# CSE2330 Introduction to Computational Neuroscience 

## Associative Memory Networks

Tutorial 4 Duration: two weeks

### 4.1 About this tutorial

The objective of this tutorial is to introduce the concept of Associative or Contents-Addressable Memory and study its various implementations and properties.
In addition we will study a simple model of Alzheimer's disease.

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## getting started

The amount of information and the number of technical terms, computational methods and concepts that you are expected to master increase quickly, therefore, you are strongly advised to come well prepared to the practical classes.
Read at least the previous prac manuals, but you cannot go wrong if you also read related lecture notes and book chapters.
I also encourage you to execute MATLAB scripts line-by-line in order to build-up your understanding of computational methods involved.

### 4.2 Introductory concepts

Consider the way we are able to retrieve a pattern from a partial key as in Figure 1.


Figure 1: A key (left image) and a complete retrieved pattern (right image)
Imagine a question "what is it" in relation to the right image.

- The hood of the Volkswagen is the key to our associative memory neural network and the stored representation of the whole Volkswagen can be thought of as an network attractor for all similar keys.
- The key starts a retrieval process which ends in an attractor which contained both the whole car and its name (maybe you go only for the name since the question is "what is it")
- Storing a memory (an image) like the shape of a Volkswagen in an associative memory network and retrieving it, starting with a key, i.e. an incomplete version of the stored memory is the topic of this chapter.

There are two fundamental types of the associate memory networks:

- Feedforward associative memory networks in which retrieval of a stored memory is a one-step procedure.
- Recurrent associative memory networks in which retrieval of a stored memory is a multi-step relaxation procedure.

Recurrent binary associative memory networks are often referred to as the Hopfield networks.

- For simplicity we will be working mainly with binary patterns, each element of the pattern having values $\{-1,+1\}$.
- Example of a simple binary pattern:

$$
\boldsymbol{\xi}_{M}=\begin{gathered}
6 \\
8 \\
\hline
\end{gathered}
$$

The pattern $\boldsymbol{\xi}$ is a column-scan of the matrix $\boldsymbol{\xi}_{M},\{0,1\}$ being replaced with $\{-1,+1\}$.

### 4.2.1 Encoding and decoding single memories

The concept of creating a memory in a neural network, that is, memorizing a pattern in synaptic weights and its subsequent retrieval is based on the "read-out" property of the outer product of two vectors that we have studied earlier.
Assume that we have a pair of column vectors:

$$
\begin{array}{ccl}
n \text {-component vector } & \boldsymbol{\xi} & \text { representing the input pattern } \\
m \text {-component vector } & \mathrm{q} & \text { representing the desired output association with the input pattern }
\end{array}
$$

The pair $\{\boldsymbol{\xi}, \mathbf{q}\}$ to be stored is called a fundamental memory.

## Encoding a single memory

We store or encode this pair in a matrix $\boldsymbol{W}$ which is calculated as an outer product (column $\times$ row) of these two vectors

$$
\begin{equation*}
W=\mathrm{q} \cdot \xi^{T} \tag{1}
\end{equation*}
$$

## Decoding a single memory

The retrieval or decoding of the store pattern is based on application of the input pattern x to the weight matrix $\boldsymbol{W}$. The result can be calculated as follows:

$$
\begin{equation*}
\mathrm{y}=W \cdot \xi=\mathrm{q} \cdot \xi^{T} \cdot \xi=\|\xi\| \cdot \mathrm{q} \tag{2}
\end{equation*}
$$

The equation says that the decoded vector $\mathbf{y}$ for a given input pattern $\boldsymbol{\xi}$ (the key) is proportional to the encoded vector $\mathbf{q}$, the length of the input pattern $\boldsymbol{\xi}$ being the proportionality constant.


Figure 2: The structure of a feed-forward linear associator: $\mathrm{y}=\boldsymbol{\sigma}(\boldsymbol{W} \cdot \mathrm{x})$

### 4.2.2 Feedforward Associative Memory

The above considerations give rise to a simple feed-forward associative memory known also as the linear associator.
It is a well-known single layer feed-forward network with $\boldsymbol{m}$ neurons each with $\boldsymbol{p}$ synapses as illustrated in Figure 2.
For such a simple network to work as an associative memory, the input/output signal are
binary signals with

$$
\{0,1\} \text { being mapped to }\{-1,+1\}
$$

- During the encoding phase the fundamental memories are stored (being encoded) in the weight matrix $\boldsymbol{W}$
- During the decoding or retrieval phase for a given input vector x , which is the key to the memory, a specific output vector y is decoded.


### 4.2.3 Encoding multiple memories

Extending the introductory concepts let us assume that we would like to store/encode $\boldsymbol{K}$ pairs of column vectors (fundamental memories) arranged in the two matrices:

$$
\begin{aligned}
\Xi & =\xi(1) \ldots \xi(K) \quad \text { a matrix of } n \text {-component vectors representing the desired input patterns } \\
Q & =\mathrm{q}(1) \ldots \mathrm{q}(\boldsymbol{K}) \quad \text { a matrix of } m \text {-component vectors representing the desired output associations with }
\end{aligned}
$$

In order to encode the $\{\boldsymbol{\Xi}, \boldsymbol{Q}\}$ patterns we sum outer products of all pattern pairs:

$$
\begin{equation*}
W=\frac{1}{K} \sum_{k=1}^{K} \mathrm{q}(k) \cdot \xi^{T}(k)=\frac{1}{K} Q \cdot \Xi^{T} \tag{3}
\end{equation*}
$$

The sum of the outer products can be conveniently replaced by product of two matrices consisting of the pattern vectors.
The resulting $\boldsymbol{m} \times \boldsymbol{n}$ matrix $\boldsymbol{W}$ encodes all the desired $\boldsymbol{K}$ pattern pairs $\mathrm{x}(\boldsymbol{k}), \mathrm{q}(\boldsymbol{k})$.

Note that eqn (3) can be seen as an extension of the Hebb's learning law in which we multiply afferent and efferent signals to form the synaptic weights.

### 4.2.4 Decoding operation

Retrieval of a pattern is equally simple and involves acting with the weight matrix on the input pattern (the key)

$$
\begin{equation*}
\mathrm{y}=\sigma(\boldsymbol{W} \cdot \mathrm{x}) \tag{4}
\end{equation*}
$$

where the function $\sigma$ is the two-valued sign function:

$$
y_{j}=\sigma\left(v_{j}\right)= \begin{cases}+1 & \text { if } v_{j} \geq 0  \tag{5}\\ -1 & \text { otherwise }\end{cases}
$$

It is expected that

1. $\mathrm{x}=\boldsymbol{\xi}$

If the key (input vector) x is equal to one of the fundamental memory vectors $\boldsymbol{\xi}$, then the decoded pattern $\mathbf{y}$ will be equal to the stored/encoded pattern q for the related fundamental memory.
2.

$$
\mathrm{x}=\xi+\eta
$$

If the key (input vector) x can be considered as one of the fundamental memory vectors $\boldsymbol{\xi}$, corrupted by noise $\boldsymbol{\eta}$ then the decoded pattern y will be also equal to the stored/encoded pattern $\mathbf{q}$ for the related fundamental memory.
3.

$$
\mathrm{x} \neq \xi+\eta
$$

If the key (input vector) x is definitely different to any of the fundamental memory vectors $\boldsymbol{\xi}$ , then the decoded pattern $\mathbf{y}$ is a spurious pattern.

- The above expectations are difficult to satisfy in a feedforward associative memory network if the number of stored patterns $\boldsymbol{K}$ is more than a fraction of $\boldsymbol{m}$ and $\boldsymbol{n}$.
- It means that the memory capacity of the feedforward associative memory network is low relative to the dimension of the weight matrix $\boldsymbol{W}$.

In general, associative memories also known as content-addressable memories (CAM) are divided in two groups:
Auto-associative: In this case the desired patterns $\boldsymbol{\Xi}$ are identical to the input patterns $\boldsymbol{X}$, that is, $Q=\Xi$. Also $n=m$.
Eqn (3) describing encoding of the fundamental memories can be now written as:

$$
\begin{equation*}
W=\frac{1}{K} \sum_{k=1}^{K} \xi(k) \cdot \xi^{T}(k)=\frac{1}{K} \Xi \cdot \Xi^{T} \tag{6}
\end{equation*}
$$

Such a matrix $\boldsymbol{W}$ is also known as the auto-correlation matrix.

Hetero-associative: In this case the input $\boldsymbol{\Xi}$ and stored patterns $\boldsymbol{Q}$ and are different.

### 4.2.5 A simple script for the linear associator

In the first example we test how the linear associator encodes and decodes binary patterns.

```
% p4ffAM.m
% Feed-Forward Auto-Associative Memory
clear
% Generation of binary patterns to be stored
n = 4 ; % number of bits. 2^n possible patterns
    % dimensionality of patterns and number of neurons
K = 5 ; % number of patterns
% generate nn = 2^n - 1 integer numbers
nn = 2^n - 1 ;
Xd = randperm(nn) % generate nn integer random numbers
Xd = Xd(I:K) ; % taking the first K numbers
Xd = [2 4 4 10 11 6] ; % for example
X = (dec2bin(Xd, n) - 'O')' % convert integers to binary strings
% recoding {0, 1} to {-1, +1}
X = 2*X - 1 % the matrix of patterns to be stored
% encoding
W = (X* X')/K ; % storing the patterns in the weight matrix
W_ = num2str (W)
% decoding
% Creating all possible n-bit patterns coded {-1, +1}
Xall = 2*(((dec2bin(1:nn) -' 0')') -1 ;
Xall_ = num2str(Xall)
% Recalling all possible binary patterns
Yb = (W*Xall >= 0) ; % in {0, 1} form
Yb__ = num2str(Yb)
Y = 2*Yb -1 ; % recoded to {-1, +1} form
Y_ = num2str(Y)
% recoding recalled patterns to decimals
pwrs2 = fliplr(2.^ ((1:n)-1)) % powers of 2
Yd = pwrs2*Yb ;
X_Yd = num2str([1:nn ;Yd], '%3d')
Xd__ = num2str(sort (Xd(1:K)))
figure(1)
plot(1:nn, Yd, '*', Xd, Xd, 'o'),
grid on, axis([0 2^n 0 2^n]),
xlabel('input patterns'), ylabel('o-stored and *-retrieved patterns')
% print -f1 -depsc2 p4ffAMp
kk = sum(Xd == Yd(Xd)) ;
sprintf('Correct recalls: %d out of %d', kk, K)
spuriousPatterns = setdiff(Yd, Xd)
```

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Results can be similar to the following:

$$
\text { Xall_ }=
$$

$$
\begin{array}{rrrrrrrrrrrrrrr}
-1 & -1 & -1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
-1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1
\end{array}
$$

Yb_ =

$$
\begin{array}{lllllllllllllll}
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0
\end{array}
$$

$$
\begin{array}{lllllllllllllll}
0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1
\end{array}
$$

$$
\begin{array}{lllllllllllllll}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1
\end{array}
$$

$$
Y_{-}=
$$

$$
\begin{array}{lllllllllllllll}
-1 & -1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & 1 & 1 & 1
\end{array}
$$

$$
\begin{array}{lllllllllllllll}
-1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & -1
\end{array}
$$

$$
\begin{array}{lllllllllllllll}
-1 & 1 & 1 & -1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 & -1 & 1 & 1
\end{array}
$$

$$
\begin{array}{lllllllllllllll}
1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 & -1 & 1 & -1 & 1
\end{array}
$$

$$
\begin{array}{lrrrrrrrrrrrrrrr}
\mathrm{X} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
\mathrm{Yd} & 1 & 2 & 11 & 4 & 5 & 6 & 5 & 11 & 9 & 10 & 11 & 4 & 13 & 14 & 11
\end{array}
$$

$$
\begin{array}{lllll}
X d=2 & 4 & 6 & 10 & 11
\end{array}
$$

$$
\text { Correct recalls: } 5 \text { out of } 5
$$

$$
\text { spuriousPatterns }=1 \begin{array}{lllll}
14 & 5 & 9 & 13 & 14
\end{array}
$$

$$
\begin{aligned}
& \mathrm{Xd}=\begin{array}{lllll}
2 & 4 & 6 & 10 & 11
\end{array}
\end{aligned}
$$

$$
\begin{aligned}
& W_{-}=\begin{array}{cccr}
1 & -0.6 & 0.2 & 0.6 \\
-0.6 & 1 & -0.6 & -0.2 \\
& 0.2 & -0.6 & 1
\end{array}-\frac{-0.2}{} \begin{array}{c}
1
\end{array}
\end{aligned}
$$



Figure 3: Encoding and decoding binary patterns in a feed-forward associative memory
Note that the pattern $11=(1011)_{2}$ is recalled also for patterns $3=(0011)_{2}$ and $15=(1111)_{2}$ that can be considered as a noisy version of 11 (difference on only one position), but also for $8=(1000)_{2}$ that differs no two positions.
There are also five spurious patterns that have not been originally encoded.

## Exercise 4. 1

Run the above script for $\boldsymbol{n}=5$ and different values of $\boldsymbol{K}$, ten times for each $\boldsymbol{K}$, and record in a table the percentage of correct recalls and spurious patterns.

### 4.3 Recurrent Associative Memory - Discrete Hopfield networks

- The capacity of the feedforward associative memory is relatively low, a fraction of the number of neurons.
- When we encode many patterns often the retrieval results in a corrupted version of the fundamental memory.
- However, if we use again the corrupted pattern as a key, the next retrieved pattern is usually closer to the fundamental memory.
- This feature is exploited in the recurrent associative memory networks.


Figure 4: A dendritic and block diagram of a recurrent associative memory

### 4.3.1 Structure

- A recurrent network is built in such a way that the output signals are fed back to become the network inputs at the next time step, $\boldsymbol{k}$
- The working of the network is described by the following expressions:

$$
\mathrm{y}(k+1)=\sigma\left(W \cdot(\mathrm{x} \cdot \delta(k)+\mathrm{y}(k))=\left\{\begin{align*}
\sigma(W \cdot \mathrm{x}) & \text { for } k=0  \tag{7}\\
\sigma(W \cdot \mathrm{y}(k)) & \text { for } k=1,2, \ldots
\end{align*}\right.\right.
$$

- The function $\boldsymbol{\delta}(\boldsymbol{k})$ is called the Kronecker delta and is equal to one for $\boldsymbol{k}=0$, and zero otherwise.
It is a convenient way of describing the initial conditions, in this case, the initial values of the input signals are equal to $\mathrm{x}(0)$.
- A discrete Hopfield network is a model of an associative memory which works with binary patterns coded with $\{-1,+1\}$
Note that if $v \in\{0,1\}$ then $u=2 v-1 \in\{-1,+1\}$
- The feedback signals $\mathbf{y}$ are often called the state signals.
- During the storage (encoding) phase the set of $\boldsymbol{N} \boldsymbol{m}$-dimensional fundamental memories:

$$
\Xi=[\xi(1), \xi(2), \ldots, \xi(K)]
$$

is stored in a matrix $\boldsymbol{W}$ in a way similar to the feedforward auto-associative memory networks, namely:

$$
\begin{equation*}
W=\frac{1}{m} \sum_{k=1}^{K} \xi(k) \cdot \xi(k)^{T}-K \cdot I_{m}=\frac{1}{m} \Xi \cdot \Xi^{T}-K \cdot I_{m} \tag{8}
\end{equation*}
$$

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- By subtracting the appropriately scaled identity matrix $\boldsymbol{I}_{\boldsymbol{m}}$ the diagonal terms of the weight matrix are made equal to zero, $\left(w_{j j}=0\right)$.
This is required for a stable behaviour of the Hopfield network.
- During the retrieval (decoding) phase the key vector x is imposed on the network as an initial state of the network

$$
y(0)=x
$$

The network then evolves towards a stable state (also called a fixed point), such that,

$$
\mathrm{y}(k+1)=\mathrm{y}(k)=\mathrm{y}_{s}
$$

It is expected that the $\mathrm{y}_{\boldsymbol{s}}$ will be equal to the fundamental memory $\boldsymbol{\xi}$ closest to the key x

### 4.3.2 Example of the Hopfield network behaviour for $m=3$

(based on Haykin, Neural Networks)
Consider a discrete Hopfield network with three neurons as in Figure 5


Figure 5: Example of a discrete Hopfield network with $m=3$ neurons: its structure and the weight matrix

- With $m=3$ binary neurons, the network can be only in $2^{3}=8$ different states.
- It can be shown that out of 8 states only two states are stable, namely: $(1,-1,1)$ and $(-1,1,-1)$.
In other words the network stores two fundamental memories
- Starting the retrieval with any of the eight possible states, the successive states are as depicted in Figure 6.


Figure 6: Evolution of states for two stable states

Let us calculate the network state for all possible initial states

$$
X=\left[\begin{array}{l}
y_{3} \\
y_{2} \\
y_{1}
\end{array}\right]=\left[\begin{array}{rrrrrrrr}
-1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
-1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
-1 & 1 & -1 & 1 & -1 & 1 & -1 & 1
\end{array}\right]
$$

(The following MATLAB command does the trick: $\left.\quad x=2 *\left(\operatorname{dec} 2 b i n(0: 7)-^{\prime} 0^{\prime}\right)^{\prime}-1\right)$

$$
\begin{aligned}
Y & =\sigma(W \cdot X)=\frac{1}{3}\left[\begin{array}{rrrr}
0 & -2 & +2 \\
-2 & 0 & -2 \\
+2 & -2 & 0
\end{array}\right] \cdot\left[\begin{array}{rrrrrrrr}
-1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
-1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 \\
-1 & 1 & -1 & 1 & -1 & 1 & -1 & 1
\end{array}\right] \\
& =\sigma\left(\frac{1}{3}\left[\begin{array}{rrrrrrrr}
0 & 4 & -4 & 0 & 0 & 4 & -4 & 0 \\
4 & 0 & 4 & 0 & 0 & -4 & 0 & -4 \\
0 & 0 & -4 & -4 & 4 & 4 & 0 & 0
\end{array}\right]\right)=\left[\begin{array}{rrrrrrrr}
1 & 1 & -1 & 1 & 1 & 1 & -1 & 1 \\
1 & 1 & 1 & 1 & 1 & -1 & 1 & -1 \\
1 & 1 & -1 & -1 & 1 & 1 & 1 & 1
\end{array}\right]
\end{aligned}
$$

It is expected that after a number of relaxation steps

$$
\boldsymbol{Y}=\boldsymbol{W} \cdot \boldsymbol{Y}
$$

all patterns converge to one of two fundamental memories as in Figure 6.

## Exercise 4.2

For the Hopfield network described in sec. 4.3.2 write a MATLAB script that generates all attractors (stable states) of the network and verify the network states presented in Figure 6

8 marks

### 4.4 Simple model of Alzheimer's disease

### 4.4.1 Background information

Alzheimer's disease (AD) is the most common type of dementia in the elderly. The disease is characterised by a gradual loss of higher cognitive functions typically involving memory loss [1]

In our simple model of Alzheimer's disease we use a binary Hopfield recurrent neural network (as illustrated in Figure 4) as a model of human memory and the memory loss will be then modelled by gradual killing of synapses.

It must be however said that [1]
Studies using neural network models of associative memory demonstrate very clearly that cell death cannot produce amnesia similar to that observed in AD [...]
Furthermore, call death is not a consistent correlate of cognitive decline in the AD brain.
Do not be discouraged by the above statements. Our simple model was one of the first that was used to model Alzheimer's disease. The big advantage of this model is its simplicity.

### 4.4.2 Specification of the fundamental memories to be stored

The patterns that will be stored in synapses of the network (in the matrix $\boldsymbol{W}$ ) are compositions by Tex (Swedish painter). Each composition consists of three equilateral colored triangles. The length of edges is constant and carefully measured to retain details of the painting.

Location of each triangle is specified by the $(\boldsymbol{x}, \boldsymbol{y})$ coordinates of the leftmost corner and rotation angle around that corner. The $\boldsymbol{x}, \boldsymbol{y}$ resolution is 0.5 mm and each coordinate is represented by an eight bit number.

Rotation angles $\phi$ are represented by seven bit numbers $\alpha=90-\phi$. The angular resolution is approximately one degree.

Each triangle is colored red, green or blue. The intensity for each color is given by two bits. A red triangle thus has the color vector $\left[\begin{array}{lll}3 & 0 & 0\end{array}\right]=\left[\begin{array}{llllll}1 & 1 & 0 & 0 & 0 & 0\end{array}\right]$, a green triangle has the color vector $\left[\begin{array}{lll}0 & 3 & 0\end{array}\right]=\left[\begin{array}{llllll}0 & 0 & 1 & 1 & 0 & 0\end{array}\right]$, ablue triangle has the color vector $\left[\begin{array}{lll}0 & 0 & 3\end{array}\right]=\left[\begin{array}{ll}0 & 0\end{array}\right.$ 0 0 11].

Hence, each composition is coded by $m=3(8+8+7+6)=87$ bits. The network consists of $m$ neurons, each with $m$ synapses. In other words the weight matrix $W$ is $87 \times 87$.

Here and there we use a complex-number notation. A complex number is a two-dimensional vector written in the form $\boldsymbol{x}+\boldsymbol{j} \boldsymbol{y}$, where $j=\sqrt{-1}$. Do not be stunned by that: it is just a convenient way of writing a pair of coordinates $\boldsymbol{x}, \boldsymbol{y}$ as a "single" number.

You can download the following script from
http://www.csse.monash.edu.au/courseware/cse2330

```
% trnglDefB.m
```

clear

```
edg = [12 25 41]; % lengths of triangle sides in mm
vv = [0; 1; exp(j*pi/3)] ; % vertices of a unity equilateral triangle
    % written as complex numbers
% vertices of all three triangles before rotation and translation:
vrt = vv*edg ;
m = 87 ; % number of bits in triangle specification (number of neurons)
% TB set contains a variety of red, green and blue triangles
% x y deg r g b
TBd = [\begin{array}{llllllll}{14}&{57.5}&{37}&{3}&{0}&{0}\end{array}]
            36 55 101 0
            17.5 16.5 43 0 3 0
%
            51 32 45 0
            25.5 14 101 0 3 0
            21 19 38 3 0
%
            53 44.5 106 0
```



```
            6 37.5 67 0
%
    8.5 14.5 70 3 0 0
    45 46.5 80 0 3 0
    29.5 28 19 0 0 3
%
\begin{tabular}{lllllllllll}
16 & 84 & 0 & 3 & 0 & 0 & & & & \\
21 & 22 & 60 & 0 & 0 & 3 & & & & & \\
21 & 59.5 & 58 & 0 & 3 & 0 & \(1 ;\) & \(\%\) & 15 & x & 6
\end{tabular}
% scans TBd row-wise and converts each number to an 8-bit string
% check first the following sequence of functions to see the structure
% Bt = dec2bin(TBd',8) , bin2dec(Bt)
Bt = (dec2bin(2*TBd', 8) - '0')'; % is is 8 x 5* 3*6
B = reshape(Bt, }8*3*6, 5) ; % is 8* 3*6 x 5
% We need only the following bits (rows of B)
k = [1:8 (1:8)+8 (1:7)+2*8 (6:7) +3*8 (6:7)+4*8 (6:7)+5*8] ;
k = [k k+48 k+2*48] ;
TB = B(k,:) ; % is 87 x 5
% recoding each set from {0, 1} to {-1, +1}
TB = 2*TB - 1;
```


## Exercise 4. 3

Explain details of the script trnglDefB.m. Comment on the structure and the role of each variable

To see the above compositions (four first) execute the script pltSetB.m that you can also download from the web. To execute the script you have to type in:

```
pltSetB(vrt, TB, 'can you see')
% pltSetB.m
function pltSetB(vrt, TB, ttl)
for k = 1:4
    Trb = TB(:,k) ;
    Tr = reshape((Trb+1)/2, 29, 3) ;
    xx = pow2(6:-1:-1)*Tr(1:8,:) ;
    yy = pow2(6:-1:-1) *Tr((1:8)+8,:) ;
    zt = xx+j*yy ;
    fi = pow2(6:-1:0)*Tr((1:7)+2*8,:) ;
    rot = exp(j*pi*(90-fi)/180) ;
    r = [2 1]*Tr((1:2)+23,:) ;
    g = [2 1]*Tr((1:2)+25,:) ;
    b = [2 1]*Tr((1:2)+27,:) ;
    cc = [r ; g ; b]/3 ;
    % rotation and translation of three tiangles vrt
    z = vrt.*rot([1 1 1],:) + zt([1 1 1],:) ;
    X = real(z) ; Y = imag(z) ;
    subplot(2,2,k)
    fill(X(:,1), Y(:,1), cc(1,:), ...
                X(:,2), Y(:,2), cc(2,:), ...
                X(:,3), Y(:,3), cc(3,:))
    axis([0 80 0 80]), axis equal, grid on
    if k ==1, title(ttl), end
end
```


## Exercise 4. 4

Modify the script trnglDefB.minto trnglDef6.m by adding your own composition (three triangles).
Modify the script pltSetB.m into pltSet $6 . m$ so that you could see all six compositions.
8 marks

### 4.4.3 First implementation of the model

This is a demo of a simple model of Alzheimer's disease based on the recurrent Hopfield neural network with autoassociative memory, represented by the weight matrix. The Alzheimer is modelled by "killing synapses", that is, by setting individual synaptic weights to zero.

```
% Alzh.m
clear
trnglDefB % definition of triangles
    % test plot of the triangle set
```

```
figure(1), clf
ttl = 'Initial set' ;
pltSetB(vrt, TB, ttl)
% matrix TB (87 x 5) contains specification of
% Ncomp = 5 triangle compositions, i.e,
% five fundamental memories, each described by 87 bits
m = 87 ;
Ncomp = 5 ;
SurvF = 0.9 ; % survival factor determines the number of killed synapses
% encoding fundamental memories
WW = (TB*TB')/Ncomp ; % outer products of fundamental memories
NKilledSynapses = zeros(1, Ncomp); % numbers of killed synapses
relKill = zeros(1, Ncomp) ; % ratio of killed synapses
cntS = zeros(1, Ncomp) ; % counters will be stored here
for k = 1:Ncomp % composition loop
    % We now gradually destroy the memory matrix by killing a given
    % proportion of synapses. We multiply each weight by a randomly
    % generated 0 or 1
    rKill = round(SurvF*rand(size(WW)));
    W = rKill.*WW; % a weight matrix with "killed" synapses
    % Now we want to know how many synapses we have killed.
    % and their fraction
    NofKilledSynapses(k) = sum(sum(abs(sign(W-WW)))) ;
    relKill(k) = NofKilledSynapses(k)/(m^2) ;
    % The Hopfield relaxation loop should converge to an attractor,
    % that is, should find a pattern (triangle) stored in a memory
    % matrix starting from a given initial state.
    % Naturally we may end up in a spurious attractor.
    % The memory matrix is given as W. The initial state is given
    % by a vector TB(:,k).
    % To know how long the while loop runs we also set a counter.
    cnt = 1;
    yTr = sign(W*TB(:,k)); % calculate the output pattern
        % for the initial pattern
    dyTr = yTr; % change in the pattern
    while (max(abs(dyTr))>0) & (cnt < 250)
        yTr22 = sign(W*yTr);
        dyTr = yTr22 - yTr ;
        yTr = yTr22 ;
        cnt = cnt+1 ;
    end
```

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```
% the attractors yTr are stored in a matrix Attr
Y(:, k) = yTr ;
% the counter is also stored
cntS(k) = cnt ;
end
% Now we calculate the number of vector elements where
% the attractors and the compositions differ.
NofDiffElements = sum(abs(sign(TB(:,1:Ncomp)- Y))) ;
```

```
figure(2), clf
```

figure(2), clf
ttl = 'Final set' ;
ttl = 'Final set' ;
pltSetB(vrt, Y, ttl) % plotting the decoded set
pltSetB(vrt, Y, ttl) % plotting the decoded set
['\# Killed synapses: ', ...
['\# Killed synapses: ', ...
num2str(NofKilledSynapses), sprintf('\n'), ...
num2str(NofKilledSynapses), sprintf('\n'), ...
'Fraction of killed synapses: ', ...
'Fraction of killed synapses: ', ...
num2str(relKill, '%1.2f '), sprintf('\n'), ...
num2str(relKill, '%1.2f '), sprintf('\n'), ...
'\# Different Elements: ', ...
'\# Different Elements: ', ...
num2str(NofDiffElements), sprintf('\n'), ...
num2str(NofDiffElements), sprintf('\n'), ...
'\# Relaxation runs: ', num2str(cntS)]
'\# Relaxation runs: ', num2str(cntS)]
% print -f1 -depsc2 AlzhInit
% print -f1 -depsc2 AlzhInit
% print -f2 -depsc2 AlzhFinal

```
% print -f2 -depsc2 AlzhFinal
```

The script produces figures similar to the following:


## Exercise 4. 5

Run the above script a number of times and record in a table: SurvF, NofKilledSynapses, relKill, cntS. Comment on the obtained results.

5 marks

### 4.4.4 Second implementation of the model

Second implementation of the model follows the same principle of operation. For convenience there is a Graphical User Interface (GUI) built that makes running the script easier.
Download the following scripts: AlzhGUI.m, AlzhGUI.fig, trnglDef.m, HopfConv.m, plotSet.m

Invoke the main script AlzhGUI.m that should open new window in which you can set up parameters run a simulation.

## Exercise 4. 6

Run simulations for different triangle sets and different parameters and record the results in a table as in the previous exercise.
Comment on the obtained results.

5marks

## Written Submission

Your written submission should include results of all exercises you have performed (relevant MATLAB scripts, figures, numerical results, etc) with brief comments and explanations. It should be in a form ready for electronic submission when requested.

## References

[1] David H. Small, "Do acetylcholinesterase inhibitors boost synaptic scaling in Alzheimer's disease," TRENDS in Neuroscience, Vol. 27, No. 5, May 2004

