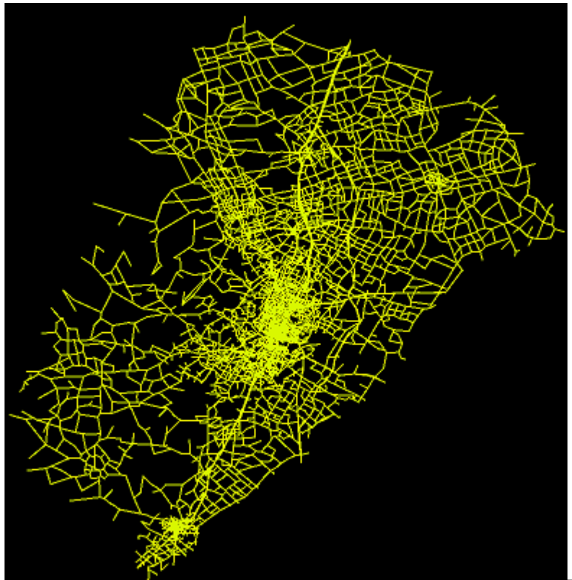

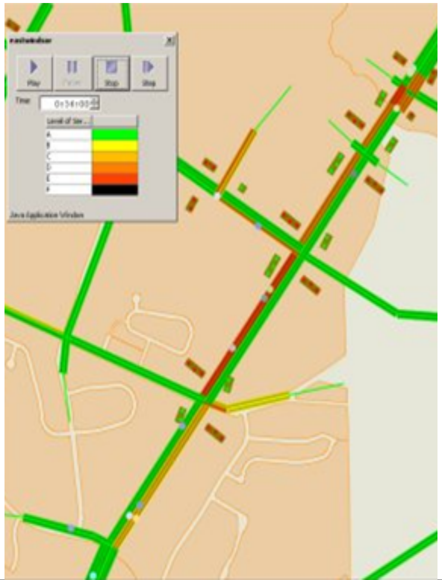

Transport network equilibrium models incorporating adaptivity and volatility

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Acknowledgements
(some past & current PhD students for some of these topics)

Satish Ukkusuri	(Associate Professor – Purdue)
Avinash Unnikrishnan	(Assistant Professor – WVU)
Steve Boyles	(Assistant Professor – UT Austin)
ManWo Ng	(Assistant Professor – Old Dominion Univ.)
Dung-Ying Ling	(Assistant Prof. – National Cheng Kung Univ.)
Ampol Karoonsoontawong	(Assistant Prof. – King Mongkut's Univ.)
Nezam Nezamudding	(Assistant Professor – Valparaiso University)
Jennifer Duthie	(NMC Center Director – UT Austin)
Lauren Gardner	(Lecturer – UNSW)
Natalia Ruiz Juri	(NMC Research Fellow)
David Fajardo	(Research Associate – UNSW)
Nan Jian	(NMC Research Fellow)
Roshan Kumar	(PB Consulting)
David Suecun	(PhD Student – UT Austin)
Kanthi Sathasivan	(PhD Student – UT Austin)
Ti Zhang	(PhD Student – UT Austin)
Shoupeng Tang	(PhD Student – UT Austin)
Melissa Duell	(PhD Student – UNSW)
Tao Wen	(PhD Student – UNSW)
Kasun Wijayaratna	(PhD Student – UNSW)

Objectives

- Brief summary and motivation of traditional transport network modelling ***for planning***

- Highlight some mathematical advances in this field related to:
  - Dynamics (very brief)
  - Strategic decision making (quite brief)
  - Adaptive behaviour (more detailed)

Transport Planning/Modelling

- In essence, mathematically represent individual **travel choice** and resulting **system impacts**
  - Trip/activity Destination Departure-time
  - Mode Toll Usage Route
  - Lane Acceleration

  - Congestion Emissions Safety
  - Energy Use Reliability Accessibility

  - **And the list continues to grow**

Transport Network Modelling

- Most transport applications contain network structure
- Numerous application characteristics
  - Operational vs planning
- Domain-specific network issues
  - Physics of traffic/transit
  - Individual operational behaviour (e.g., reaction time, distraction ,stress)
  - Individual strategic behaviour (eg,route/mode/toll/trip choice)

Today, we will note some advances in **dynamics**, **volatility**, **and** **adaptivity**

Our Network Model Deployments for planning

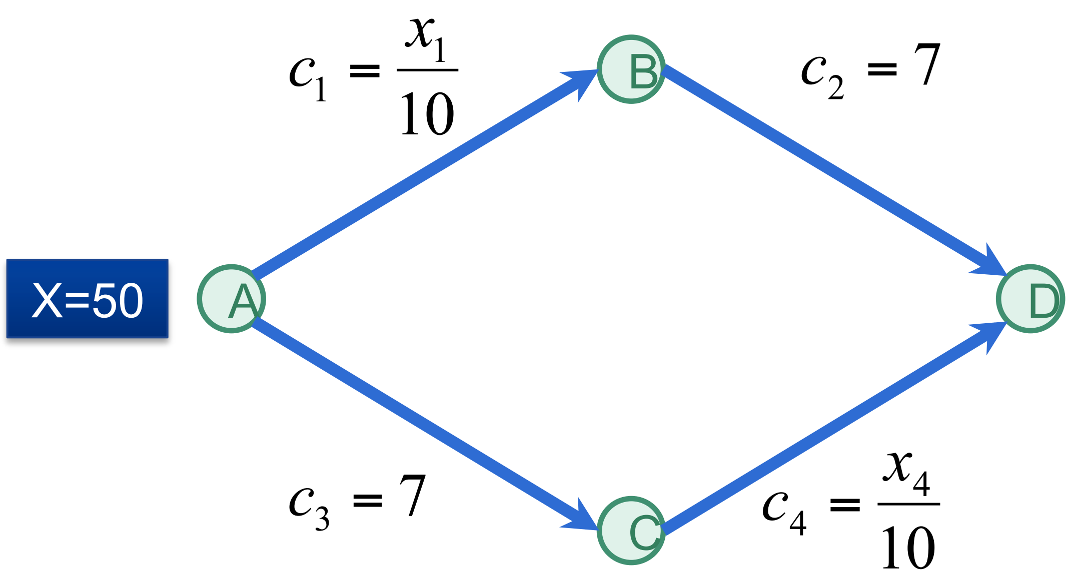
- **Ongoing and previous project involvements**

Sydney, NSW	Austin,TX	Dallas,TX
El Paso, TX	Houston, TX	Chicago, IL
New York, NY	Atlanta, GA	Phoenix, AZ
San Francisco, CA	New Jersey, NJ	Columbus, OH
Jacksonville, FL	Nicosia, Cyprus	Orlando, FL
New Orleans, LA		

- Over 40 specific externally funded projects in last decade

But, what is the point of the basic model?

Simplified Static Equilibrium Model
Braess's Paradox (simplified example)



2 Paths

- P_1= A-B-D
- P_2 = A-C-D

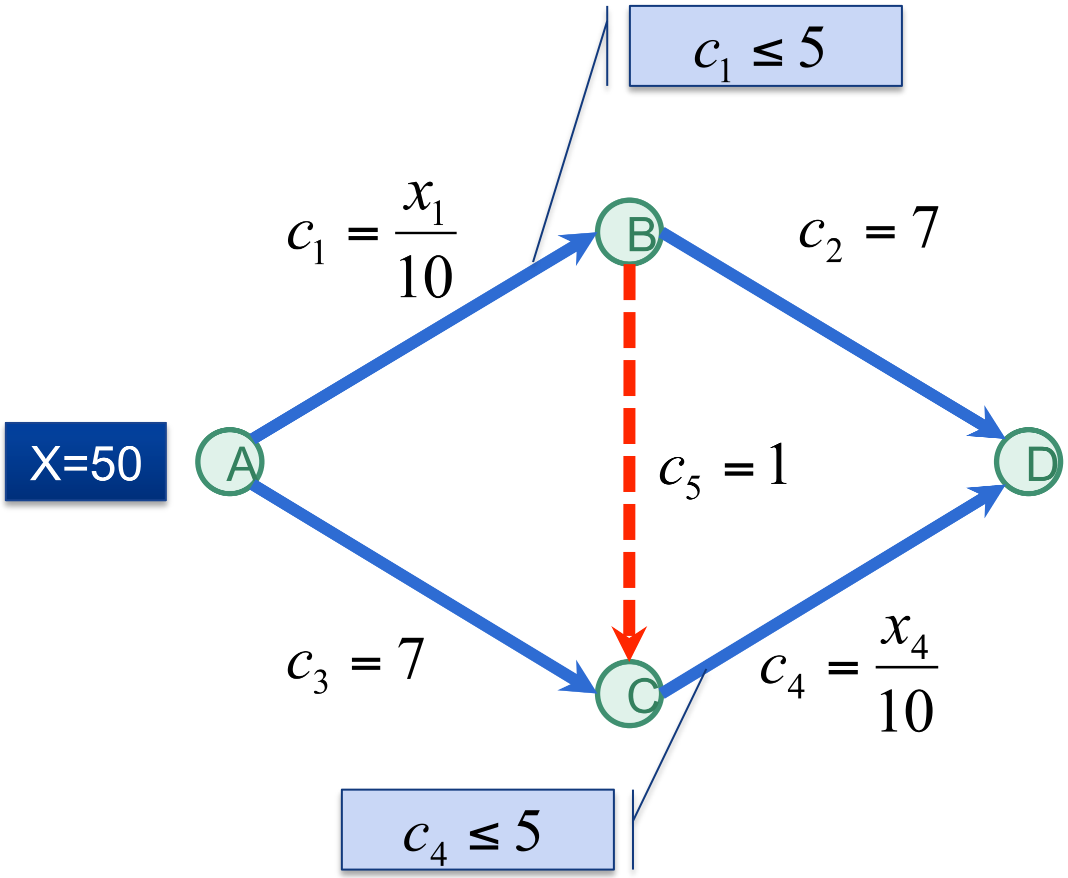
Equilibrium flows

- $P_1 = P_2 = 25$

$c_1+c_2 = c_3+c_4 =$ **9.5**

Total cost = 475

Braess's Paradox Example



3 Paths

- P_1 = A-B-D
- P_2 = A-C-D
- P_3 = A-B-C-D

Equilibrium flows

$P_3 = 50, P_1 = 0, P_2 = 0$

$C_1 + C_5 + C_4 =$ **11**

Total cost = 550

"Static" Traffic Assignment

- Formulation (Beckman, 1956)

$$\min \sum_a \int_0^{x_a} c_a(\omega)d\omega$$

$$s.t.$$

$$\sum_k h_k^{rs} = q_{rs} \qquad \forall r, s$$

$$h_k^{rs} \geq 0 \qquad \forall k, r, s$$

$$x_a = \sum_r \sum_s \sum_k h_k^{rs} \delta_{a,k}^{rs} \qquad \forall a$$

Advances in Network Realities

- Numerous advances over the past 60 years
  - Stochasticity
  - Dynamics
  - Multiple classes of travel behaviour
  - Pricing
  - Network design
  - Signal design
  - Information
  - Demand/Supply integration
  - Many others

DTA and Travel Demand Formulation
Lin, Eluru, Waller and Bhat (2007)

$$DTA : \Psi(\Xi^*)^T(\Xi - \Xi^*) \geq 0 \qquad \forall \Xi \in D$$

$$DEMAND : \Psi(\Xi^*) = S(P(Z(\Psi(\Xi^*))))$$

Ξ = Any feasible DTA solution(vector)

Ξ^* = Optimal DTA solution(vector)

$\Psi(\Xi)$ = Path cost vector resulting from DTA Ξ

$Z(\Psi(\Xi))$ = Dynamic trip table resulting from path cost vector $\Psi(\Xi)$

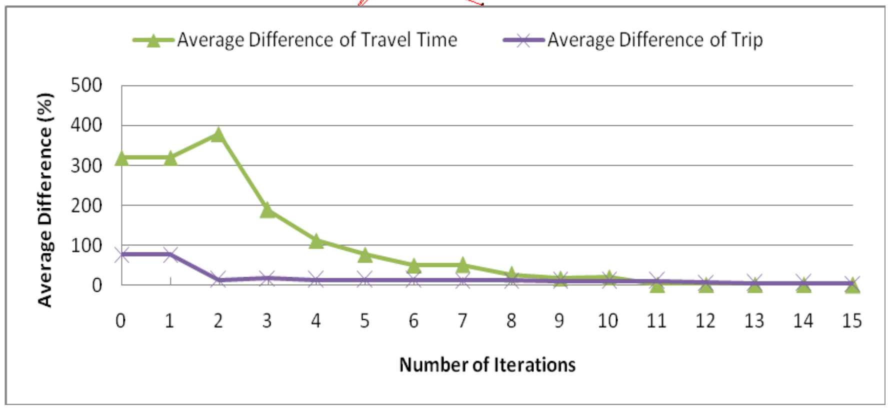
$P(Z(\Psi(\Xi)))$ = User paths vector from assigning trip table $Z(\Psi(\Xi))$

$S(P(Z(\Psi(\Xi))))$ = Path cost vector obtained from simulating user paths $P(Z(\Psi(\Xi)))$

Dallas, TX CBD Deployment
Lin, Eluru, Waller, and Bhat (2008)

- Previous formulation implemented in software packages CEMDAP (ABM demand) and VISTA (network DTA).

- Computational performance and convergence properties examined

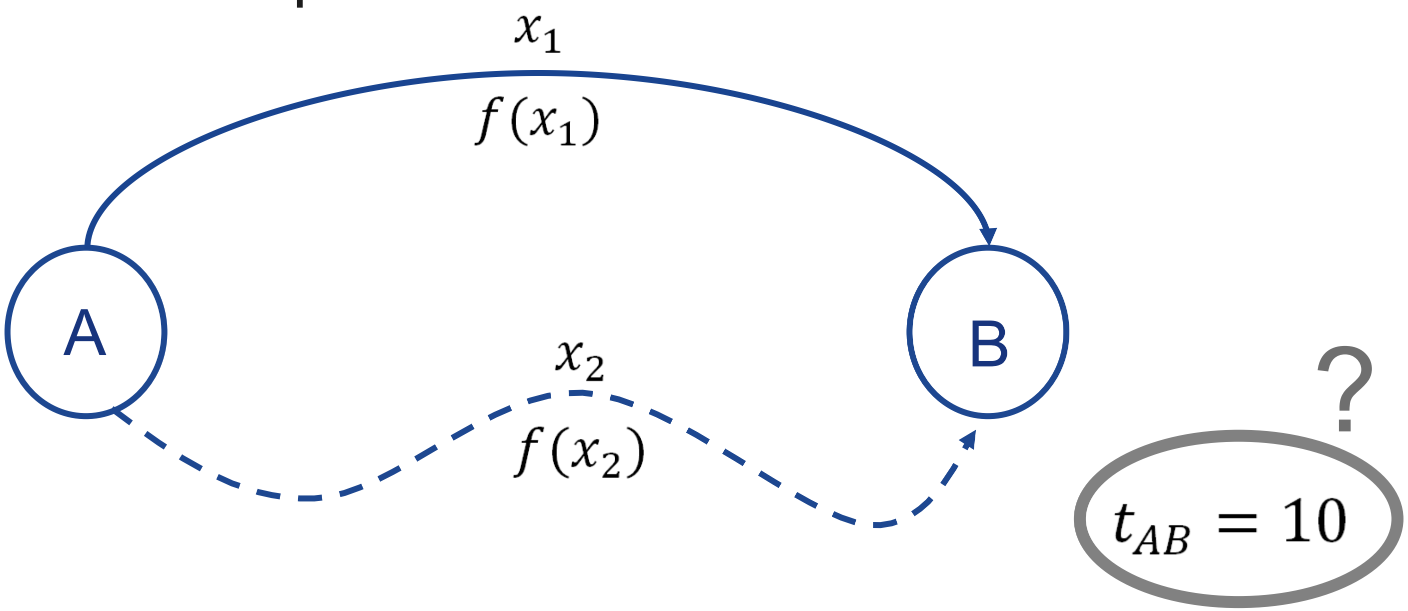


Strategic Assignment

- Altered assumption
  - Travellers make stable routing decisions considering daily volatility
  - First model, only consider demand uncertainty

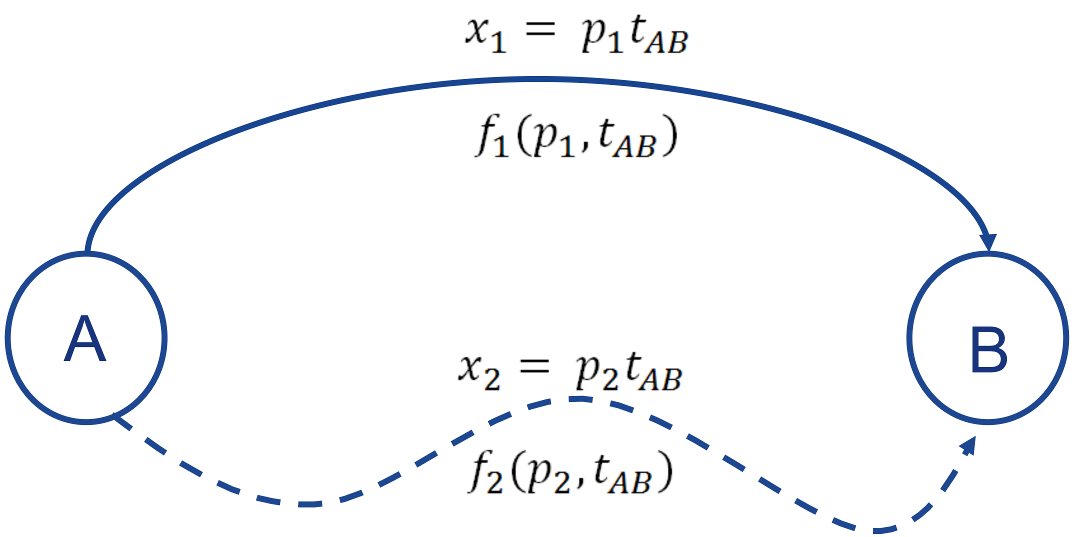
Simple Concept – Assignment with demand uncertainty

- How to account for demand uncertainty
  - User equilibrium
    - Expected costs equilibrate
  - System optimal
    - Minimize total expected cost



Strategic traffic assignment

- Path proportions
  - What becomes uncertain is simply number of travelers
- User equilibrium
  - People equilibrate according to ***expected*** cost
- System optimal
  - Minimize ***expected*** total system cost



$t_{AB} = t_{rs}(pt)$

Literature Sample

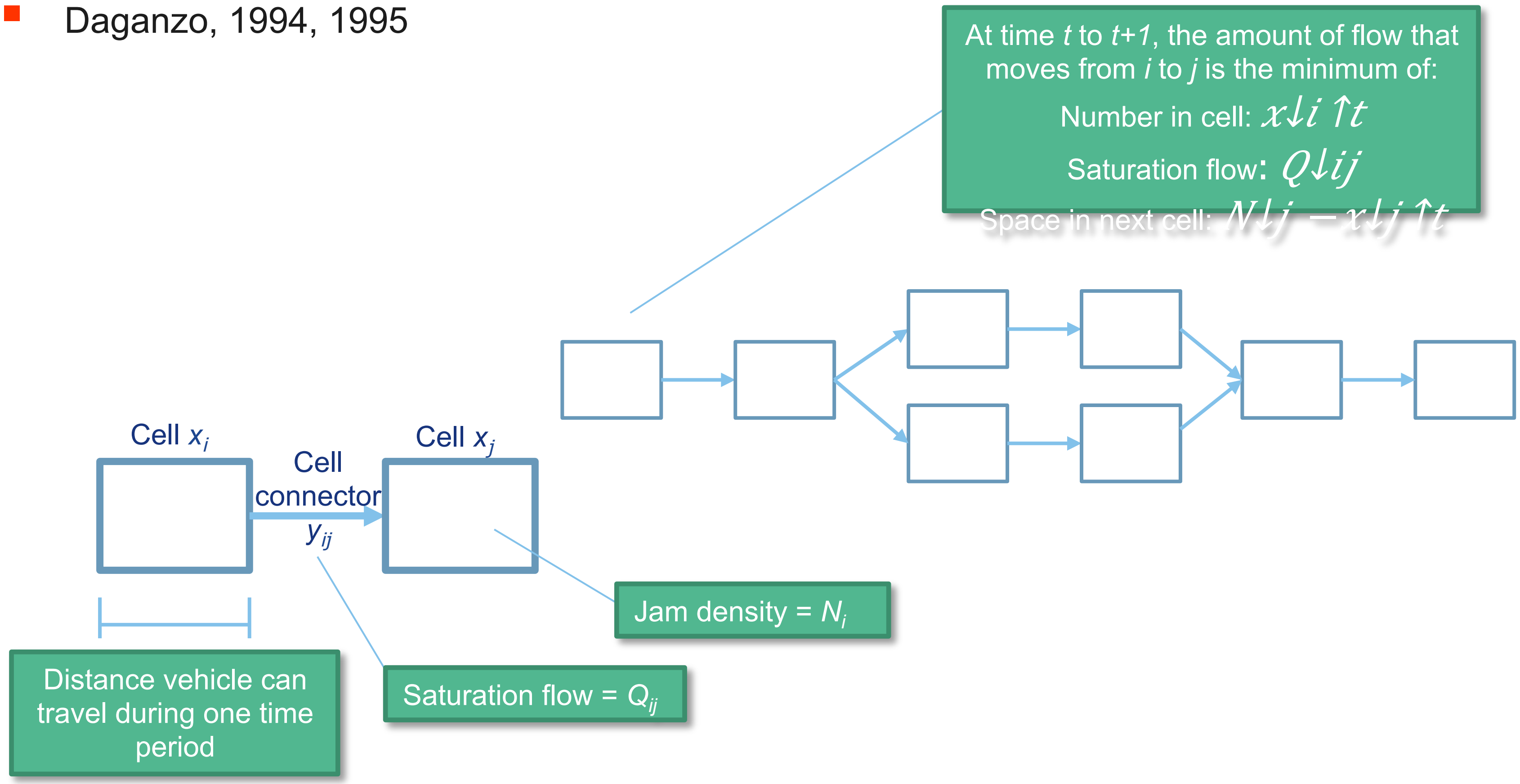
- Day-to-day travel
  - Asakura and Kashiwadani, 1991; Clark and Watling, 2005
  - Watling and Hazelmen, 2003; Hamdouch et al, 2004
- Strategic/Policy Based Approaches
  - Chriqui and Robillard, 1975; Marcotte and Nguyen, 1998
  - Marcotte et al, 2004; Hamdouch et al, 2004
  - Gao, and Chabini, 2006; Unnikrishnan and Waller, 2009
- Stochastic User Equilibrium
  - Daganzo and Sheffi, 1977; Sheffi and Powell, 1982
  - Maher and Hughes, 1997; Horowitz, 1984

Contribution

- New SO-DTA LP formulation for strategic path choice considering demand uncertainty
- Analysis of resulting path flows and cell densities
- Cues to future work and possible directions

The cell transmission model (CTM)

- Represents network structure in small “street” segments (cells)
- Efficient model that propagates traffic according to hydrodynamic flow equations
- Dynamic, simple, intuitive
- Daganzo, 1994, 1995



Strategic SO DTA LP

- Based on Ziliaskopolous (2000): Linear programming formulation of system optimal dynamic traffic assignment that embeds the CTM
  - Benefits
    - Propagates traffic without the use of a link performance function
    - Linear program
  - Challenges
    - Computationally costly
- Benefits of strategic approach
  - Encompasses the concept of strategies
  - Use path flows instead of link flows
  - Stochastic demand
- Challenges of strategic approach
  - Complex formulation
  - Still preliminary

Basic SO-DTA LP
Ziliaskopoulos (2000)

$\forall i \in C \setminus \{C_R, C_S\}, \forall t \in T,$

$\forall i \in C, \forall t \in T$

$\forall j \in C \setminus \{C_R, C_S\}, \forall t \in T$

$\forall j \in C \setminus C_R, \forall t \in T,$

$\forall i \in C \setminus C_S, \forall t \in T$

$\forall j \in S(i), \forall i \in C,$

Strategic Assignment:
Need to maintain path proportions and demand scenarios

Set of demand scenarios.

Demand scenario index.

Demand between OD pair (r,s) at departure time τ in demand scenario ξ

Set of all paths , $\phi\downarrow 1\uparrow rs$ …, $\phi\downarrow i\uparrow rs$ connecting OD pair (r,s)

Number of vehicles at time interval t on cell i which departed at time τ, following path ϕ between origin r and destination s in demand scenario ξ

Number of vehicles contained in cell i at time interval t in demand scenario ξ

Flow from cell i to cell j at time interval t for OD pair rs with departure time τ and travelling along path ϕ in demand scenario ξ

Total flow into cell i at time t in demand scenario ξ

Total flow out of cell i at time t in demand scenario ξ

Indicator equal to 1 if cell connector (i,j) is included along path ϕ, and 0 otherwise

LP formulation
Waller, Fajardo, Duell, and Dixit (2013)

- Can be intuitively interpreted as many SO DTA LPs all connected by the same path proportions

$$Minimize \sum_{\forall \xi \in \Xi} \sum_{\forall t \in T} \sum_{\forall i \in C \setminus C_s} p^{\xi} x_i^{t\xi}$$

Minimize the *expected* total system travel time, which equates to the sum of the densities for each cell over all time periods and demand scenarios

$s.t.$

$$x_{i,\phi,\tau}^{t,rs,\xi} - x_{i,\phi,\tau}^{t-1,rs,\xi} - \sum_{k \in P(i)} \delta_{\phi}^{ki} y_{ki,\phi,\tau}^{t-1,rs,\xi} + \sum_{j \in S(i)} \delta_{\phi}^{ij} y_{ij,\phi,\tau}^{t-1,rs,\xi} = 0, \quad \forall i \in C \setminus \{C_R, C_S\}, \forall t \in T, \forall \phi \in \Phi(rs), \forall rs \in RS, \forall \xi \in \Xi \qquad (2)$$

$$x_{i,\phi,\tau}^{t,rs,\xi} - x_{i,\phi,\tau}^{t-1,rs,\xi} - \sum_{k \in P(i)} \delta_{\phi}^{ki} y_{ki,\phi,\tau}^{t-1,rs,\xi} = 0 \quad \forall s \in C_S, \forall t \in T, \forall \phi \in \Phi(rs), \forall rs \in RS, \forall \xi \in \Xi \qquad (3)$$

$$x_{r,\phi,\tau}^{t,rs,\xi} - x_{r,\phi,\tau}^{t-1,rs,\xi} + \sum_{j \in S(r)} \delta_{\phi}^{rj} y_{rj,\phi,\tau}^{t-1,rs,\xi} = p_{\phi\tau}^{rs} d_{t-1}^{rs,\xi} \quad \forall r \in C_R, \forall t \in T, \forall \phi \in \Phi(rs), rs \in RS, \forall \xi \in \Xi \qquad (4)$$

$$\sum_{j \in S(i)} \delta_{\phi}^{ij} y_{ij,\phi,\tau}^{t,rs,\xi} - x_{i,\phi,\tau}^{t,rs,\xi} \leq 0, \quad \forall i \in C \setminus C_S, \forall t \in T, \forall \xi \in \Xi, \forall \phi \in \Phi(rs), \forall (rs) \in RS, \forall \xi \in \Xi \qquad (5)$$

$$\omega_i^{t,\xi} + x_i^{t,\xi} \leq N_i^t, \quad \forall i \in C \setminus \{C_R, C_S\}, \forall t \in T, \forall \xi \in \Xi \qquad (6)$$

$$\omega_i^{t,\xi} \leq Q_i^t, \quad \forall i \in C, \forall t \in T, \forall \xi \in \Xi \qquad (7)$$

$$\psi_i^{t,\xi} \leq Q_i^t, \quad \forall i \in C \setminus C_S, \forall t \in T, \forall \xi \in \Xi \qquad (8)$$

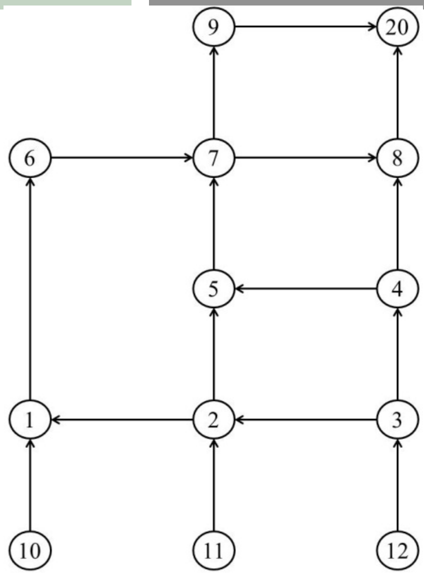
$$\sum_{\forall \phi \in \Phi(r,s)} p_{\phi t}^{rs} = 1 \quad \forall\, rs \in RS, \forall t \in T, \forall \xi \in \Xi \qquad (9)$$

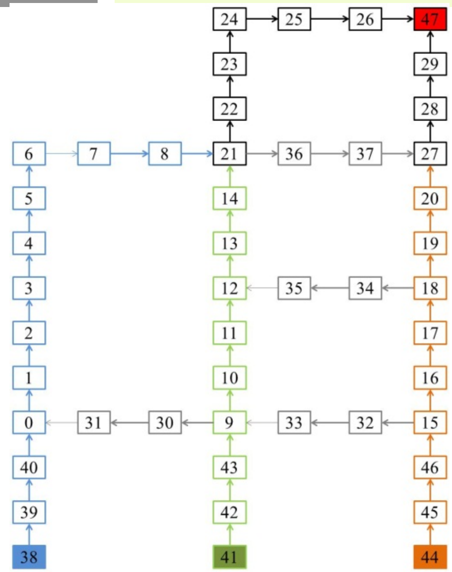
Flow conservation constraints (2)–(4)

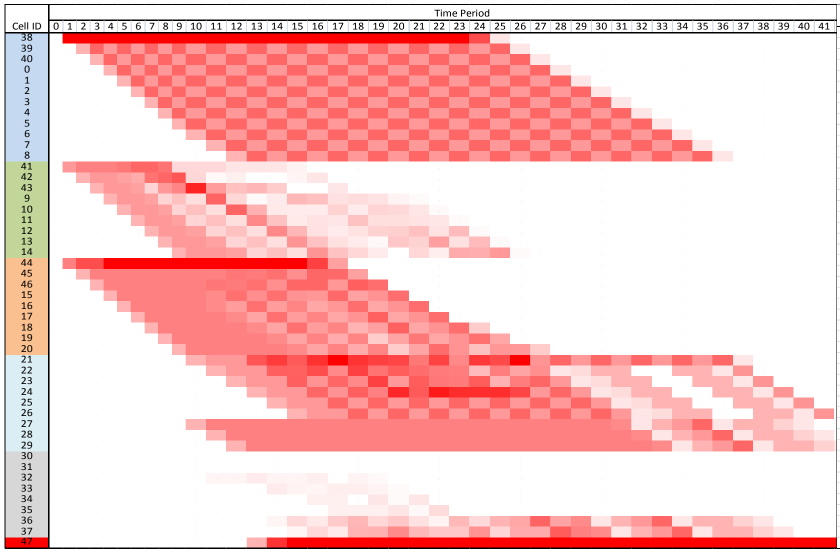
Cell capacity/ connector constraints (6)–(8)

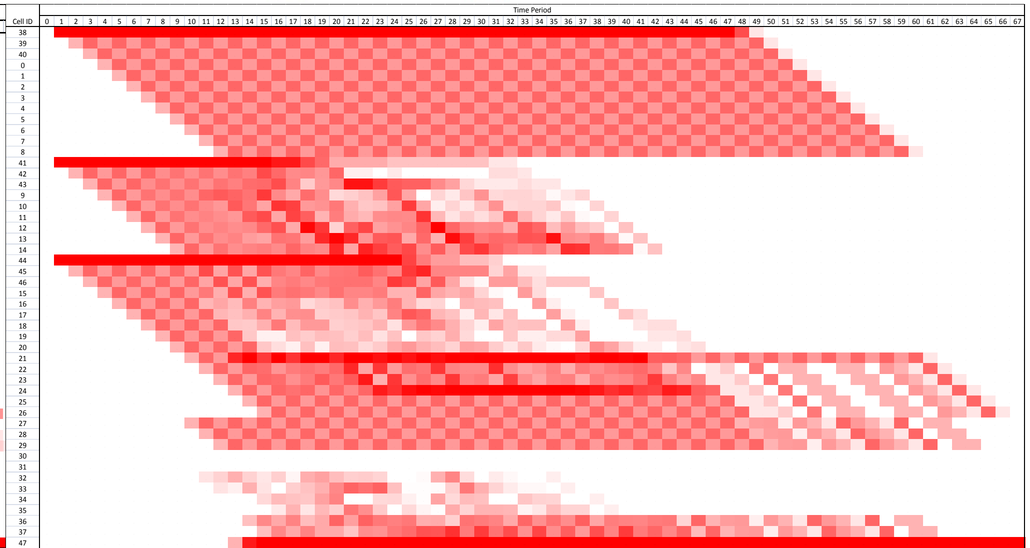
+8 more constraint sets: aggregate link flow to path flow, initial demands to zero, non-negativity

Results from the LP SO DTA Approach





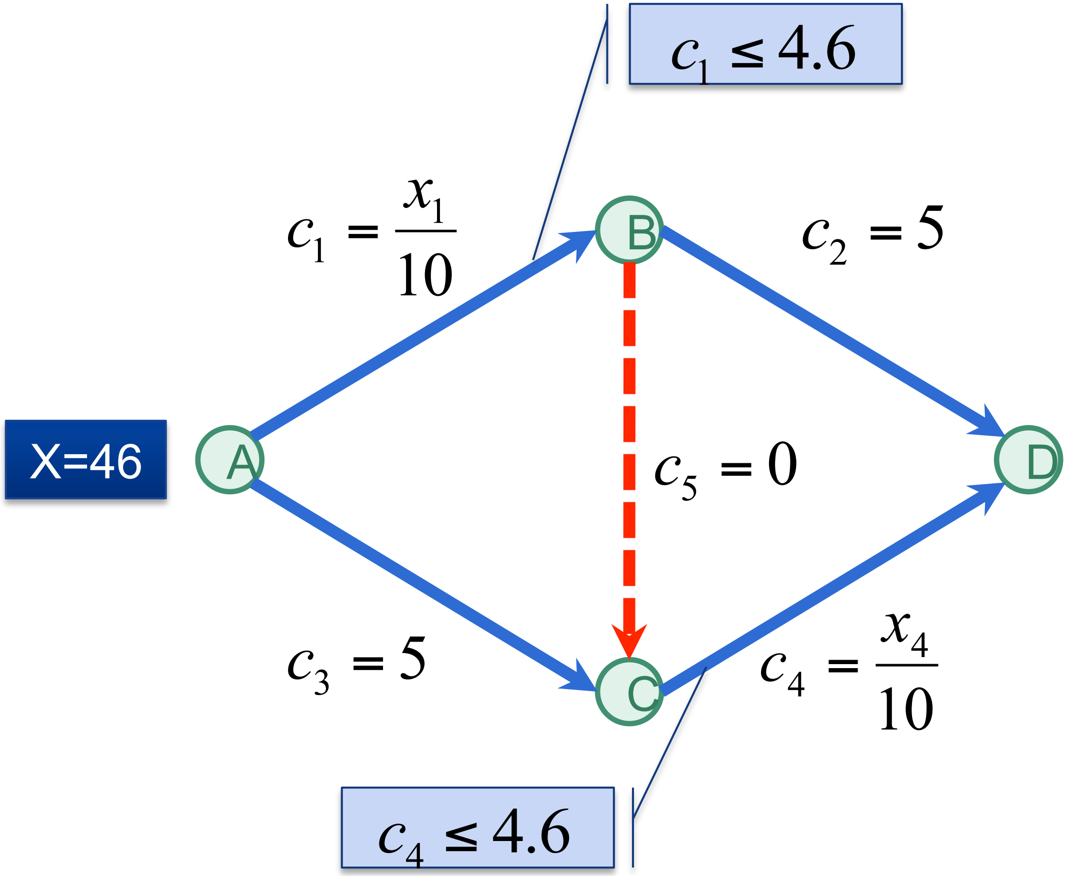




Current Work

- Developing novel numerical solution method
  - Marginal cost and dual numerical approximation
- Deriving static equilibrium formulation and solution algorithm
- Expanding dynamic formulation for network design and other forms of uncertainty
- NEXT: Adaptivity

Recall: Braess's Paradox Example



3 Paths

- A-B-D (***y_1***)
- A-C-D (***y_2***)
- A-B-C-D (***y_3***)

$y'_1 = y'_2 = 0$

$y'_3 = 46$

c'1+c'5+c'4=9.2

Z'=423.2

Z'-Z=87.4

Need such a model for adaptivity

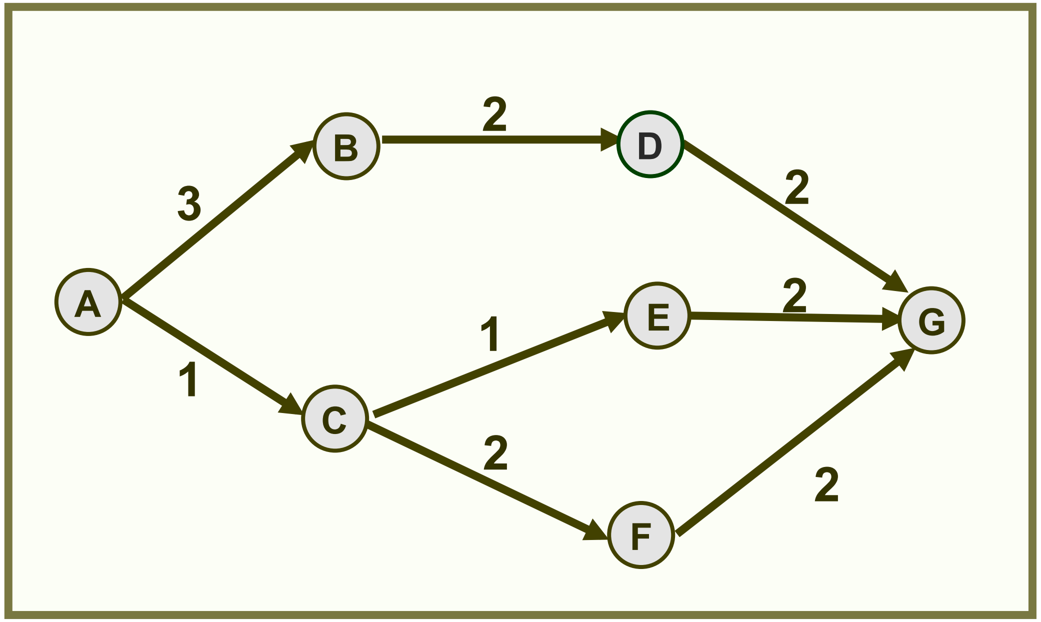
- We need similar models for **information** and **uncertainty** evaluation

- True impact of real-time ITS?
  - Fundamental **behavior**, including **anticipation**, will change

- We will begin with an examination of *individual routing under information*

Deterministic Costs:
Example Network

Path Costs
ABDG: 7
ACEG: 4
ACFG: 5

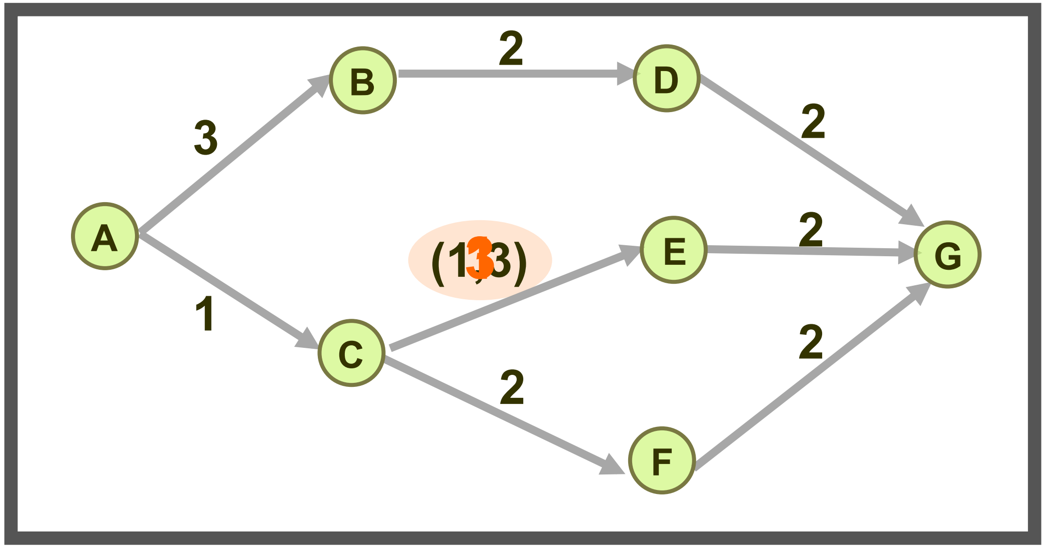


A user travel from A to G

Costs do not change with flow

Three elementary paths

Stochastic Costs:
Arc States & Hyper-paths



2 states

State 1 with cost 1

State 2 with cost 3

Both states have equal probability

Online Routing: Users learn the state of CE when they reach C

Recourse : Users change their paths en-route depending on the information received

State 1: ACEG

State 2: ACFG

Solution : Model assigns users to **hyperpaths**

AC/1-EG

AC/2-FG

Online Shortest Path (OSP)

- Numerous issues exist for even simple OSPs

- A couple quick examples and solution properties

Notation

o = origin node d = destination node

$S_{a,b}$ = Set of possible states for arc (a,b)

E[b|a,s]= expected cost to d from b, given that arc (a,b) is traversed at state s

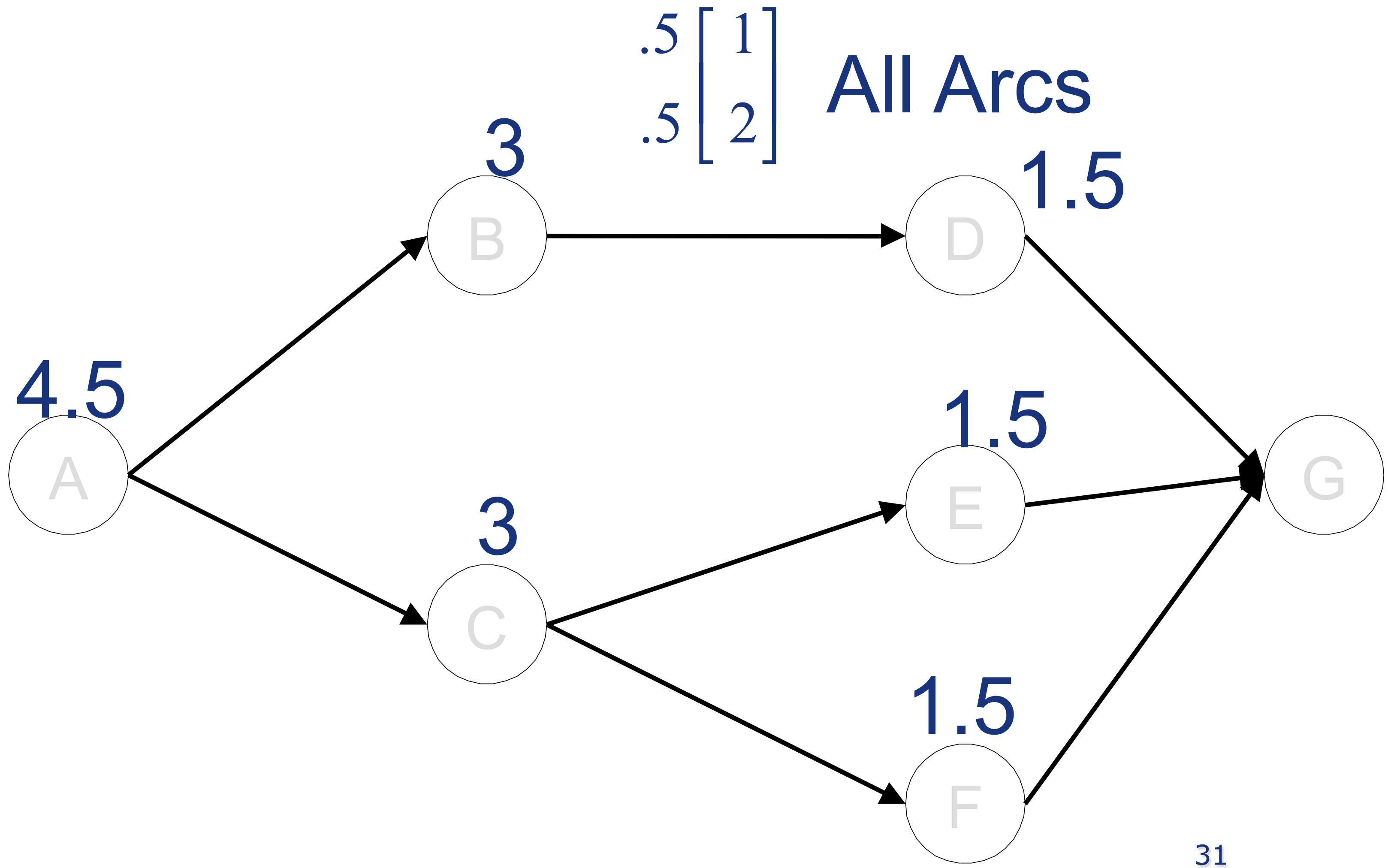
$p_{s,k}^{a,b,c}$ = probability that arc (b, c) is in state k, given that arc (a, b) was in state s

SE = scan eligible list

Γ^{-1}(a) = set of all predecessor nodes of a

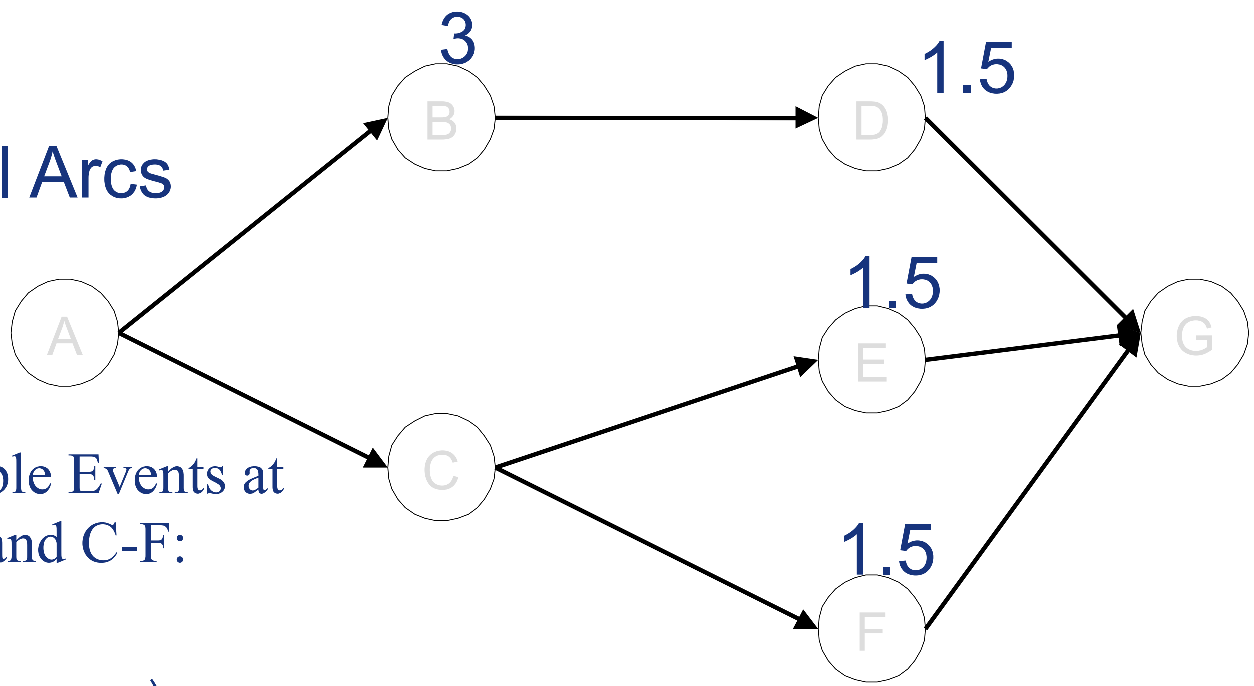
Γ(a) = set of all sucessor nodes of a

A Priori (offline) Example



On-line Example

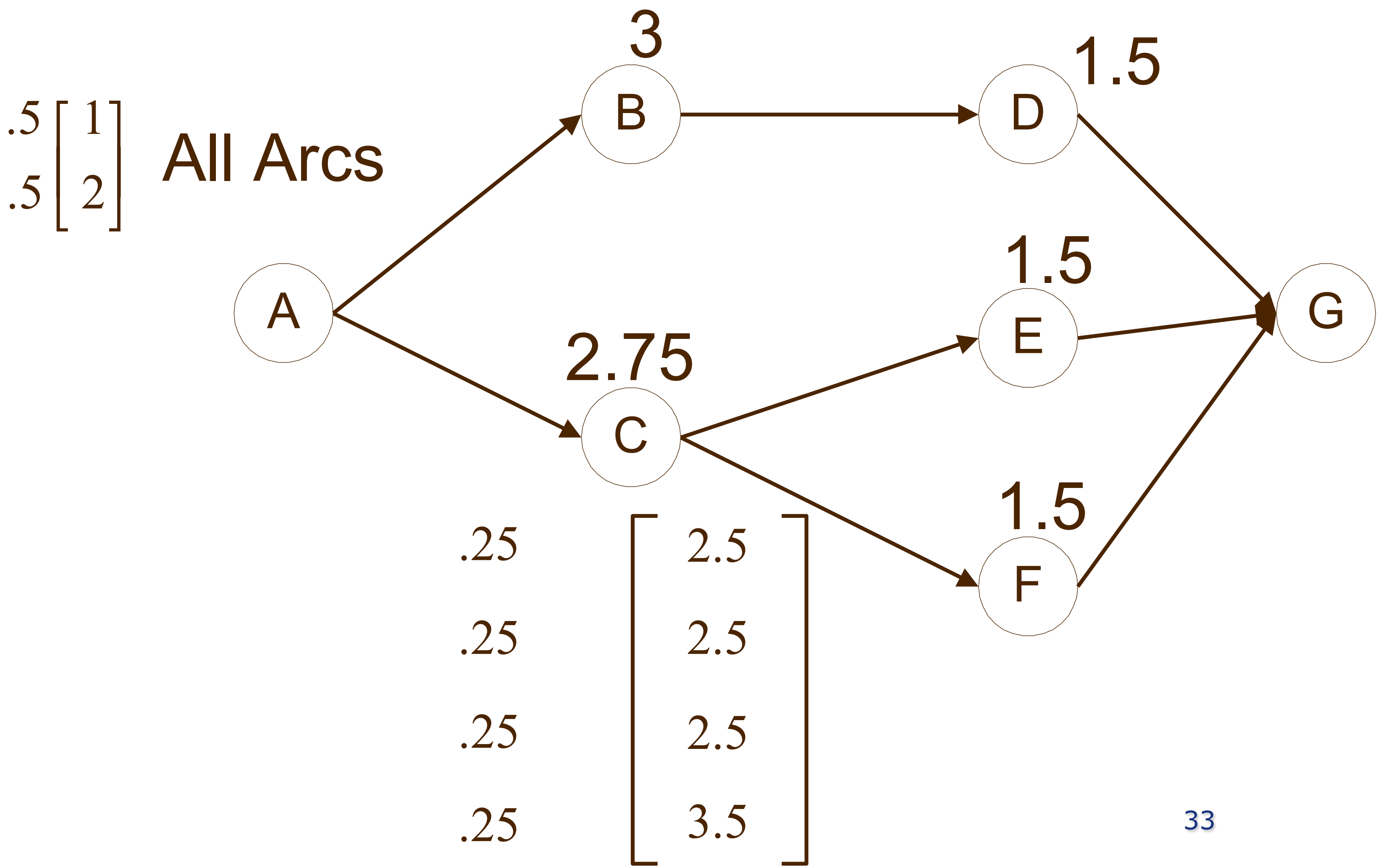
$$\begin{matrix} .5 \\ .5 \end{matrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$$ All Arcs



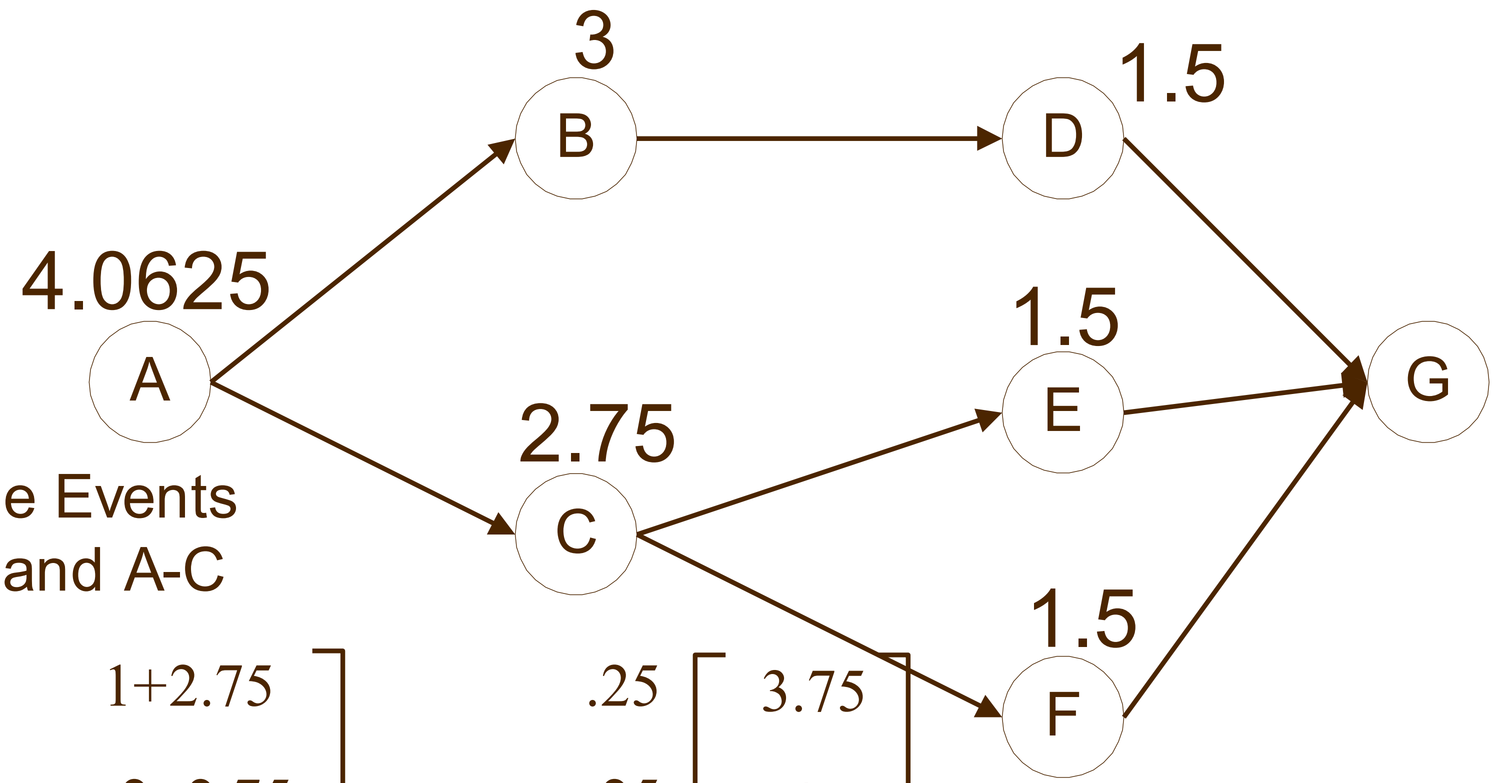
Possible Events at C-E and C-F:

$$\begin{matrix} .25 \\ .25 \\ .25 \\ .25 \end{matrix}\begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \\ 2 & 2 \end{pmatrix}$$

On-line Example



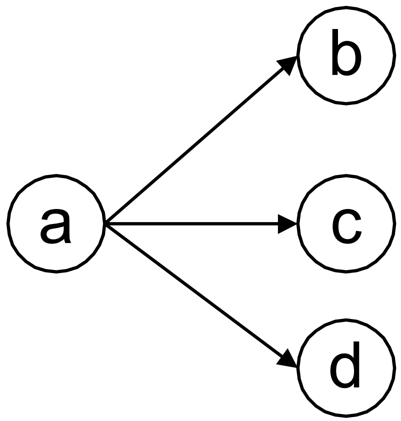
On-Line Example



Possible Events
at A-B and A-C

$$\begin{matrix} .25 \\ .25 \\ .25 \\ .25 \end{matrix} \begin{bmatrix} 1+3 & 1+2.75 \\ 1+3 & 2+2.75 \\ 2+3 & 1+2.75 \\ 2+3 & 2+2.75 \end{bmatrix} \Rightarrow \begin{matrix} .25 \\ .25 \\ .25 \\ .25 \end{matrix} \begin{bmatrix} 3.75 \\ 4 \\ 3.75 \\ 4.75 \end{bmatrix}$$

Simple Combined Probability Matrix



$$P^{a,b} = \begin{matrix}.333\\.333\\.333\end{matrix}\begin{bmatrix}1\\4\\8\end{bmatrix}, P^{a,c} = \begin{matrix}.5\\.5\end{matrix}\begin{bmatrix}2\\6\end{bmatrix},$$

$$P^{a,d} = \begin{matrix}.5\\.5\end{matrix}\begin{bmatrix}3\\5\end{bmatrix}$$

$$E = \begin{matrix}.0833\\.0833\\.0833\\.0833\\.0833\\.0833\\.0833\\.0833\\.0833\\.0833\\.0833\\.0833\end{matrix}\begin{bmatrix}\min(1+E[b],2+E[c],3+E[d])\\ \min(1+E[b],6+E[c],3+E[d])\\ \min(1+E[b],2+E[c],5+E[d])\\ \min(1+E[b],6+E[c],5+E[d])\\ \min(4+E[b],2+E[c],3+E[d])\\ \min(4+E[b],6+E[c],3+E[d])\\ \min(4+E[b],2+E[c],5+E[d])\\ \min(4+E[b],2+E[c],5+E[d])\\ \min(8+E[b],2+E[c],3+E[d])\\ \min(8+E[b],6+E[c],3+E[d])\\ \min(8+E[b],2+E[c],5+E[d])\\ \min(8+E[b],6+E[c],5+E[d])\end{bmatrix}$$

Pair-Wise Combination

- Combine first two arcs:

$$\begin{bmatrix} .166 & \min(1+E[b],2+E[c]) \\ .166 & \min(1+E[b],6+E[c]) \\ .166 & \min(4+E[b],2+E[c]) \\ .166 & \min(4+E[b],6+E[c]) \\ .166 & \min(8+E[b],2+E[c]) \\ .166 & \min(8+E[b],6+E[c]) \end{bmatrix}$$

- There can be at most 5 unique states in this matrix.
- Therefore, this matrix can be reduced and then combined with another arc.

Matrix Reduction

- 1)Create an empty dynamic Linked List (LL)
- 2)Remove row (a), consisting of a state cost and probability, from the original matrix
- 3)Perform a Binary Search on LL for the state of (a)
- 4)If it exists, add the probability from (a) to element in LL
- 5)If it does not exist, insert (a) into LL at the place pointed to by the binary search

Complexity of Reduction

- Take S to be the maximum number of States on any arc.
- This procedure must be carried out until the original combined matrix is empty, at most S^2 times.
- Each steps takes O(1) except 3.
- The maximum size of a reduced matrix is nS.
- Step 3 can be completed in log(nS).
- Reduction takes S^2 log(nS). For each pair-wise combination

Probability Bounds, Positive Costs

- C = Minimum Arc Cost, M = Maximum Arc Cost
- N = Number of Nodes,   E=Expected # of Arcs
- p(i) = Probability of exactly i cycles
- F = Cumulative distribution for # of Arcs

- C * E[# of Arcs] ≤ NM

$$E = \sum_{i=0}^{\infty} i * p(i)$$

Probability Bounds

- C * E ≤ NM

$$E = \sum_{i=0}^{\infty} i * p(i)$$

- Take ε(j) as a lower bound on E:
- $$\varepsilon(j) = \sum_{i=j}^{\infty} j * p(i)$$ where j ≥0 integer

- ε(j) = j*(1-F(j))
- Since ε(j) ≤ E ≤ NM/C
- => 1-F(j) ≤ NM/(Cj)

Properties and Complexity

- Cumulative probability F() that the optimal solution will contain j arcs is bounded:
  - $1-F(j) \leq nM/(Cj)$

- State space matrices can be iteratively bounded and reduced

- Yields algorithm complexity, given error ε
  - $O(n^2 m S^2 M(nM-C) / (C^2 \varepsilon))$

Online Algorithm 1 (of 3)

Waller and Ziliaskopoulos (2002)

Step 1.

$E[d|i,s]=0 \quad \forall\ i\in \Gamma^{-1}(d),\ s\in S_{i,t}$

$E[n|i,s]=\infty \quad \forall\ n\in N/d, \quad i\in\Gamma^{-1}(n),\ s\in S_{i,n}$

SE:= d

Algorithms are presented for variants of spatial, temporal and combined dependency

Step 2.

while SE $\neq \varnothing$

Remove an element, n, from the SE

for each $i\in\Gamma^{-1}(n)$, $s\in S_{i,n}$, $j\in\Gamma(n)$

$$\pi[n\,|\,i,s] = \sum_{k\in S_{n,j}} p_{s,k}^{i,n,j}\left(c_k^{n,j} + E[j\,|\,n,k]\right)$$

If $\pi[n|i,s] < E[n|i,s]$, then $E[n|i,s] := \pi[n|i,s]$

SE:=SE $\cup \{j\in\Gamma^{-1}(i)\}$

UER Network Assignment Model

Equilibrium Formulation

- Accounts for **congestion effect**
- Costs are a function of flow & network state

Model Assumptions

- Cost **functional form varies** according to the network state
- Travelers **learn the cost** functional form of an arc when they reach **upstream node**

Network Equilibrium with Recourse

Develop **analytical formulation** for traffic network assignment problem under online information provision

User Equilibrium

System Optimal

Develop a Frank-Wolfe based **solution algorithm** for solving the problem

Static network assignment

Limited one-step information

UER Model Definitions & Assumptions

Arc states follow a **discrete probability distribution**

When a **traveler reaches node *i*** they learn the cost functional form for **ALL arcs *(i,j)***

Special case: travelers learn the **capacity** on each arc

$C_{ijs}(\ )$ is the state-dependent cost function
$s \in S_{ij}$
S_{ij} is the set of possible states for arc *(i,j)*

Model A: All users see the same node state
Model B: Users see different node states

Model A : Expected Hyperpath Cost

Node State: combination of emanating link state realizations

System State: combination of node state realizations

Hyperpath Flow: H^k (for hyperpath ***k***)

Link/Hyperpath incidence: $\gamma^k_{i-j/u} \begin{cases} \mathbf{1} \text{ if hyperpath } \boldsymbol{k} \text{ uses arc } \boldsymbol{(ij)} \text{ under state } \boldsymbol{u}) \\ \mathbf{0} \text{ otherwise} \end{cases}$

Hyperarc Flow: $f_{i-j/u} = \sum_k \gamma^k_{i-j/u} H^k$ (given system state ***u***)

Hyperarc Flow Vector: $F = \Delta H$

Hyperpath flow vector (H)

Node-hyperpath accessibility matrix (Δ)

Hyperpath-Hyperarc Accessibility Matrix: $P_{l,k} = p_u \gamma^k_{i-j/u}$

Probability of system state u (p_u)

Expected Hyperpath Cost Vector: $P^T C[\Delta H]$

Model A: Formulation & Solution Algorithm

Unnikrishnan and Waller (2009)

CONVEX FORMULATION

$$Min\ \ Z[F(H)] = \sum_{iju} \int_{x=0}^{f_{i-j/u}} p_u . C_{i-j/u}(x)dx$$

$$\text{Subject to} \quad F = \Delta H \quad t = BH \quad H \geq 0$$

SOLUTION ALGORITHM : FRANK-WOLFE

Step 1: At iteration *n*, fix the costs on the arcs $C_{i-j/u}(f^n_{i-j/u})$

Step 2: Determine the optimal hyperpath H

Step 3: Conduct all-or-nothing assignment on H

Step 4: Determine the auxiliary link flows $y^{n+1}_{i-j/u}$

Step 5: Determine $f^{n+1}_{i-j/u}$ by a linear combination of $y^{n+1}_{i-j/u}, f^n_{i-j/u}$

Step 6: Test for convergence. If no set *n=n+1*, go to Step 1

Model A: Equilibrium Condition

Property: *A traffic network is in UER if each user follows a hyperpath that guarantees the minimum expected cost and no user can unilaterally change his/her hyperpath to improve their expected travel time*

EQUILIBRIUM CONDITION

$$H^T[P^T C[\Delta H] - B^T u] = 0$$

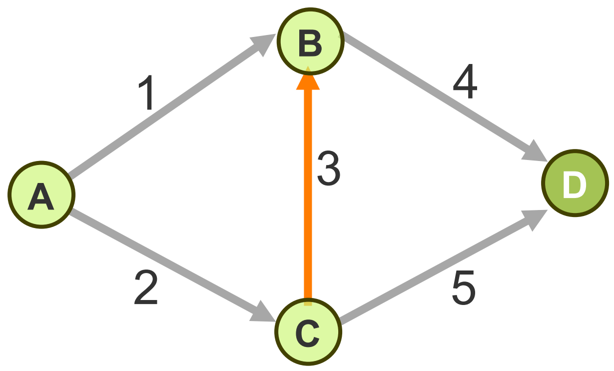
$$P^T C[\Delta H] - B^T u \geq 0$$

$$H \geq 0$$

INSIGHTS

- All used hyperpaths will have equal (and minimum) expected cost.
- This implies that those network users who follow a UER solution without options, still receive precisely the same benefit as those users who actually experience the options.

Without information



- ***Arc CB*** has **2 *STATES***:
  - ***State 1:*** $C_3(x)=1000$ (wp 0.2)
  - ***State 2:*** $C_3(x)=1$ (wp 0.8)
- Other arcs: ***single states***
  - $C_1(x)=5$, $C_2(x)=x/10$ (wp 1)
  - $C_4(x)=X/10$, $C_5(x)=5$ (wp 1)

PATHS

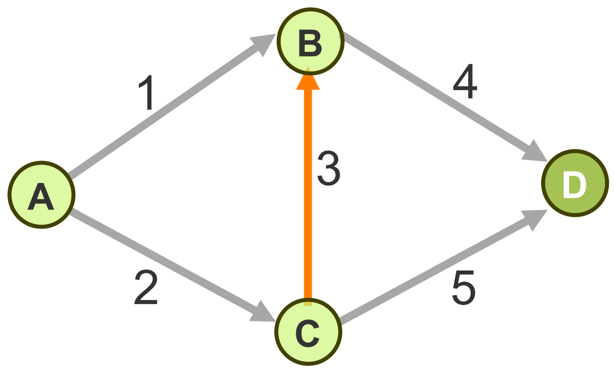
P1: A-B-D

P2: A-C-D

P3: A-C-B-D

- **DEMAND:** ***40 users*** want to travel from A to D
- **Solution:** all users split over paths P1 and P2 (*P3 too risky*)
- P1 = P2 = 20
- **User Cost = 7**

UER Example



- ***Arc CB*** has **2 *STATES***:
  - ***State 1:*** $C_3(x)$=1000 (wp 0.2)
  - ***State 2:*** $C_3(x)$=1 (wp 0.8)
- Other arcs: ***single states***
  - $C_1(x)$=5, $C_2(x)$=x/10 (wp 1)
  - $C_4(x)$=X/10, $C_5(x)$=5 (wp 1)

HYPERPATHS

H1: A-B-D

H2: A-C/1-B-D & A-C/2-B-D

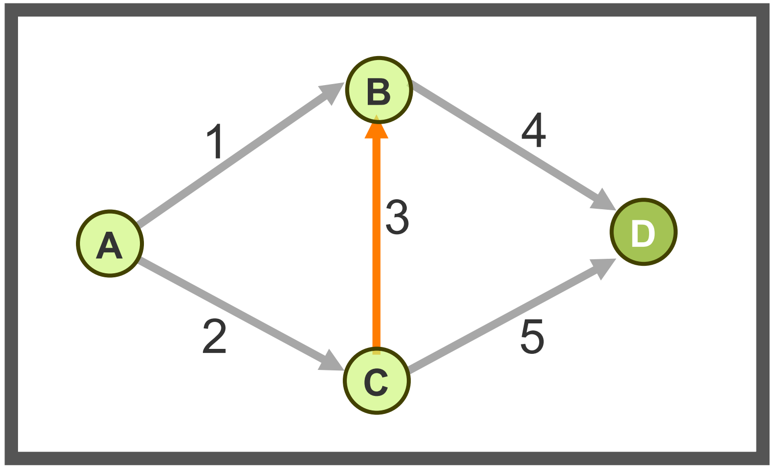
H3: A-C/1-B-D & A-C/2-D

H4: A-C/1-D & A-C/2-D

H5: A-C/1-D & A-C/2-B-D

- **DEMAND:** ***40 users*** want to travel from A to D
- Users assigned to ***HYPERPATHS***

UER Example



HYPERPATH	FLOW	EXP COST
H1	8.33	8.1666
H2	0	207.1333
H3	0	208.3333
H4	2.5	8.1666
H5	29.166	8.1666

All used hyperpaths have equal and minimal expected costs

*Flow on **BD** depends on **state of C**. Even though states are not correlated, the flow induces dependency*

UER vs UE Without Information: Braess Paradox

Expected User Cost
UER : 8.1666

Expected User Cost
No Information: 7

If **everybody** has access to the network state information, system performance may be **worse** than under a no-Information scenario

Fundamental implications when **planning for information provision** through **ITS devices**

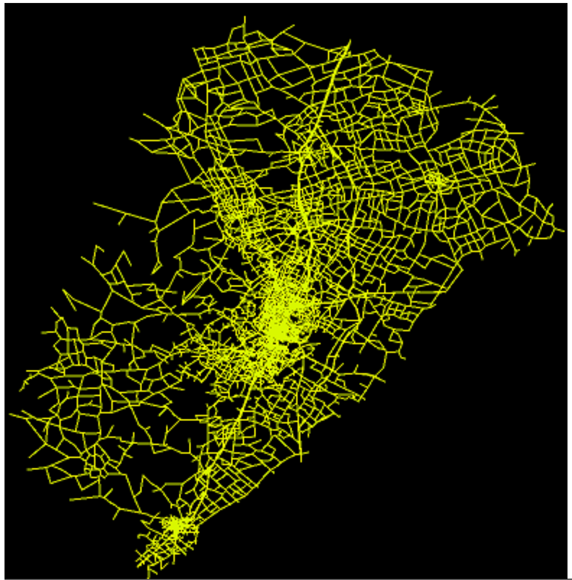
These analytical models form the next generation of deployable practical models

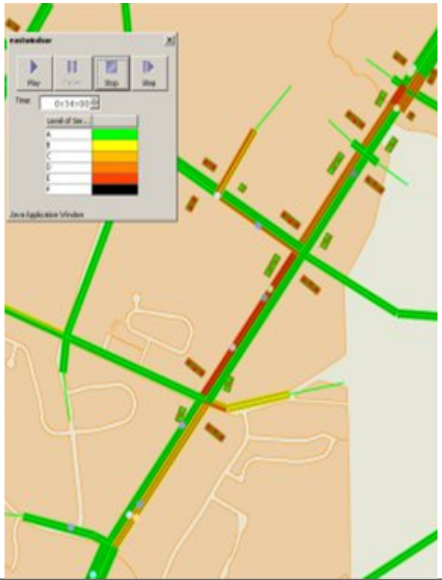
We need additional algorithmic computational improvement

Summary

- Overview of traditional network equilibrium for planning
- New models for strategic behavior
  - Including some explanatory capability for dis-equilibria
- New algorithms for online shortest path
- New models for user equilibrium with recourse

These models form only one specific piece of the bigger planning picture.





Questions?

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