Linear-Quadratic Model Predictive Control for Urban Traffic Networks

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Motivation

- Growing demand for transportation.
- Increasing congestion in urban area.
- The need for a real time large scale road traffic control system.
Aims and Outcomes

- To develop a better centralized control
- New framework to coordinate the green time split and turning fraction to maximize throughput in urban networks.
  - We explicitly consider travel time and spill-back.
  - The state prediction is linear so the model is suitable for large networks in real time control.
  - Show via simulation that our model is superior in reducing congestion in compare with other control schemes in the studied scenarios.
Outline

- Background and literature review.
- Proposed MPC framework.
  - Predictive model.
  - Optimization problem.
- Simulation and results.
Review of Intersection Control

- Traffic signals at intersections is the main method of control in road traffic networks.
  - Have big influence on overall throughput.

- Control variables at intersections
  - Phase combination.
  - Cycle time.
  - Split.
  - Offset.
Control strategy classification

Fixed-time coordinated

Coordinated traffic responsive
- SCOOT (Bretherson 1982) and SCAT (Lowrie 1982).
- TUC (C. Diakaki 1999).
- Rolling-horizon optimization.

Isolated fixed-time
- SIGSET (Allsop 1971) and SIGCAP (Allsop 1976) systems.

Isolated traffic-responsive
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Fixed-time


Traffic-responsive

- SCOOT and SCAT systems. Control splits, offset and cycle time.
  Use real time measurement to evaluate the effects of changes.
- TUC system.
  Control splits.
  Store and forward model: Assume vertical queue, continuous flow.
  Linear Quadratic Regulator problem.
- Rolling-horizon optimization.

Isolated

- Store and forward model: Assume vertical queue, continuous flow.
- Linear Quadratic Regulator problem.
- Rolling-horizon optimization.
Rolling horizon optimization

Optimization over finite horizon $N$

- OPAC (Gartner (1983)), PRODYN (Henry (1983)), RHODES (Mirchandani (2001)) and CRONOS (Boillot (1992)).

Notations

- Time evolves in discrete steps $n=0,1,2,...$
- Continuous flows of traffic.
- Network state maintains continuous vehicle count in delay, route, queue and sink classes.
Notations

- Vehicles flowing out of a class move to downstream classes through links. We denote the links $j=1,\ldots,L$.
- Each link is associated with a flow rate $f_j$, which is the maximum number of vehicles that can go through that link in an unit of time.
Notations

- Control variables: $u_j(n)$ is the fraction of time unit during which link $j$ is active.
State dynamics

\[ x_k(n + 1) = x_k(n) + a_k(n) + \sum_{\{j:d_j=k\}} u_j(n)f_j - \sum_{\{j:s_j=k\}} u_j(n)f_j, \quad k \in K_{D,R,Q,S}. \]

- \( x_k(n) \) is the queue length of class \( k \) at time \( n \)
- \( a_k(n) \) is exogenous arrival
- \( \sum_{\{j:d_j=k\}} u_j(n)f_j \) is the number of vehicles arriving from other classes.
- \( \sum_{\{j:s_j=k\}} u_j(n)f_j \) is the number of vehicles leaving class \( k \).
Constraints

- Control constraints:
  \[ 0 \leq u_j(n) \leq 1, \quad 1 \leq j \leq J. \]

- Traffic light cycle constraints:
  \[ u_{L+1+(i-1)\times2}(n) + u_{L+1+(i-1)\times2+1}(n) \leq 1, \quad i \in \mathcal{M}. \]

- Green duration constraints:
  \[ u_j(n) \leq u_{L+1+(i-1)\times2}(n) \]

- Flow conflict constraints:
  \[ \sum_{j \in C_{WE}^p(i)} u_j(n) \leq u_{L+1+(i-1)\times2}(n) \]
Constraints

- Non-negative queue constraints:
  \[ \sum_{\{j : s_j = k\}} u_j(n) f_j \leq x_k(n), \quad k \in \mathcal{K}_{D,R,Q}. \]

- Capacity constraints:
  \[ x_k(n + 1) \leq c_k, \]
  \[ x_k(n) + a_k(n) + \sum_{\{j : d_j = k\}} u_j(n) f_j - \sum_{\{j : s_j = k\}} u_j(n) f_j \leq c_k. \]
Constraints illustration

- Control constraints:
  \[ 0 \leq u_j(n) \leq 1, \quad 1 \leq j \leq J. \]

- Traffic light cycle constraints:
  \[ u_7 + u_8 \leq 1. \]

- Green duration constraints:
  \[ u_j \leq u_7, \quad j = 1, \ldots, 7. \]

- Flow conflict constraints:
  \[ u_2 + u_4 \leq u_7, \quad u_5 + u + 3 \leq u_7. \]
The MPC framework

\[ d(n) \rightarrow X(n) \rightarrow X(n + 1) \rightarrow \ldots \rightarrow X(n + N) \]

\[ U(n) \rightarrow d(n + 1) \rightarrow X(n + 1) \rightarrow U(n + 1) \]

Optimization over finite horizon N
Quadratic Programming

\[ \min \sum_{i=n}^{n+N-1} \hat{X}_{D,R,Q}(i + 1)'Q\hat{X}_{D,R,Q}(i + 1) + RU(i) \]

\[ \text{s.t.} \]
\[ F_x\hat{X}_{D,R,Q}(i) + F_u U(i) \leq g(i), \quad i = n, \ldots, n + N - 1. \]

- \( Q \) is all \( 1 \) matrix, positive semi-definite. We minimize the quadratic cost which is square of sum of all queues over horizon \( N \).

- Holding back problem: vehicles are held back even when there is available space in the downstream queues.

- The linear cost \( RU(i) \) gives small incentive for the controller to move vehicle forward.
Matrix representation of the QP

\[
\min_{\mathbf{U}} \quad \mathbf{U}^T \mathbf{B} \mathbf{Q} \mathbf{B} \mathbf{U} + \left( 2(\mathbf{X}_0^T \mathbf{A} + \mathbf{d}^T \mathbf{A}_1^T) \mathbf{Q} \mathbf{B} + \mathbf{R} \right) \mathbf{U} + (\mathbf{X}_0^T \mathbf{A} + \mathbf{d}^T \mathbf{A}_1^T) \mathbf{Q} (\mathbf{A} \mathbf{X}_0 + \mathbf{A}_1 \mathbf{d})
\]

s.t.

\[
\begin{bmatrix}
F_u S_u^0 \\
F_u S_u^1 + F_x S_x^1 B \\
\vdots \\
F_u S_u^{N-1} + F_x S_x^{N-1} B
\end{bmatrix}
\leq
\begin{bmatrix}
g(0) - F_x X_0 \\
g(1) - F_x S_x^1 (\mathbf{A} X_0 + \mathbf{A}_1 \mathbf{d}) \\
\vdots \\
g(N - 1) - F_x S_x^{N-1} (\mathbf{A} X_0 + \mathbf{A}_1 \mathbf{d})
\end{bmatrix}
\]

- We prove the above QP is convex but not strictly convex. Thus, there are multiple optimal solutions.
Simulation and results

- We conduct simulations to obtain the performance of our MPC framework.
  - Significantly reduce congestion.

![Diagram showing the process of simulation and optimization]

- MATLAB or SUMO simulation → Network state → Optimization solver → Control decision
Scenario 1
Results for scenario 1

(a) Total vehicles in the network

(b) Queue length at bottleneck intersection
Scenario 2
Results for scenario 2

(a) Number of vehicles that arrive destination

(b) Queue at the top bottleneck link
Conclusion

- We present a general Model Predictive Control framework for centralized traffic signals and route guidance systems aiming to maximize network throughput.

- We explicitly consider travel time, spill back while retain linearity and tractability.

- The numerical experiments show that our proposed scheme may reduce congestion significantly while still achieving better throughput.