
- Typical in (theoretical) physics, mathematics
- Example: special theory of relativity (Einstein postulated that the speed of light is the same for all observers, regardless of their motion relative to the light source – observations proved him right)



Induction

Modeling approaches

- Starts with observations (phenomena, patterns, etc.)
- Typical in social sciences and biology
- Example: Darwin's theory of evolution by natural selection (Darwin observed populations finks diverging in different habitats and postulated natural selection as the motor – modern genetics, biology and many, many other scientific disciplines proved him right)



Traffic and Transportation Theory?

Inductive or deductive?

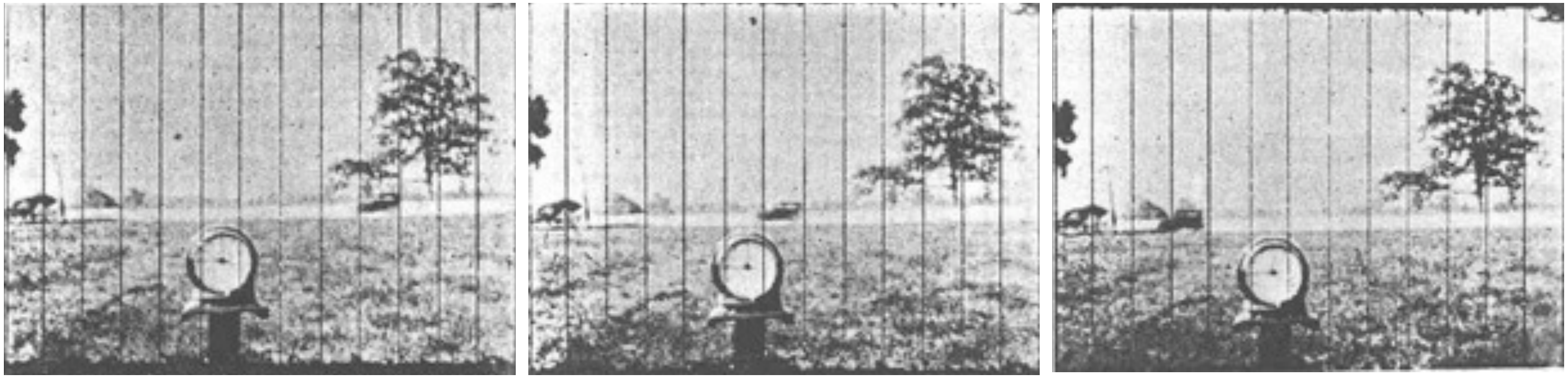
- Traffic flow theory is **largely** based on **induction** (with a bit of deduction): theory building is for a large part based on empirical or experimental observations
- Our theories and models are as good as the quality of their predictions (and should be assessed with that in mind!)
  - Do they predict the **key phenomena and traffic flow features** we observe in the real world?
  - Do they incorporate **a (mathematical) structure** that provide insight into how these phenomena emerge?

- Let us consider some of these phenomena, starting with the father of traffic flow theory...

Bruce Greenshields…

The discovery of the Fundamental Diagram

- First traffic data collection using cameras and may hours of manual labour…

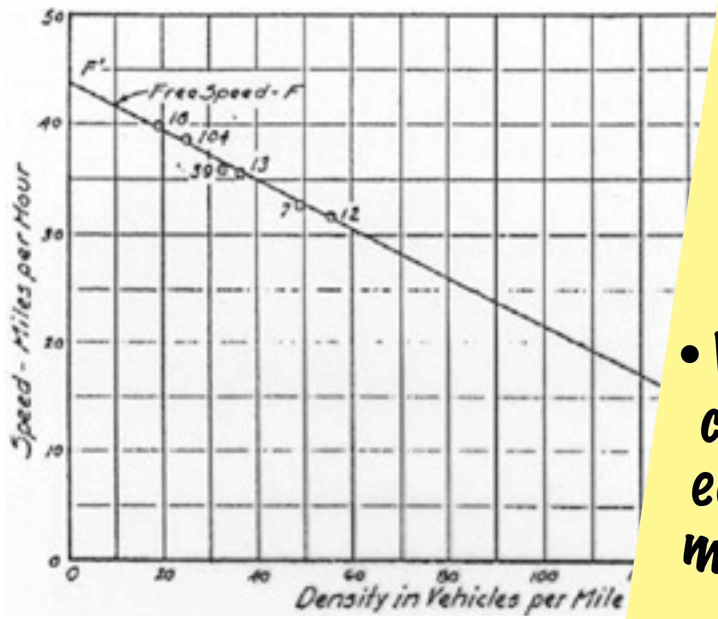


- Studied relation between average vehicle speeds and vehicle density (= average distance^{-1}) and found an important relation

Bruce Greenshields

The discovery of the Fundamental Diagram

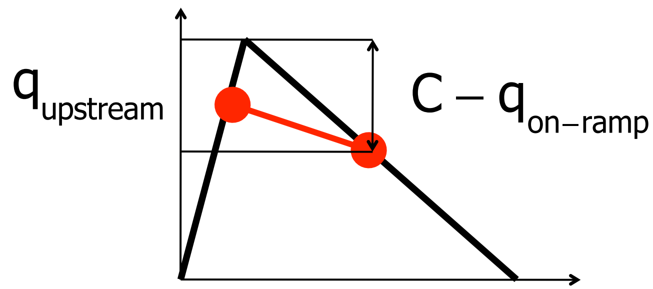
- Decreasing relation between speed and density
- When speed decreases, drivers drive closer



- Although the assumption of a linear relation turned out to be flawed, FD formed basis for contemporary traffic flow theory!
- With q = ku = Q(k) and conservation of vehicle equation we get a complete model of traffic flow!

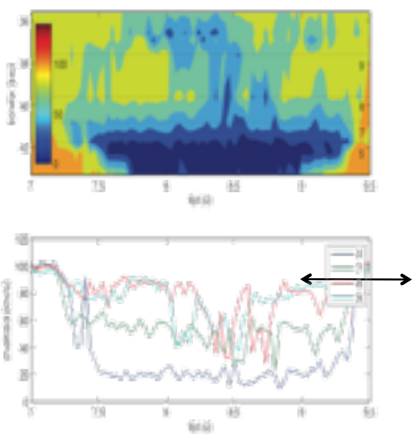
First-order theory

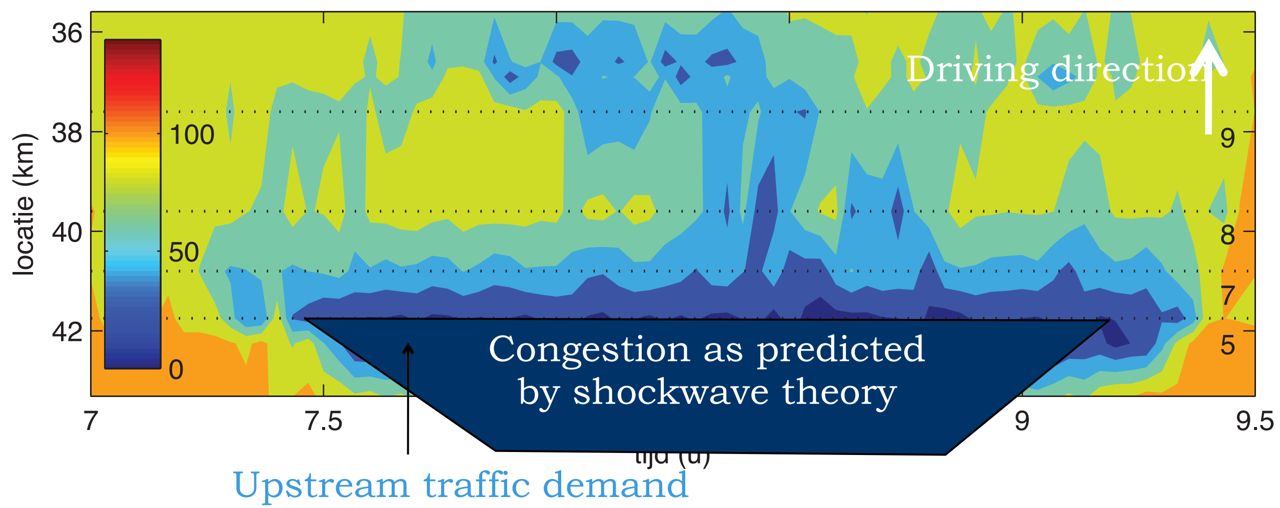
Application of the FD



- Predicting queue dynamics using first order theory
- Predicts dynamics of congestion using FD
- Flow in queue = $C - q_{on\text{-}ramp}$
- Shock speed determined by:

$$\omega_{12} = \frac{Q(k_2) - Q(k_1)}{k_2 - k_1}$$





With improved data collection to better theory!

- Data collection system for collecting high-frequency images from the air (helicopter, drones)
- Algorithms for stabilization of images and geo-referencing
- Vehicle detection and tracking, resulting in high-resolution data on revealed driving behavior (long + lat)
- 15-30 min of data, 500 m roadway, 15 Hz, 40 cm resolution, all vehicles!
- Multiple data sets for variety of circumstances (congestion, merges, incidents, etc.)

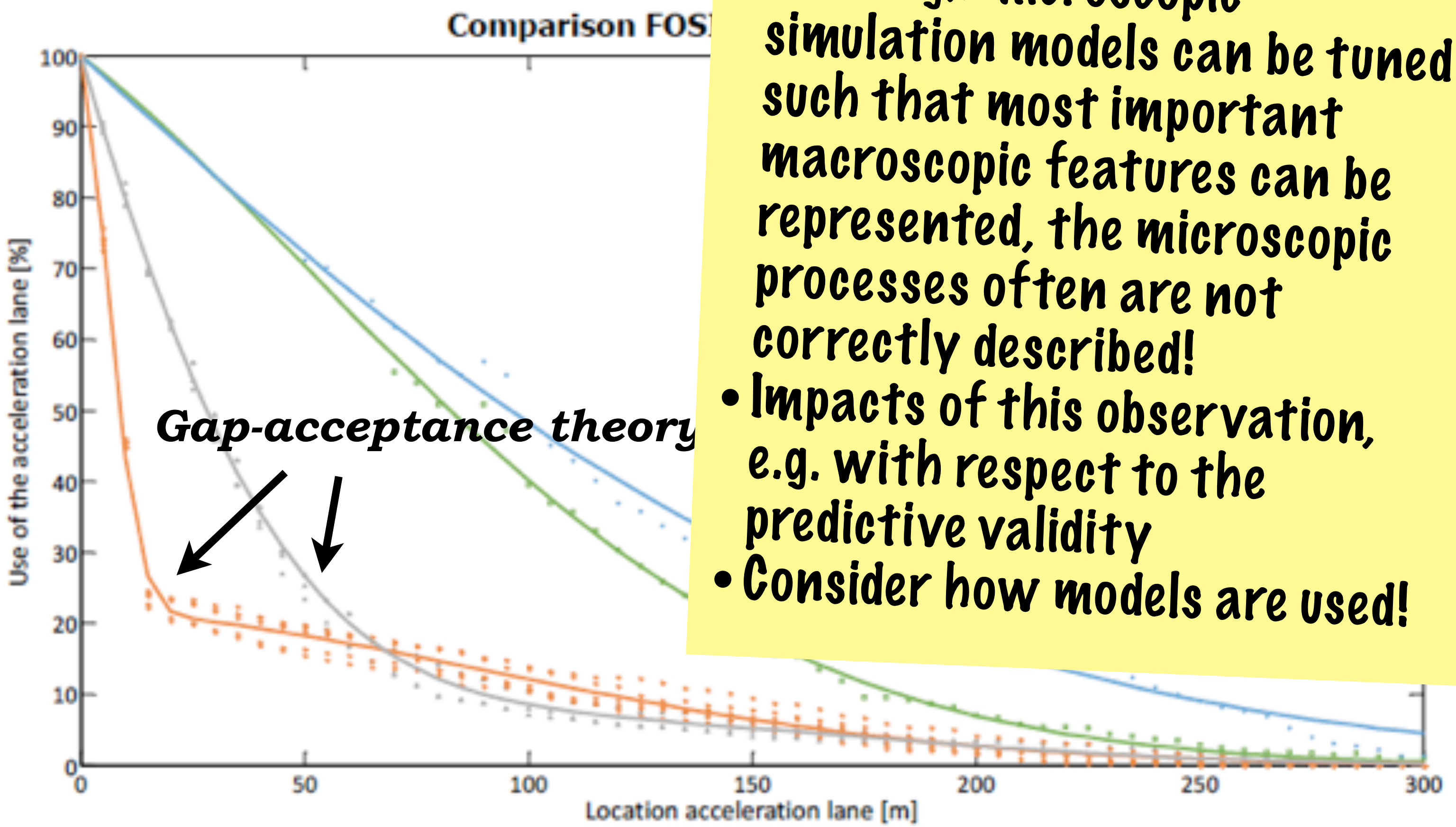
Vehicle trajectory information

Example of findings

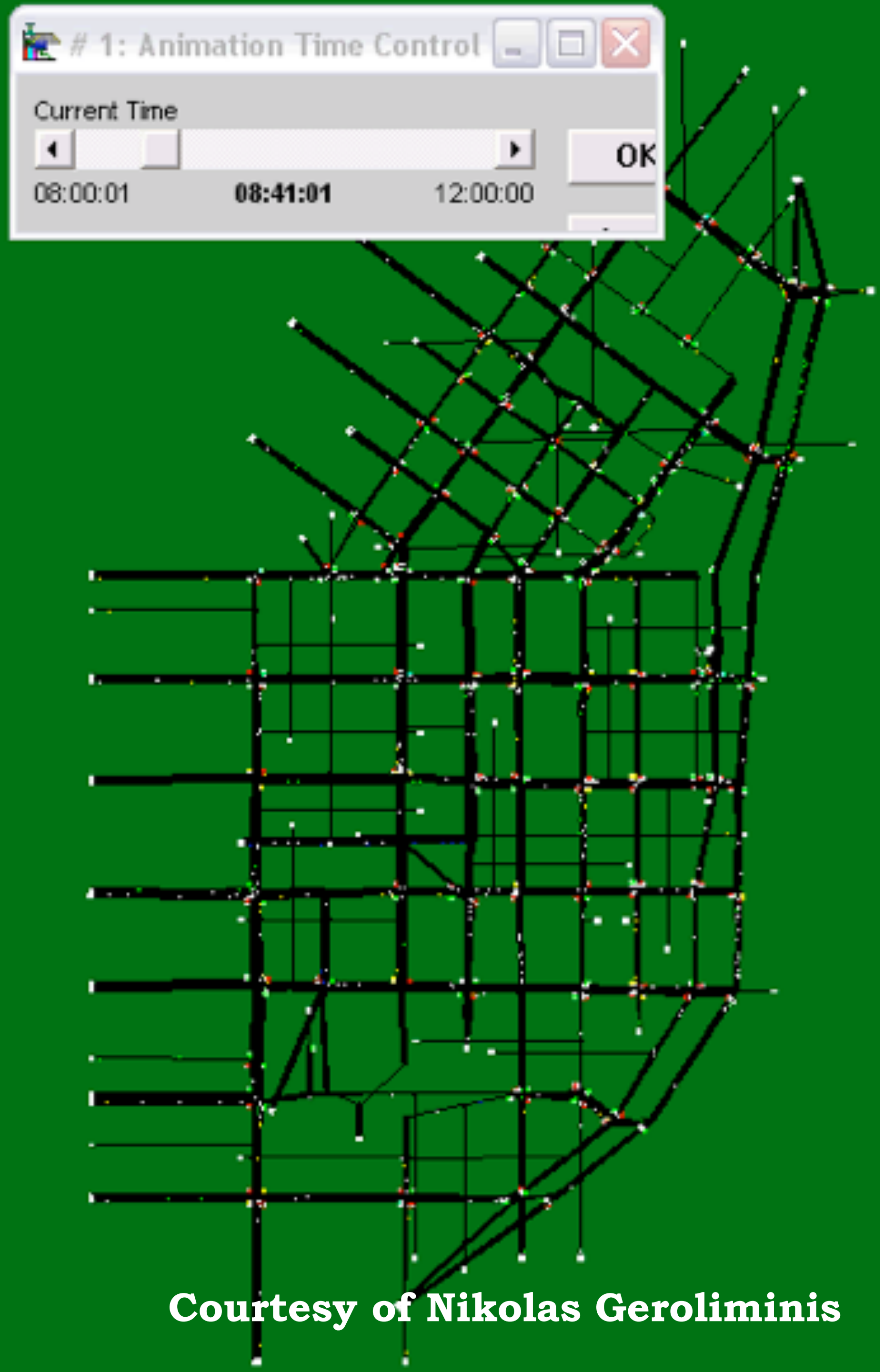
- New data has provided avalanche of new insights for regular and non-recurrent conditions:
  - Driver heterogeneity and adaptation effects (e.g. in case of incidents)
  - Benchmarking of car-following models
  - Discontinuous car-following behavior (action points)
  - Detailed analysis of lane changing and merging behavior
- Example analysis merging behavior:
  - **Accepted models for merging turn out to be flawed** since drivers actively select gap actively rather than passively accept it
  - Paradigm shift and new mathematical models yield increased predictive validity of microscopic flow models
  - Practically: distribution of merging points far less concentrated

Vehicle trajectory information

Example of findings

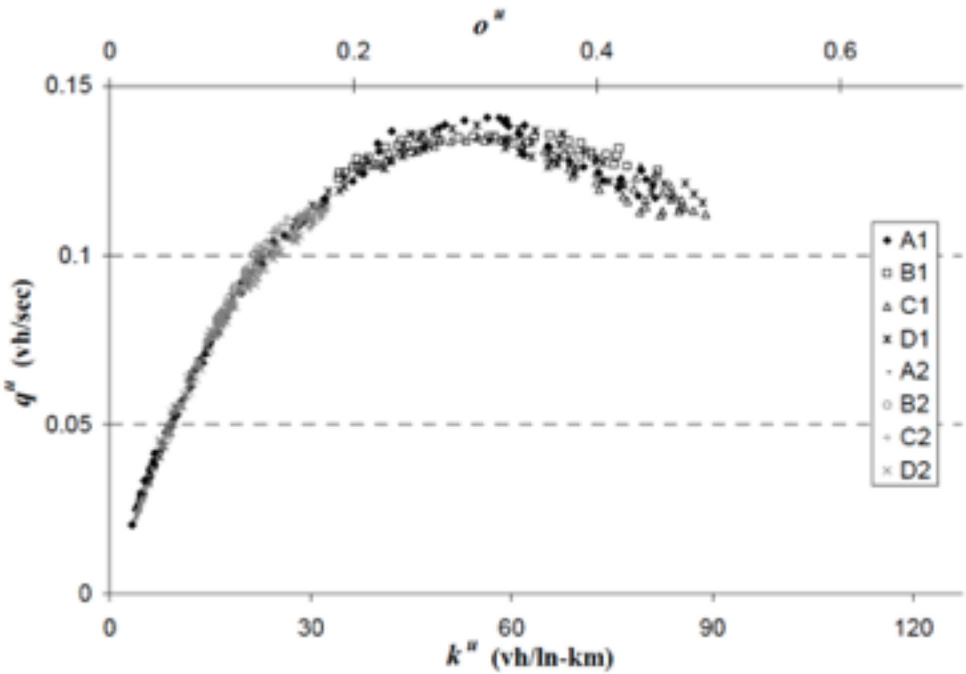


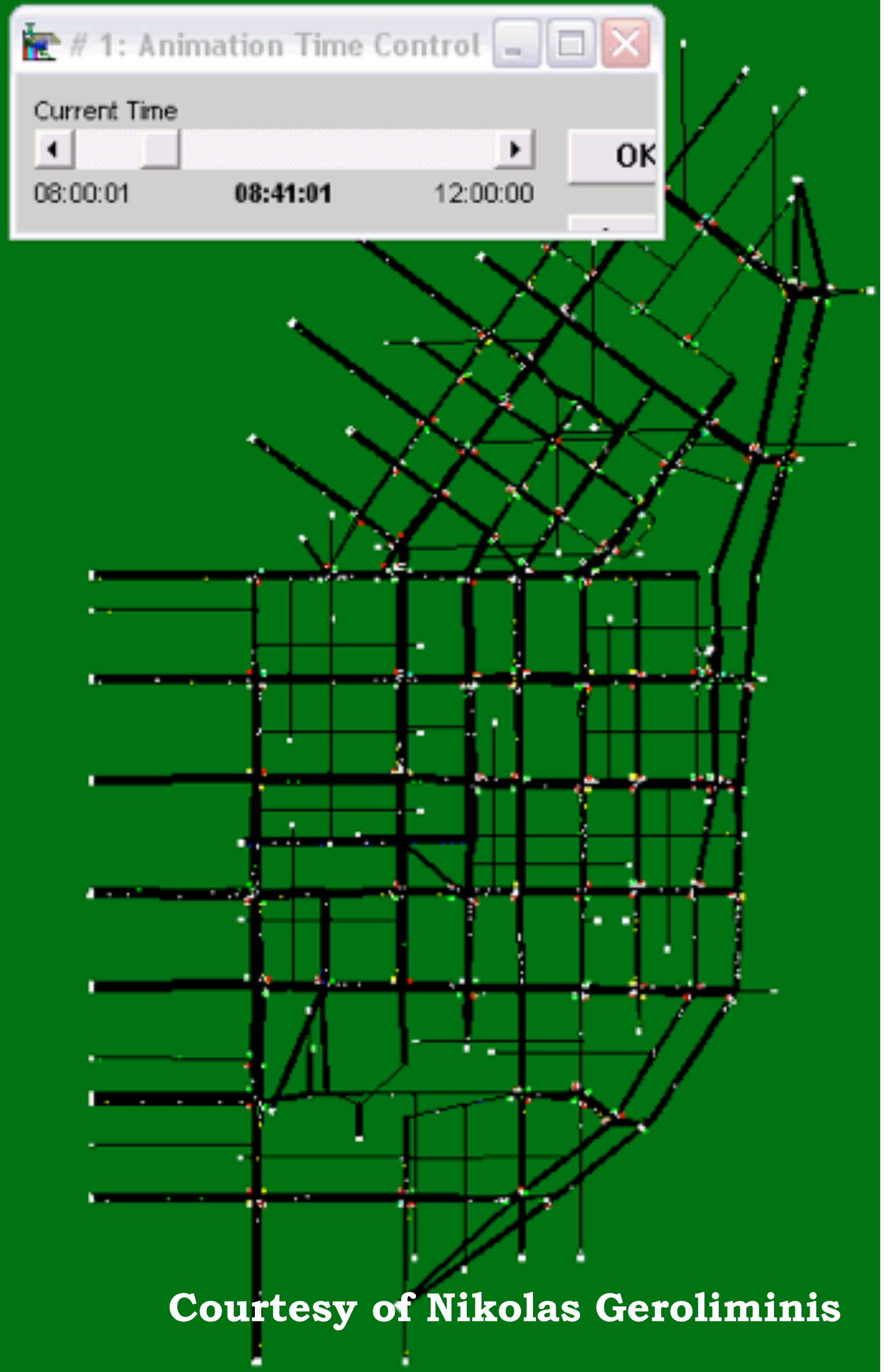
- Although microscopic simulation models can be tuned such that most important macroscopic features can be represented, the microscopic processes often are not correctly described!
- Impacts of this observation, e.g. with respect to the predictive validity
- Consider how models are used!



More (big?) data, new insights

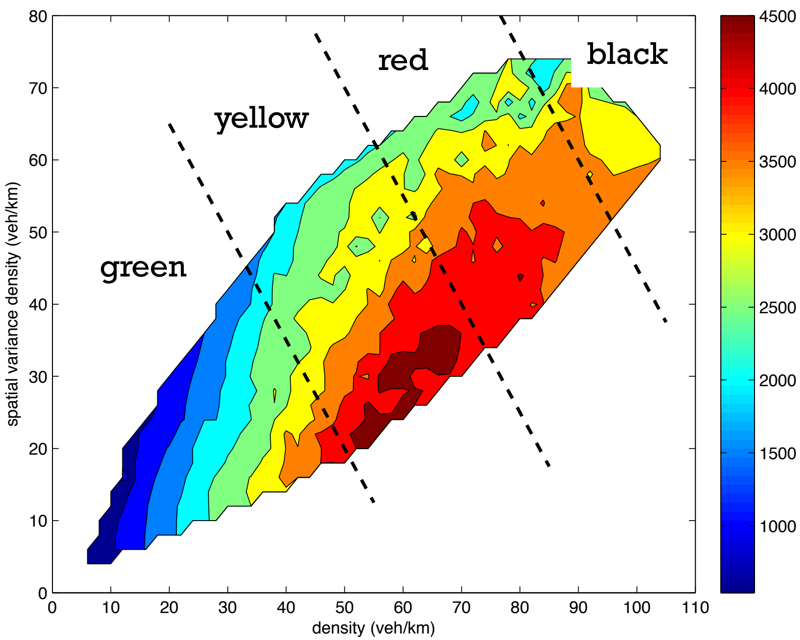
- Availability of large datasets from urban and motorway arterials leads to new insights into network dynamics
- Data from GPS (Yokohama) empirically underpins existence of Network Fundamental diagram
- Fundamental property of traffic network: production deteriorates a high loads!





More (big?) data, new insights

- Recent studies (TU Delft, ICL) show that network dynamics are a "bit more involved"
- Next to average density, spatial variation of density plays a crucial role in representing network traffic production and level of service...
- Congestion nucleation causes spatial variation to self-sustain & increase

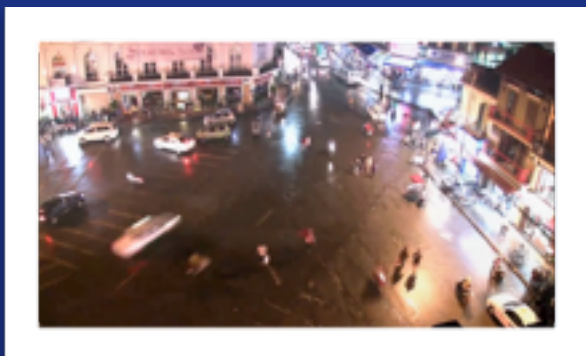


Network Dynamics

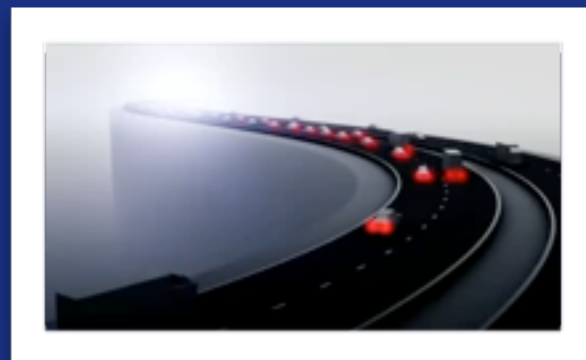
Features and phenomena that you need to capture!

There are severe limits to the self-organization capacities of the traffic system

Efficient self-organization



Capacity-drop and waves



Grid-lock and turbulence

Increasing traffic loads

Decreasing system performance

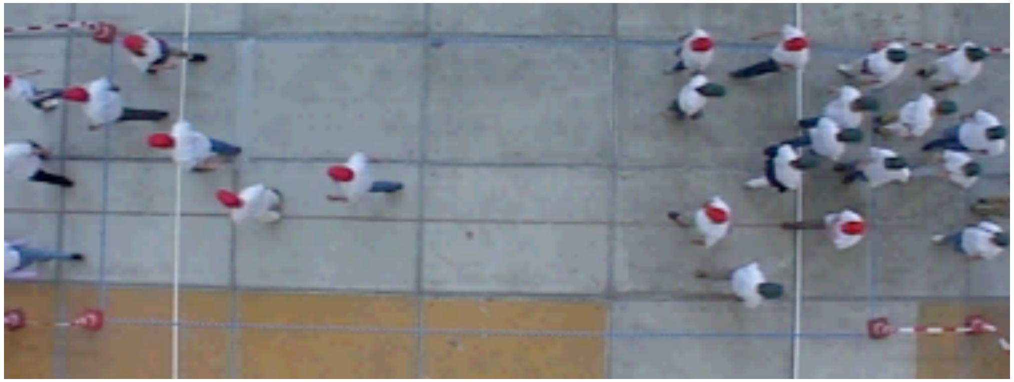
Efficient and inefficient self-organization and network degradation

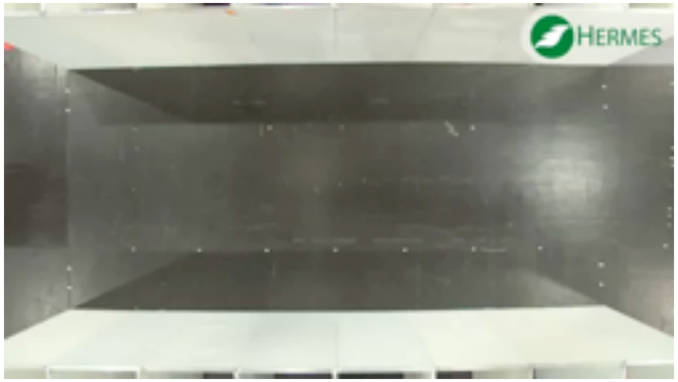
- For low network loads, interactions between traffic participants is very efficient
- For high loads, inefficient phenomena self-organize / occur reducing performance

Characteristic features of traffic flow

Efficient self-organization in dilute flow conditions

- Dynamically formed walking lanes
- High efficiency in terms of capacity and observed walking speeds
- Experiments by Hermes group show similar results
- Phenomena is characteristic of a pedestrian flow, and needs to be included in model





Interaction modeling

Use of differential game theory

- Main behavioral assumptions (loosely based on psychology):
  - Pedestrian can be described as optimal, predictive controllers who make short-term predictions of the prevailing conditions, including the **anticipated behavior of the other pedestrians**
  - Pedestrians **minimize walking effort** caused by distance between peds, deviations from desired speed / direction, and acceleration
  - Costs are discounted over time, yielding:

$$J = \int_t^{\infty} e^{-\eta t} \left[\frac{1}{2} \mathbf{a}^T \mathbf{a} + c_1 \frac{1}{2} (\mathbf{v}^0 - \mathbf{v})^T (\mathbf{v}^0 - \mathbf{v}) + c_2 \sum_q e^{-\frac{\|\mathbf{r}_q - \mathbf{r}\|}{R_0}} \right]$$

- Use of differential game theory to determine the pedestrian acceleration behavior (i.e. the acceleration **a**)

Game-theory applications

To modeling interactions of traffic participants

- Next to walker behavior, other applications of differential game theory have been put forward
  - Car-following and merging behavior modeling
  - Cooperative driving control strategies for vehicle platoons
- Recent work involves interactions of large vessels, where game theory is used to describe the behavior of the bridge team under different scenarios (cooperative and single-sided interaction, demon-ship interaction)

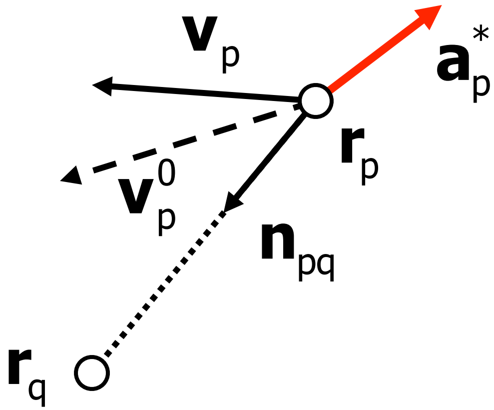
- Note that the resulting optimization problem can be solved using Pontryagin's minimum principle + dedicated numerical solver
- Computationally quite demanding!

Adding fraction terms

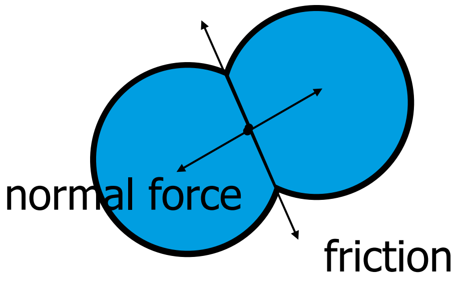
The simplest of models…

- Under the assumption that the opponent peds do not react to the considered ped, we find a closed form expression for acc vector:

$$\mathbf{a}_p(t) = \frac{\mathbf{v}_p^0 - \mathbf{v}_p}{\tau_p} - A_p^0 \sum_{q \neq p} \mathbf{n}_{pq} e^{-\|\mathbf{r}_p - \mathbf{r}_q\| / R_p^0}$$



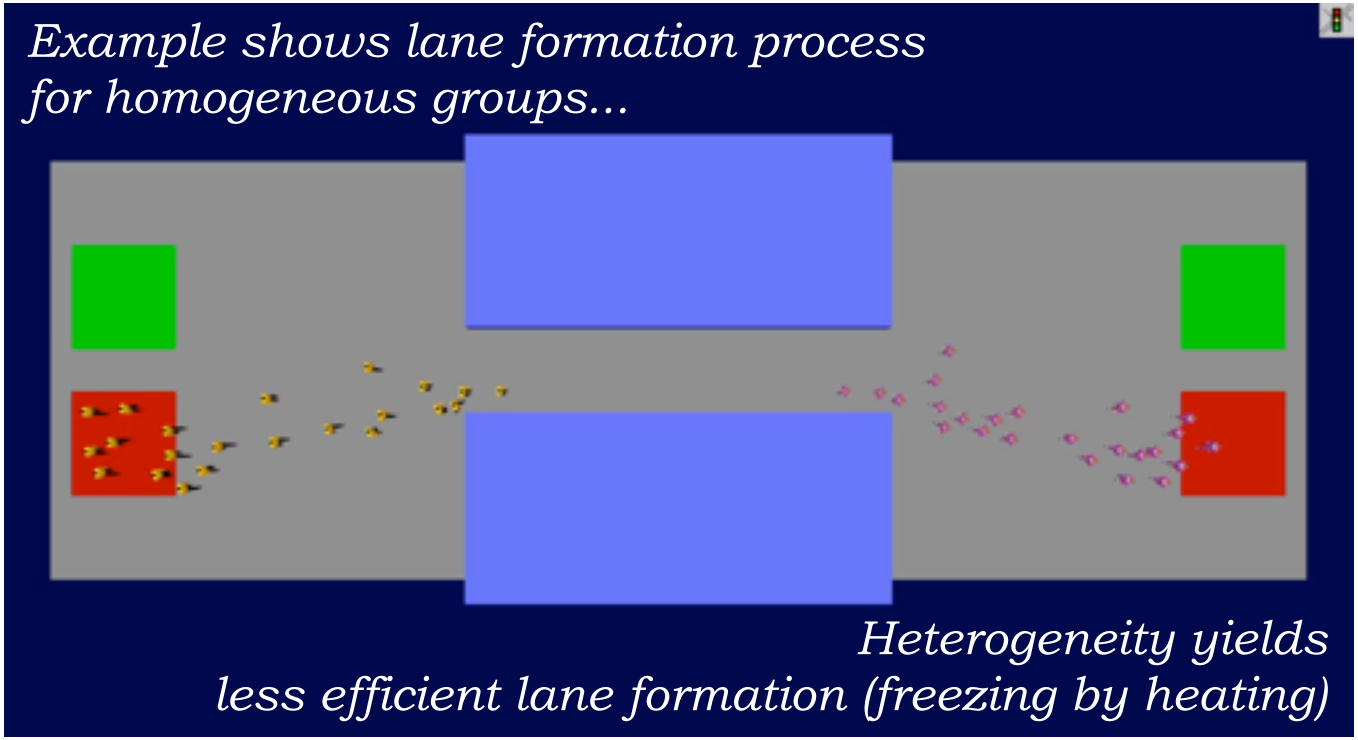
- Resulting expression is same as original Social Forces model of Helbing
- Physical interactions (physical contact, pushing) can be modeled by adding physical forces between pedestrians

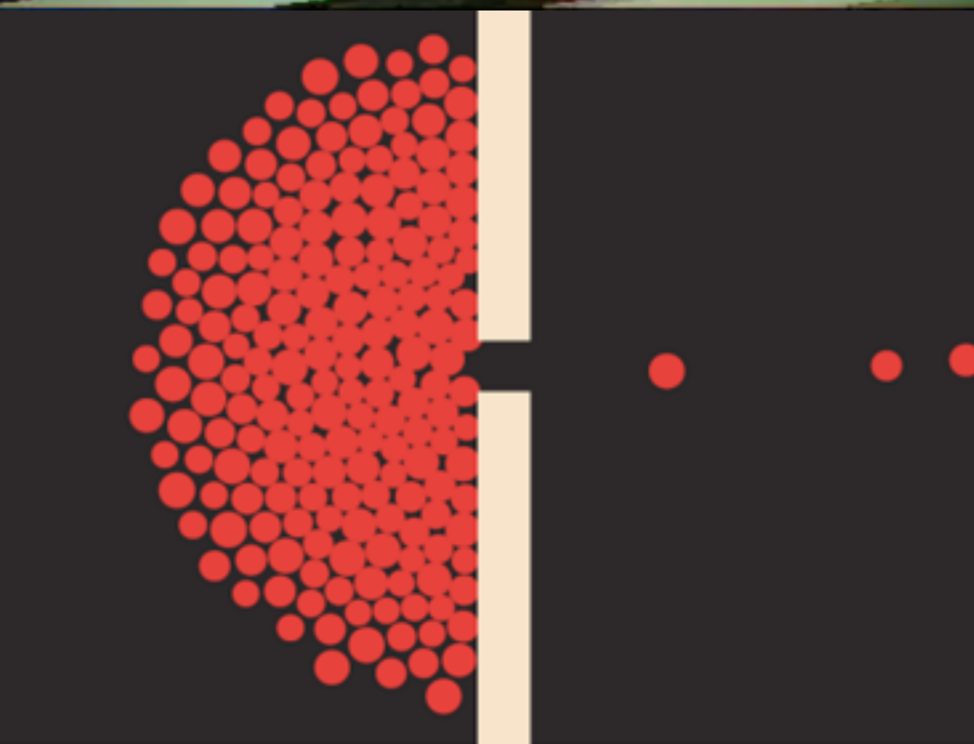


Interaction modeling

Use of differential game theory

- Simple model reproduces lane formation processes adequately

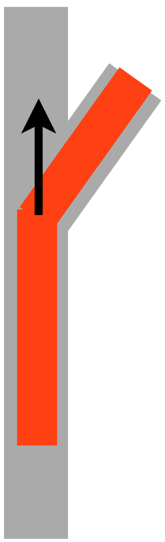


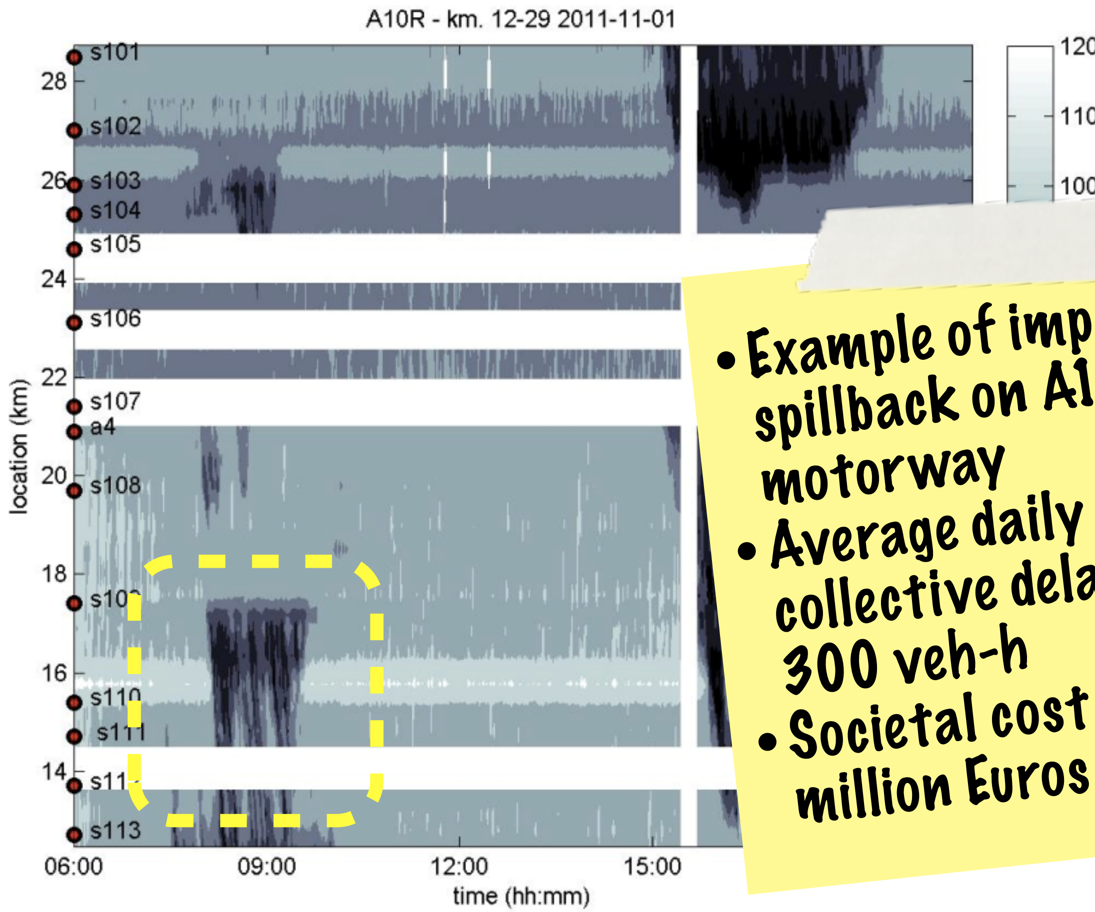


Pedestrian flow capacity drop

- Adding friction between pedestrians causes severe reduction in capacity
- Capacity drop is due to arc formation in front of exit
- Gets worse when pedestrians are more anxious to get out (Helbing et al, Nature 2000)
- In line with results from pedestrian experiments (TU Dresden, TU Delft)
- Capacity drop also occurs in car-traffic: when congestion sets in, capacity reduces with 10-15%

Impact of spillback on throughput



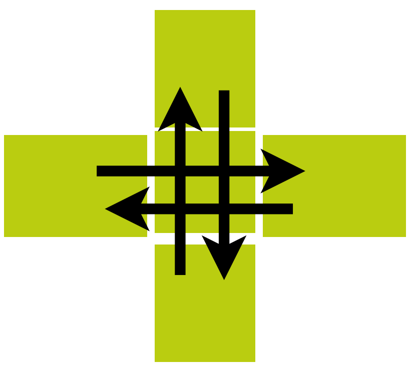


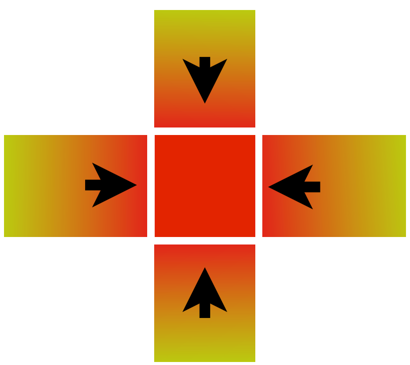
- Example of impacts of spillback on A10 motorway
- Average daily collective delay of 300 veh-h
- Societal cost about 1 million Euros per year!

Spill-back and grid-lock

Urban networks

- Spill-back easily leads to grid-lock effects, as we saw earlier...
- Similarly, grid-lock can occur in pedestrian networks when network load is too high
- In this case, self-organization fails and capacity drops





Stochasticity…

Random nature of traffic


http://www.traffic-states.com/?
www.traffic-states.com
Home
Data
Method
Phenomena
Phase Diagram
Contact
Links
Help
Search
Route
Highway
A5
Direction
please choose
Section
Time
From
To
Day
Daytime
Traffic Pattern
any
MLC
PLC
TSG
OCT
HCT
Full Text
Search
Search result
found 561 images, showing page 5 of 32
Pictures per page: 18
A5N, 2001/05/29 (Tue)
A5N, 2001/05/29 (Tue)
A5N, 2001/05/29 (Tue)
A5N, 2001/05/30 (Wed)
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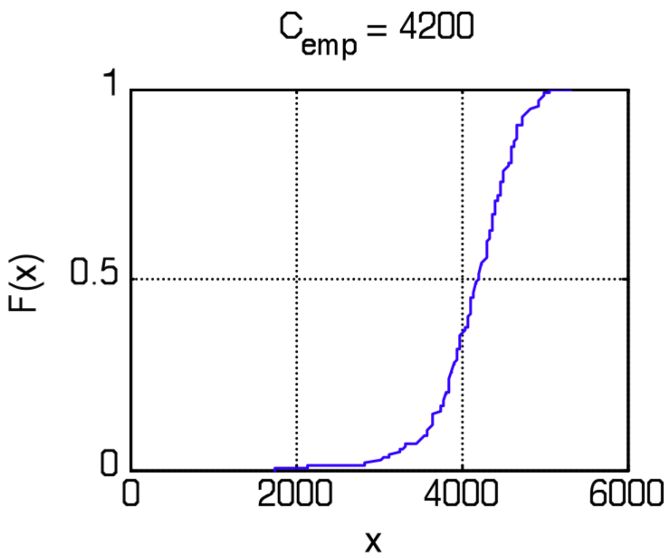
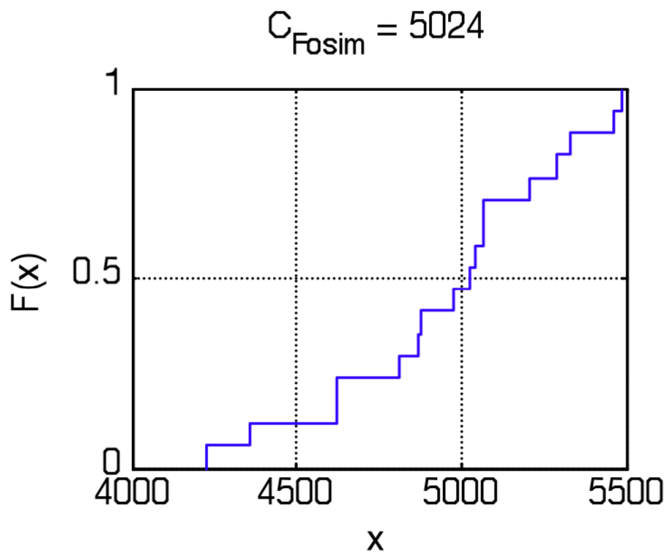
A5N, 2001/06/06 (Wed)
A5N, 2001/06/07 (Thu)
A5N, 2001/06/07 (Thu)
Legal Disclaimer
Imprint
Admin
Which is the representative day?

Stochasticity

Supply factors

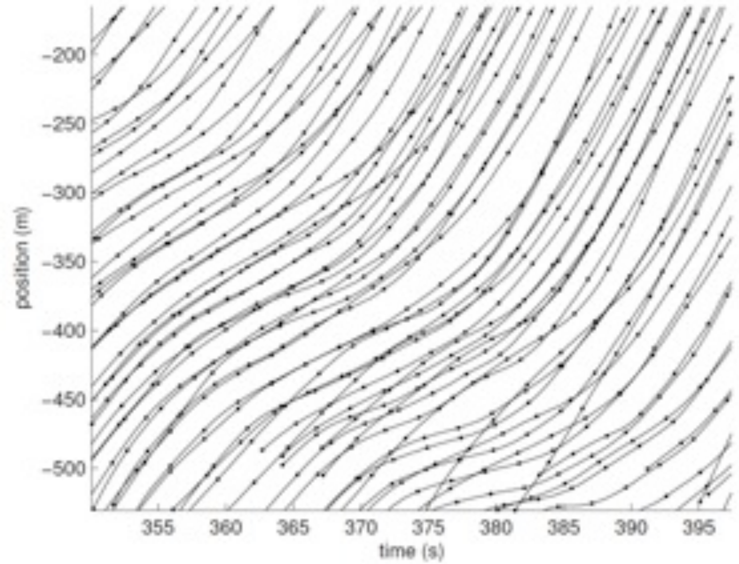
- Clearly, **traffic demand** is stochastic but what about capacity?
- Capacity = maximum (hourly) flow that can be sustained for a considerable time period
- What determines capacity?
  - Infrastructure
  - Driving behavior
  - Vehicle characteristics
  - Occurrence of incidents
- It is not reasonable to assume that capacity is deterministic!





Example: IDM

Explaining stochasticity?

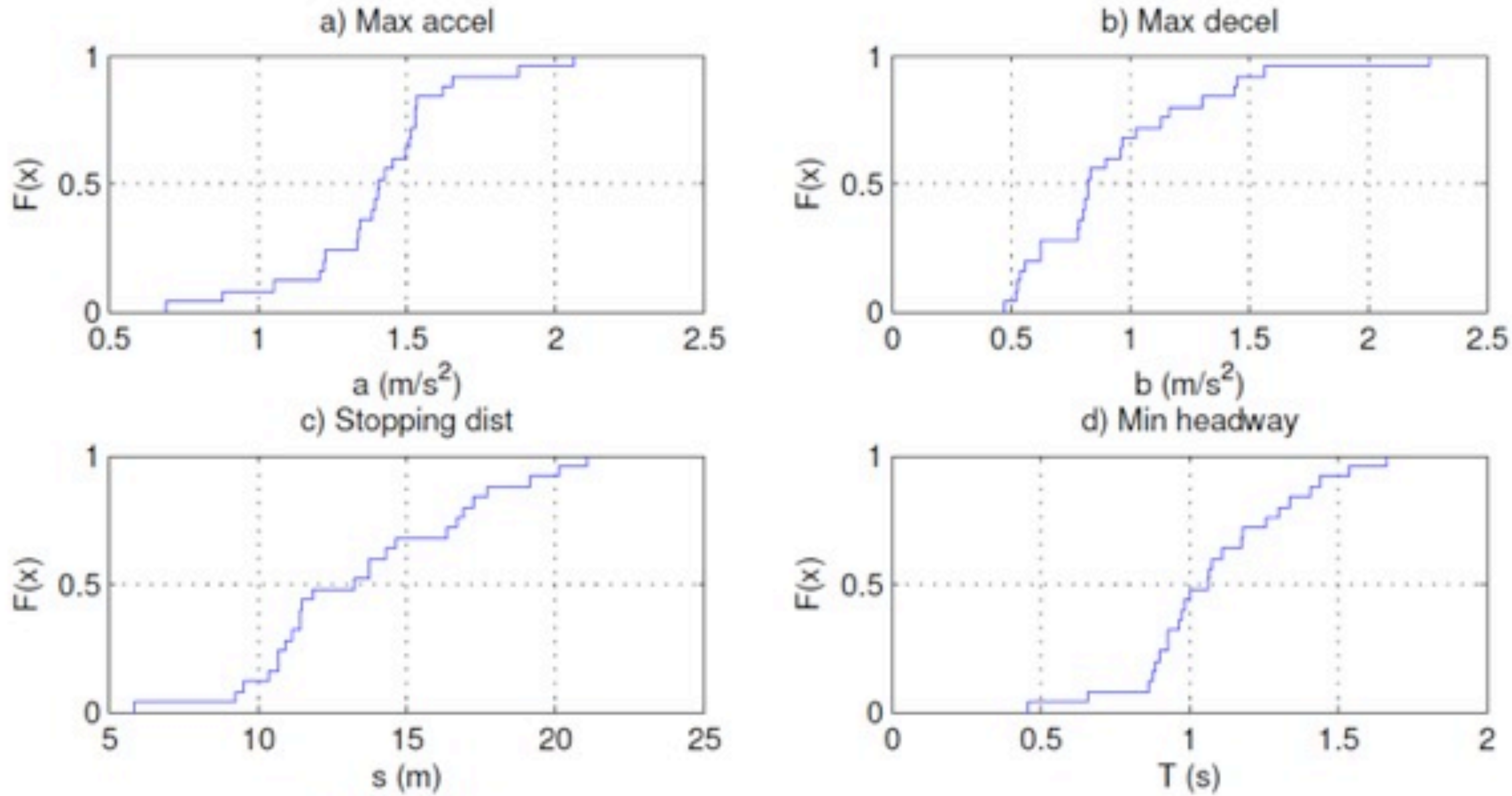


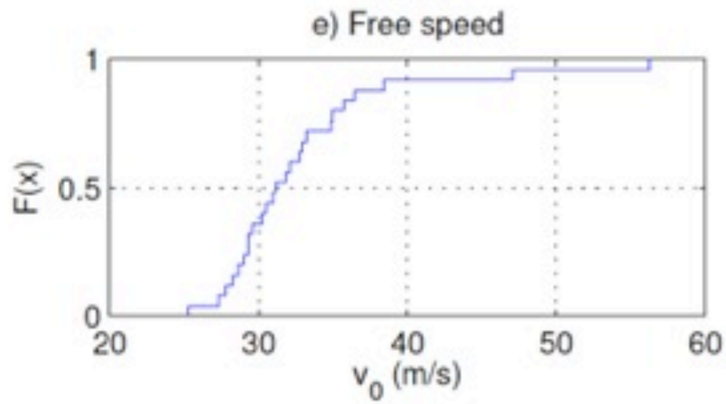
- Vehicle trajectories collected from airborne platform (helicopter)
- IDM model by Treiber and Helbing:

$$a = f(s, v, \Delta v) = a \cdot \left[1 - \left(\frac{v}{v_*} \right)^4 - \left(\frac{s_*(v, \Delta v)}{s} \right)^2 \right]$$

$$\text{where} \quad s_* = s_0 + \tau v + \frac{v \Delta v}{2\sqrt{ab}}$$

- Find estimates for parameters that maximize the likelihood L of finding the actually observed car-following behavior





Pictures show CDFs of estimated parameters showing large heterogeneity in driving behavior!

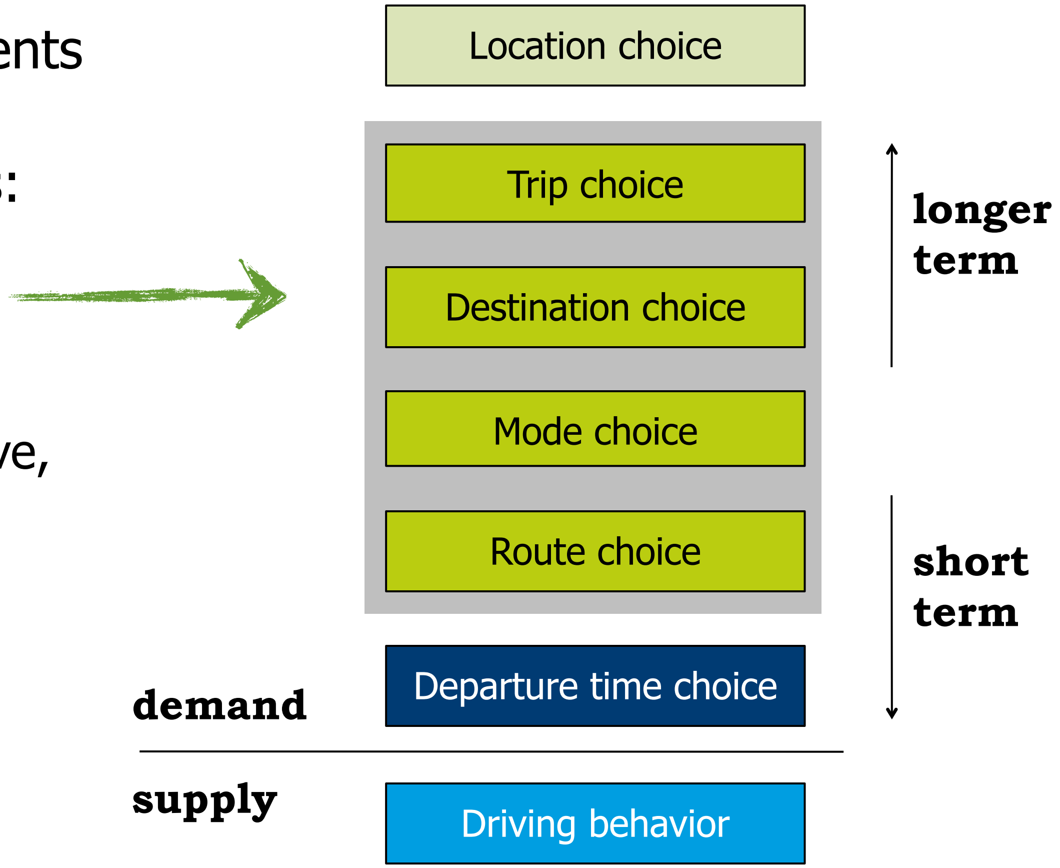
Modeling approaches

Fitting models…

Some considerations

When choosing / developing a model

- Trivial: model requirements depend on application, which in turn prescribes:
  - Which behavioral processes to include
  - Type of validity (qualitative, quantitative, reproduce or predict?)
  - Which phenomena or features need to be reproduced
  - Math / computational properties of approach



Modeling approaches

Reproducing vs predicting

- Two dimensions:
  - Representation of traffic
  - Behavioral rules

	Individual particles	Continuum
Individual behavior	Microscopic	Mesoscopic
Aggregate behavior	Mesoscopic	Macroscopic

	Individual particles	Continuum
Individual behavior	Microscopic (simulation) models	Gas-kinetic models (Boltzmann equations)
Aggregate behavior	Particle discretization models (Dynasmart)	Queuing models Macroscopic flow models

Explain and predict

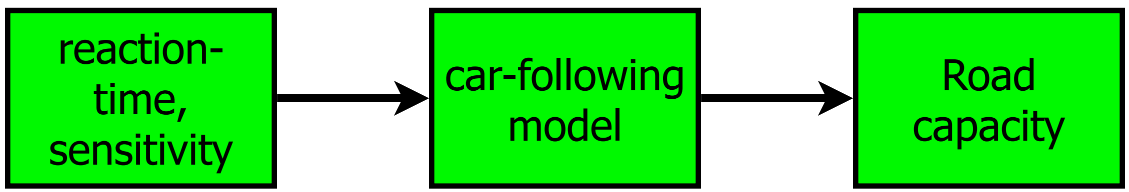
↕

Reproduce

Relation between micro and macro

Micro, meso and macro?

- Microscopic models (aim to) **explain and predict** driving behavior (car-following, lane changing, etc.)
- Macroscopic features (e.g. capacity, jam-density, etc.) are thus predicted output of these models
- **Example:**
  (CHM model)



- Ensuring correct reproduction of macroscopic features is often a difficult (calibration) process (parameters not directly observable)
- Macroscopic models generally (often) take macroscopic features as input and correct **representation** is thus ‘trivial’

How good are these models anyway?

Some example approaches…

Phenomena	BPR functions	Queuing models	First-order theory	Micro-simulation
Capacity drop	N/A	EVAQ	Infinite wave speed	Yes, but often too small
Spill-back	N/A	Extended LTM	Yes	Only if model reproduces FD
Stochastic demand and supply	N/A	Quast	Only research models	Variation often too small
Congestion instability	N/A	N/A	Only research models	No absolute validity

[Skip to final remarks](#)

Trade-offs!

It is not only accuracy that counts…

Application	Key requirements	Examples
Understanding phenomena	• Construct / face validity • Analytical properties	Flow instability, train delay propagation analysis
Off-line assessment of (ITS) measures	• Predictive validity	Evacuation assessment and optimization
State estimation (Kalman filters)	• Computational properties • Content validity	Lagrangian multi-class modeling
On-line prediction and scenario assessment	• Predictive validity • Computation speed	Fastlane Multiclass Traffic macro model
On-line optimization	• Computation speed / properties?	Reduced models, smart reformulations (Le et at,2013)

Reformulate and simplify

…or conservation of misery?

- Reformulation can lead to models with more favorable mathematical / computational properties
- Simplified models allowing favorable computational techniques:
  - Decomposition the NP-hard evacuation instruction optimization problem into three simple subproblems
  - Reformulating non-linear optimization problem for MPC control of urban networks as a LQ optimization problem (Le et al, 2013), or approximating it as a MILP problem (Bart De Schutter)
- Learning for the resulting optimal solutions:
  - Deriving heuristics for controlling motorway arterials (Specialist speed-limit controllers) or networks (Praktijkproef Amsterdam)

Instruction optimization

- Objective: get out as many inhabitants within [0,T]:

$$J(u) = \int_0^T q(t)\,dt$$

- Bi-level problem: instructions yield response from evacuees and result in traffic operations

Simplifying the problem

Using decoupling of the problem…

Optimization of turning fractions

- Intermediate optimized tur… flows

Approximation of compliance behavior

- Instructed turning fractions
- Realized turning fractions

- Upper and lower bounds on turning fractions

- Small reduction of effectiveness
- Very large impact on computation speed (upto 100 for simple Walcheren network)
- Application to other problems likely

Final words...

Stochastic nature of traffic

Some final remarks…

Almost there!

- Importance of model choice in relation to application!
  - Ensure that your model captures the phenomena that are relevant for your application (e.g. optimization of ramp-meter signal requires a model to capture the capacity drop and spill-back!)
  - Think what type of validity you need (face, content, predictive) and which trade-off you need to make between accuracy / performance
- Still many challenges left to solve:
  - in modeling (predictive validity of microscopic models, modeling for safety assessment, modeling for ITS)
  - in estimation (making sense of all these data) and prediction
  - in optimization (network-wide control approaches anticipating on behavioral adaptation)

Innovations in data collection

- Development of a Virtual Traffic and Travel laboratory (VTT-Lab) for collecting data under a variety of experimental conditions