Chapter 10 Associative Memory Networks

A higher dimensional example

Let us study an example. We begin by storing one vector as a fundamental memory. The particular choice of vector is not important , let us choose $\xi 1$ = [1 1 1 1 1 1 1 1 1 1]'.

The energy minimum has two minima at E = -90, one for x = [1 1 1 1 1 1 1 1 1 1]' and one for x = [-1 -1 -1 -1 -1 -1 -1 -1 -1 -1]', just as was stated before.

There are several more energy levels, but the levels are discrete since the elements can only assume the values 1 and –1.

If we store two vectors, $\xi 1$ = [1 1 1 1 1 1 1 1 1 1]' and $\xi 2$ = [1 1 1 1 1 -1 -1 -1 -1 -1]', then as expected we have minima at E = -80 for $\xi 1$ and $\xi 2$ and their “opposites”.

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If we store three vectors, $\xi 1$ = [1 1 1 1 1 1 1 1 1 1]' and $\xi 2$ = [1 1 1 1 1 -1 -1 -1 -1 -1]' and $\xi 3$=[1 -1 1 -1 1 -1 1 -1 1 -1]', then we have minima at E = -74 but only at $\xi 2$ and $\xi 3$. The energy at $\xi 1$ is slightly higher, at –70.

If we store four vectors, $\xi 1$ = [1 1 1 1 1 1 1 1 1 1]' and $\xi 2$ = [1 1 1 1 1 -1 -1 -1 -1 -1]', $\xi 3$ = [1 -1 1 -1 1 -1 1 -1 1 -1]' and $\xi 4$ = [1 1 -1 -1 1 1 -1 -1 -1 1]', then we have minima at E = -68 but only at $\xi 2$, $\xi 3$ and $\xi 4$. Again the energy at $\xi 1$ is slightly higher, at –60.

There are more differences between are four cases which become clear when we study the histograms for the energies. Let us do that.

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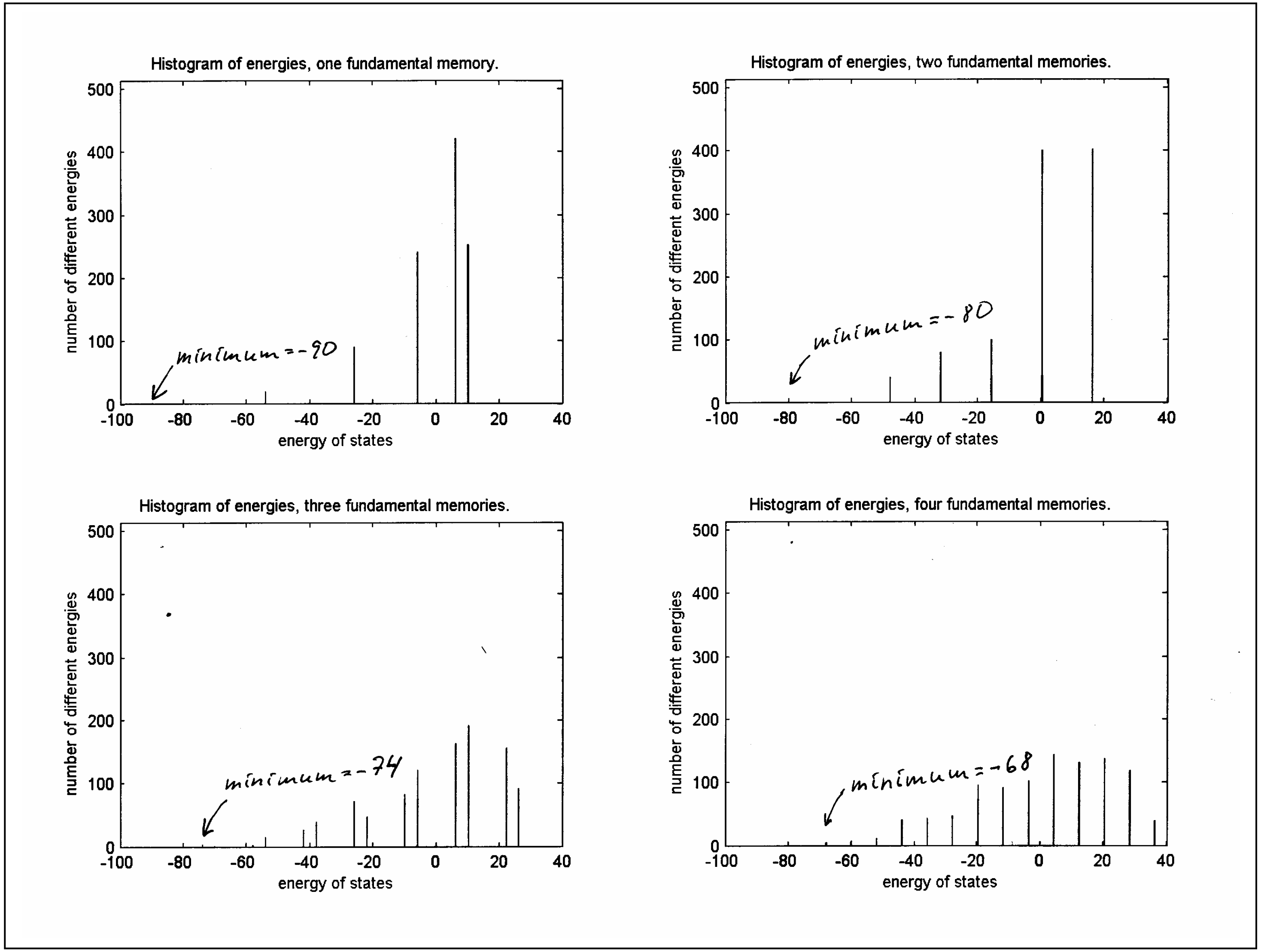

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We see that as the number of fundamental memories increase we get many more intermediate energy levels. Also there is much less difference between the minimum energy levels and other low energy levels. In other words, the energy minima of the attractors are not as distinct when we have more fundamental memories.

We can also expect some spurious attractors to have appeared, i.e. attractors that don't correspond to fundamental memories (others than opposite attractors, which are always present).

We might wonder if we have lost $\xi 1$ as a fundamental memory, but that is happily not the case. If we let the input to the network be $x = \xi 1$ then the output will be $\xi 1$. This is of course not a big feat, but we haven't lost the fundamental memory.

If we let the input be $\xi 1$ with any one element changed from 1 to –1 we can calculate the energy level at – 36, thus all these slightly corrupted versions of $\xi 1$ lie on a higher energy level than $\xi 1$ itself. We would expect to retrieve $\xi 1$ from such corrupted versions.