CSE468
Information Conflict

Lecturer: Dr Carlo Kopp, MIEEE, MAIAA, PEng

Lecture 03

Introduction to Game Theory Concepts

Reference Sources and Bibliography

- There is an abundance of websites and publications dealing with game theory.
- Examples include:
  - http://www.gametheory.net/ ***Game Theory .net***
  - http://mayet.som.yale.edu/ ***Yale School of Management***
  - http://www.courses.fas.harvard.edu/~ec1052/Lectures/ ***Harvard University Introduction to Game Theory***
  - http://stellar.mit.edu/S/course/14/fa04/14.12/materials.html ***MIT Game Theory Lectures***
  - http://faculty.haas.berkeley.edu/rjmorgan/MBA217/lecture_notes.htm ***Berkeley Game Theory Lectures***
  - http://www.pitt.edu/~jduffy/econ1200/Lectures.htm ***Game Theory Lectures***
  - http://www.agsm.edu.au/~bobm/teaching/SGTM.html ***Strategic Game Theory for Managers***
  - http://www.cse.iitd.ernet.in/~rahul/cs905/ ***Introduction to Game Theory Indian Institute of Technology Delhi***

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- Fudenberg D, Tirole J, *Game Theory*, MIT Press, Massachussetts, 1995.
- Fraser N.M, Hipel K.W., *Conflict Analysis – Models and Resolutions*, North-Holland, New York, 1984.
- J Maynard Smith, G.R. Price, The Logic of Animal Conflict, *Nature*, Vol.246, November 2, 1973.
- Grossman P.Z., The Dilemma of Prisoners, Choice During Stalin’s Great Terror, 1936-38, *Journal of Conlfict Resolution*, Vol.38, No.1, March 1994, pp 43-55.
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- Berne E, *Games People Play, The Psychology of Human Relationships*, Penguin, London, 1964.

What is Game Theory?

- Game theory is the study of games, defined as ‘conflicts involving gains and losses between two or more opponents who follow formal rules’.
- Games are often very useful models to describe behaviours observed in nature or social environments.
- Games are models and may not reflect actual observed behaviour in nature or social environments – irrational behaviours can and do arise in the physical world.
- Information plays a key role in games as it determines how players view games and make their choices.
- Game theory is thus important in studying IW since it provides robust models for representing how information is used and how its manipulation can affect outcomes.

Brief History of Game Theory

- 1713 Waldgrave; 1838 Cournot; 1871 Darwin; 1881 Edgeworth; 1913 Zermelo; 1921-27 Borel
- 1928 John Von Neumann - minimax theorem
- 1944 John von Neumann and Oskar Morgenstern - Theory of Games and Economic Behavior published.
- 1950 Dresher, Flood, Tucker, Raiffa – Prisoner's Dilemma game publications.
- 1950-53 John Nash - strategic equilibrium theory.
- 1960 Thomas Schelling - The Strategy of Conflict.
- 1972 John Maynard Smith - Evolutionarily Stable Strategy concept.
- Refer '*A Chronology of Game Theory*' @ http://www.econ.canterbury.ac.nz/personal_pages/paul_walker/gt/hist.htm

Definitions (1)

- 'A [strategic] game is defined as a conflict involving gains and losses between two or more opponents who follow formal rules' [Weisstein].
- A 'player' in a game is an entity which participates in a game.
- A 'strategy' in a game is 'a set of moves which a player plans to follow while playing a game' [Weisstein].
- A 'payoff' in a game is some utility or gain which might accrue to a player in a game.
- An 'equilibrium' in a game is a situation where no player can further improve their position by a unilateral move.
- Players are assumed to be 'rational' if their moves are directed to increasing their payoff in a game.

Definitions (2)

- A ‘cardinal game’ is a game in which a player can assess the performance of moves by comparing payoffs.
- An ‘ordinal game’ is a game is which a player can assess the performance of moves by comparing a ranking of outcomes involving other players.
- A player is said to have ‘perfect information’ if it knows at all times the state of all players in the game.
- A game is ‘cooperative’ iff players have freedom to communicate before moves to form binding agreements.
- A game is ‘non-cooperative’ iff players cannot communicate before moves to form binding agreements.
- A ‘zero sum game’ is a game in which the improvement of a player’s payoff is at the expense of other players.

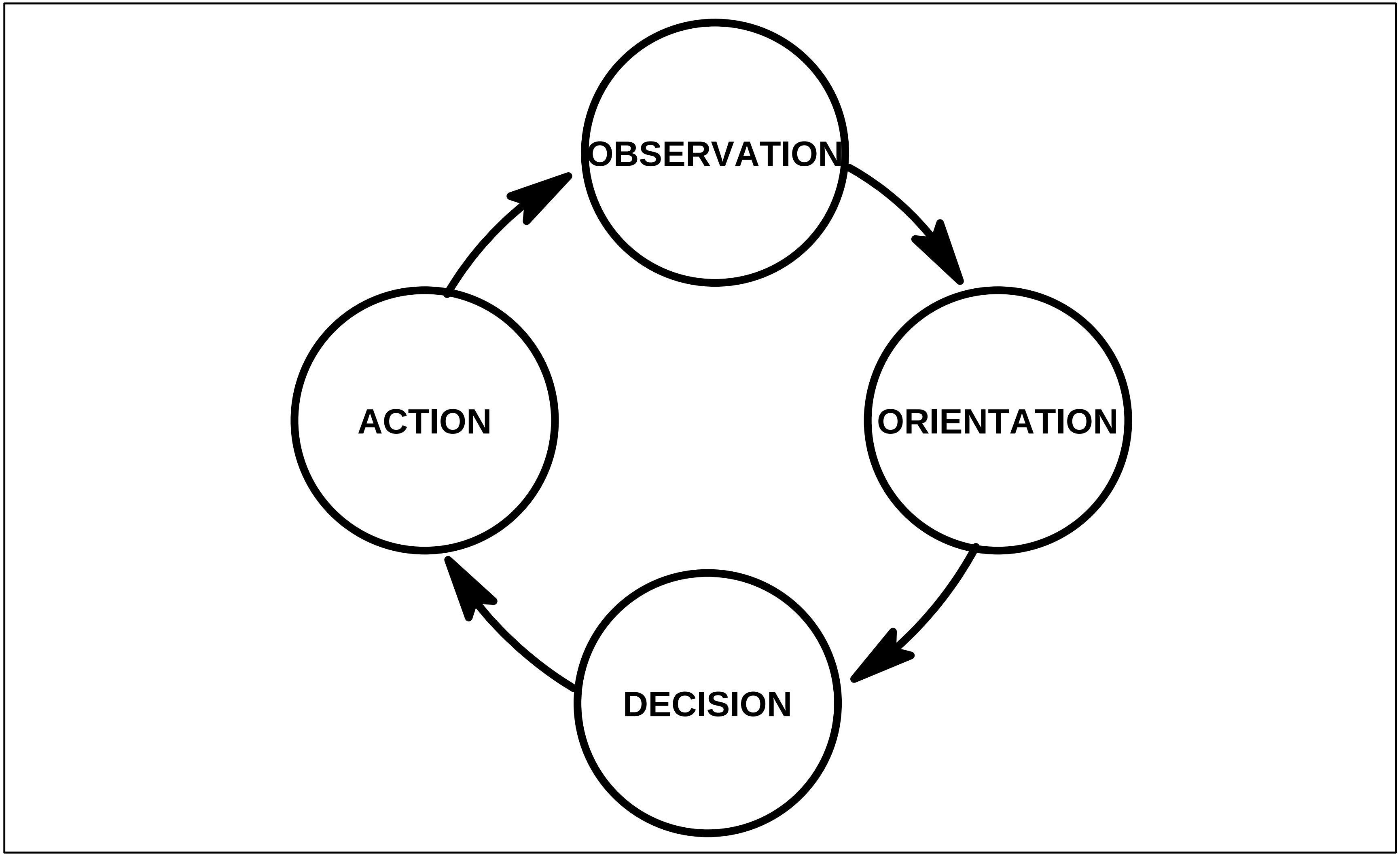
The Observation Orientation Decision Action Loop

- Boyd defined the OODA loop model during the 1970s to describe engagements in conflict, and later extended the model to more general strategy.
- The model states that players in an engagement will repeatedly cycle through four steps associated with each action (move in a game).

1. Observe the situation to gather information.
2. Orient oneself relative to the situation.
3. Make a decision.
4. Act upon that decision.

- Boyd's OODA loop is important since it models the internal functions of a player, identifies the role of information in the play, and identifies the effect of time, where it applies to a game.

OODA Loop



Defining a Game

- A cardinal strategic game with rational players has the following properties:

1. It has a finite set of players, P_i *{1, 2, 3, …N}*
2. It has some set of rules *R* constraining player moves.
3. It has some set of strategies S_i for each player P_i.
4. It has some set of outcomes *O*.
5. It has some set of payoffs $U_i(O)$ for each outcome and player.

- When the game is played, each player will execute consecutive moves in the game, defined by the rules of the game, in accordance with their respective strategies, to achieve an outcome which provides a favourable payoff.

Zero Sum Games

- Zero sum games are an important category since they frequently arise in nature and social contexts. E.g. a predator vs prey survival game – *if the predator eats the prey it does not starve, but the prey does not survive, etc.*
- A zero sum game has the property that:

$$\sum_{i=1}^{N} U_i(S) = 0$$

- The payoff gained by one player is at the expense of a reduced payoff by the other players in the game.
- Survival contests often become zero sum games.

Ordinal vs Cardinal Games

- Much of existing game theory deals with *cardinal* games, where the payoff can be readily or exactly defined for players.
- Cardinal games can present genuine difficulties in social contexts since the payoff function may be very difficult or impossible to exactly define.
- Under these circumstances *ordinal* games become much more useful, as it is possible to identify payoffs in terms of ranked player preferences in possible outcomes.
- An example is a game in which there may be N possible outcomes, where player A will prefer some ranked set of outcomes O{1,2,…N} which differs from the ranked set of outcomes preferred by player B.

Hypergames

- Hypergames are games in which the players may not be fully aware of the nature of the game they are playing, or indeed that they are actually participating in a game .
- Hypergame properties include:
  1. **Players may have false perceptions of the intent or aims of the other players.**
  2. **Players may not understand the choices available to other players.**
  3. **Players may not know who other players in the game may be.**
  4. **A player may be subject to one or more of the previous misperceptions of the game.**
- *The 'perfect information' assumption does not hold for a hypergame. Misperceptions, deceptions and surprise apply.*

The Nash Equilibrium

- A Nash Equilibrium is defined as '.. A profile of strategies such that each player's strategy is an optimal response to the other player's strategies' [Fudenberg, Tirole].
- In plain language, each choice made by a player is the best response the player can make to the anticipated play of his opponents in the game.
- If all players in the game make the same prediction of a Nash equilibrium, there is no payoff for a player choosing a different strategy.
- As a result the game should exhibit stable behaviour, when played over multiple iterations.

Pareto Optimality

- ‘The Pareto-optimal outcome is defined as such an outcome, which has the property that no other outcome of the game exists in which neither player gets a smaller payoff’ (Rapaport)
- ‘Given a set of alternative allocations and a set of individuals, a movement from one alternative allocation to another that can make at least one individual better off, without making any other individual worse off is called a *Pareto improvement* or *Pareto optimization*.’ (Wikipedia)
- Pareto optimality is widely used in game theory as well as economics modelling.

Example – Two Player Prisoner's Dilemma Game

- A crime is committed by two offenders, who are captured by the police, but no incriminating evidence is found.
- The police play the 'prisoner's dilemma' game against the offenders. Both prisoners are given the choice of incriminating the other to gain immunity from prosecution.
- If both prisoners steadfastly refuse to incriminate the other, the police have no case. If one incriminates the other, the police get one conviction. If both incriminate each other, the police get two convictions.
- The strategy of denouncing the other is termed 'defecting', the strategy of not denouncing the other is term 'colluding'.

Example – Two Player Prisoner's Dilemma Game (2)

- We can represent the respective strategies and payoffs in a table. If the prisoner is convicted his payoff is 0, if the prisoner is given immunity his payoff is 1, if the prisoners cannot be convicted the payoff is also 1 for both.
- Given these choices we would assume that the best play for the prisoners is to cooperate since both escape prosecution. However, if either player mistrusts the other, they are likely to defect to gain an advantage.
- The police may thus deceive either or both players with a claim the other has defected, to induce a defection.

Player A \ Player B	**Collude**	**Defect**
Cooperate	1,1	0,1
Defect	1,0	0,0

Example – N Player Prisoner's Dilemma Game

- The multiple player prisoner's dilemma game was devised to model situations like the Inquisition, Stalin's purges or other abuses of state power.
- In this game the prisoners are promised leniency if they denounce as many other people as possible as 'heretics', 'witches', 'counter-revolutionaries' or other conspirators against state authority.
- In the multiplayer game the matrix becomes very large due to the large number of players.
- Historical case studies suggest that this game nearly always results in overwhelming numbers of defections.
- This is because a prisoner assessing the odds of all other prisoners not defecting will conclude that this will never happen, and then opts to defect to win leniency.

Example – N Player Prisoner's Dilemma Game (2)

- Why does the prisoner reach this conclusion?
- Let us assume that there is some probability $p_i\{0,1]$ that a prisoner will not defect and that probability is close to 1.
- The condition where all prisoners opt not to defect will arise only when:

$$\prod_{i=1}^{N} p_i = 1;\ Assume\ \{p_1 = p_2 = p_3 .... = p_N\},\ then \prod_{i=1}^{N} p_i = p_i^{\ N}$$

- Even if $p_i\{0,1] \rightarrow 1$, for large N the product is <<*1*.
- Therefore a player will typically be certain that at least one of the N will defect, and therefore there is no point in not defecting to gain an advantage.

Example – The ‘Chicken’ Game

- Kahn’s ‘Chicken Game’: two hoons drive their cars in a head-on collision trajectory. The first of the two to swerve away is ‘chicken’ and considered to be a coward by the peer group. If neither swerves they collide and die.
- The game can be presented thus:

Hoon A \ Hoon B	**Do not swerve**	**Swerve**
Do not swerve	-1, -1	1,0
Swerve	0,1	0,0

- In this game, the winner is the player who delays swerving for as long as possible, incurring the risk that a collision will occur.

Example – The ‘Chicken’ Game

- The ‘chicken’ game is like prisoner’s dilemma, a game which arises frequently in the real world.
- One example is in aerial dogfighting where the fighter pilot who ‘breaks’ first ends up in a disadvantageous position and is shot down by his opponent.
- Other examples involve group behaviours where contests for leadership are played out, in which some high risk activity is pursued to demonstrate that one of N individuals in the group has the courage to become leader.
- In nature, competitive combat between males over females often amounts to a ‘chicken’ game, since wounds inflicted during the fight may result in the death of both animals contending for the female.

Example – Hawk Dove Game

- Maynard Smith defined this model to describe two animals contesting a resource such as food or territory. The payoff gained by winning this resource is *V*. If an animal is injured its payoff is reduced by amount *C*.
- When two animals meet to play they can adopt either a *Hawk* strategy or a *Dove* strategy. A *Hawk* will escalate the fight until it suffers injury or its opponent concedes. A *Dove* when faced with escalation will run for safety. The game can be modelled thus:

	Hawk	**Dove**
Hawk	$\frac{1}{2}(V-C)$	V
Dove	0	$\frac{1}{2}V$

Example – Hawk Dove Game (2)

- The game assumes repeated 'engagements' between players, therefore the payoff for two equally successful Hawks who escalate every time they meet is the average of multiple games played, and thus equal to $\frac{1}{2}(V-C)$.
- Maynard Smith modelled a large population of animals, repeatedly playing this game, with a mixed strategy where *Hawk* is played with probability p and *Dove* with probability $(1-p)$.
- He found that for a $V=2$, $C=4$ *and* $p=0.5$ the mixed strategy was an Evolutionarily Stable Strategy (ESS).
- An ESS is such a strategy, that in a population where all players play this strategy, no other strategy could outperform it and thus in an evolving population, 'outpopulate' it.

Example - Tit for Tat Game

- 'Iterated Prisoner's Dilemma' (IPD) is a game in which PD is played over multiple turns. The aim is to establish what patterns of defection or collusion yield the best long term payoff for a player.
- In a 1970s contest staged by Axelrod, Rapaport's 'Tit for tat' (TFT) strategy performed best, beating 62 other entrants.
- In an IPD game, a TFT player will reciprocate in the next move with whatever strategy his opponent played in the previous move, be it a defection or collusion.
- Axelrod describes TFT as 'strategy of cooperation based on reciprocity'. TFT players do not defect first, forgive after one move, and retaliate on each defection.

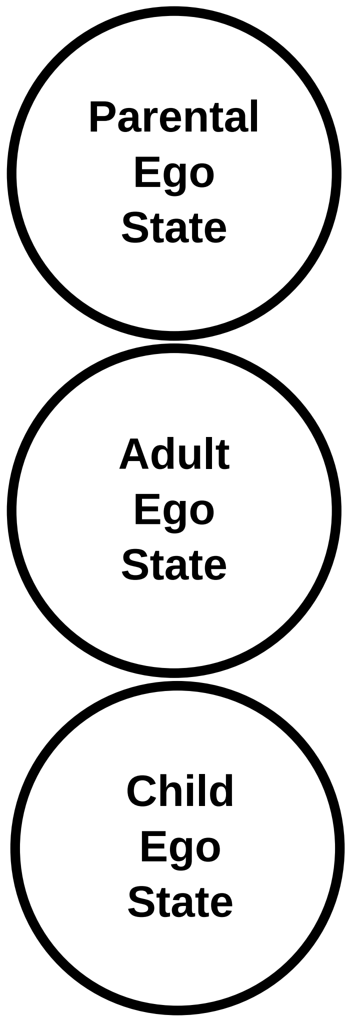
Berne’s Psychological Games

- Berne’s psychological games are part of a school of psychological research termed ‘transactional analysis’.
- Transactional analysis in not rooted in mathematical game theory, rather it is the result of empirical studies and classifications of observable human behaviour.
- The various games Berne identifies and classifies can however be modelled using mathematical game theory and are tractable should hypergame models be employed. This is as many of these games involve strategic surprise and misperception or deception.
- Berne’s work provides a good illustration of the ubiquity of games in everyday human social behaviour.

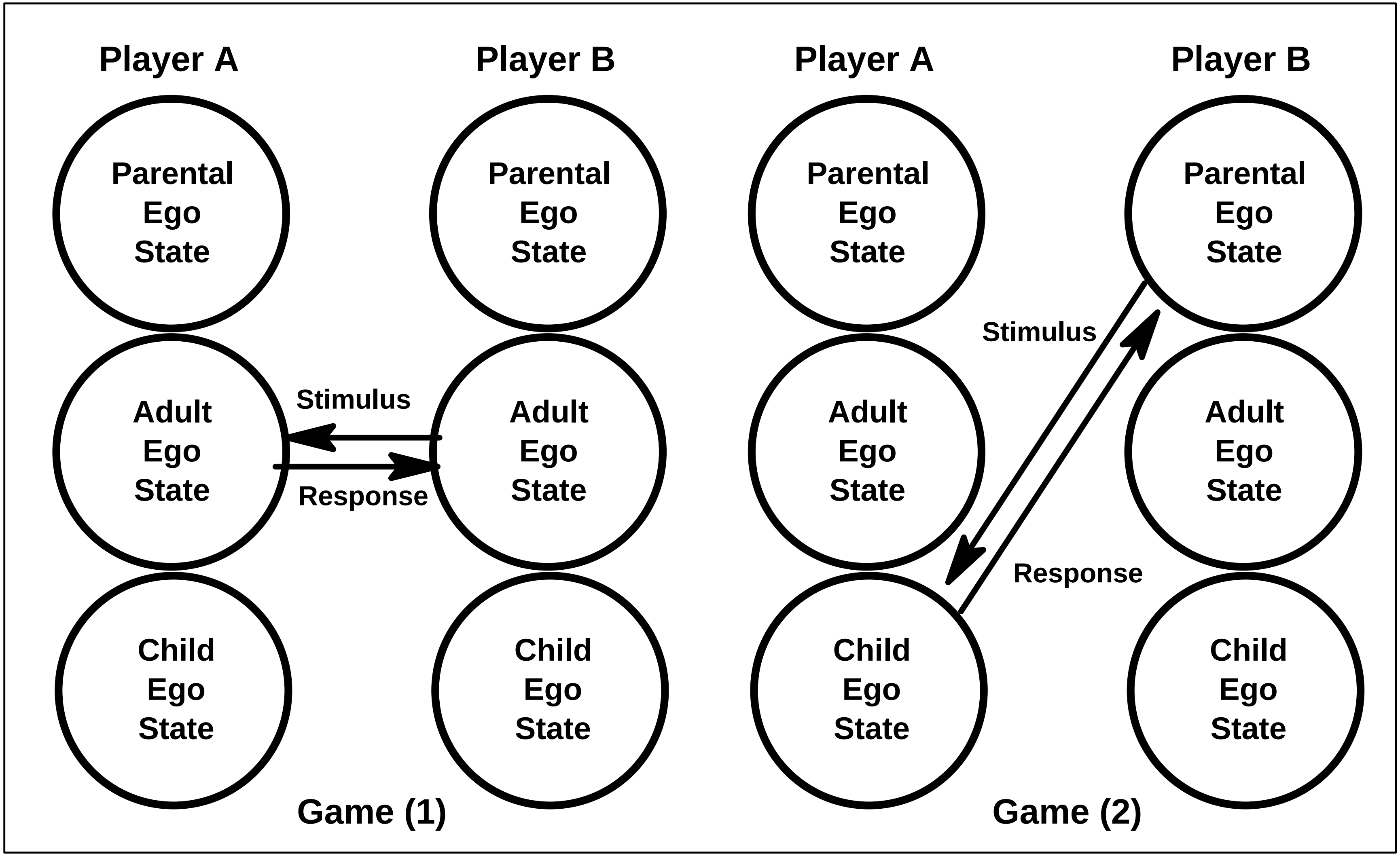
Transactional Analysis Concepts

- Berne defines the idea of an 'ego state' to determine the instantaneous manner in which an observed individual is behaving.
- This state is described as a 'coherent system of feelings' or a 'coherent behaviour pattern'.
- In Berne's model, a player can be in one of three basic 'ego states':
  1. **The 'exteropsychic' or 'Parent' state, where the player exhibits the behaviour of a parental figure.**
  2. **The 'neopsychic' or 'Adult' state, where the player exhibits rational adult behaviour.**
  3. **The 'archaeopsychic' or 'Child' state, where the player exhibits childlike behaviour.**
- During these games, players typically adopt specific ego states to derive a psychological payoff, or may 'switch' between states to derive a psychological payoff.

Berne's 'Ego States'



Structure of Berne’s Games



NIGYSOB Game (Berne)

- 'Now I've Got You, Son of a Bitch' is commonly played in social settings, Internet newsgroups and often commerce.

1. Players A and B initiate an exchange such as a commercial transaction, debate or conversation.
2. Player B looks for some transgression, implied or real, on the Part of Player A.
3. Player B explodes in rage and verbally/physically attacks Player A for his transgression.

- Psychological payoff – opportunity to exercise power over another, vent pent up anger or frustration.
- Transactions – Player B switches from Adult to Parent, Player A may switch to Child state if he cooperates.

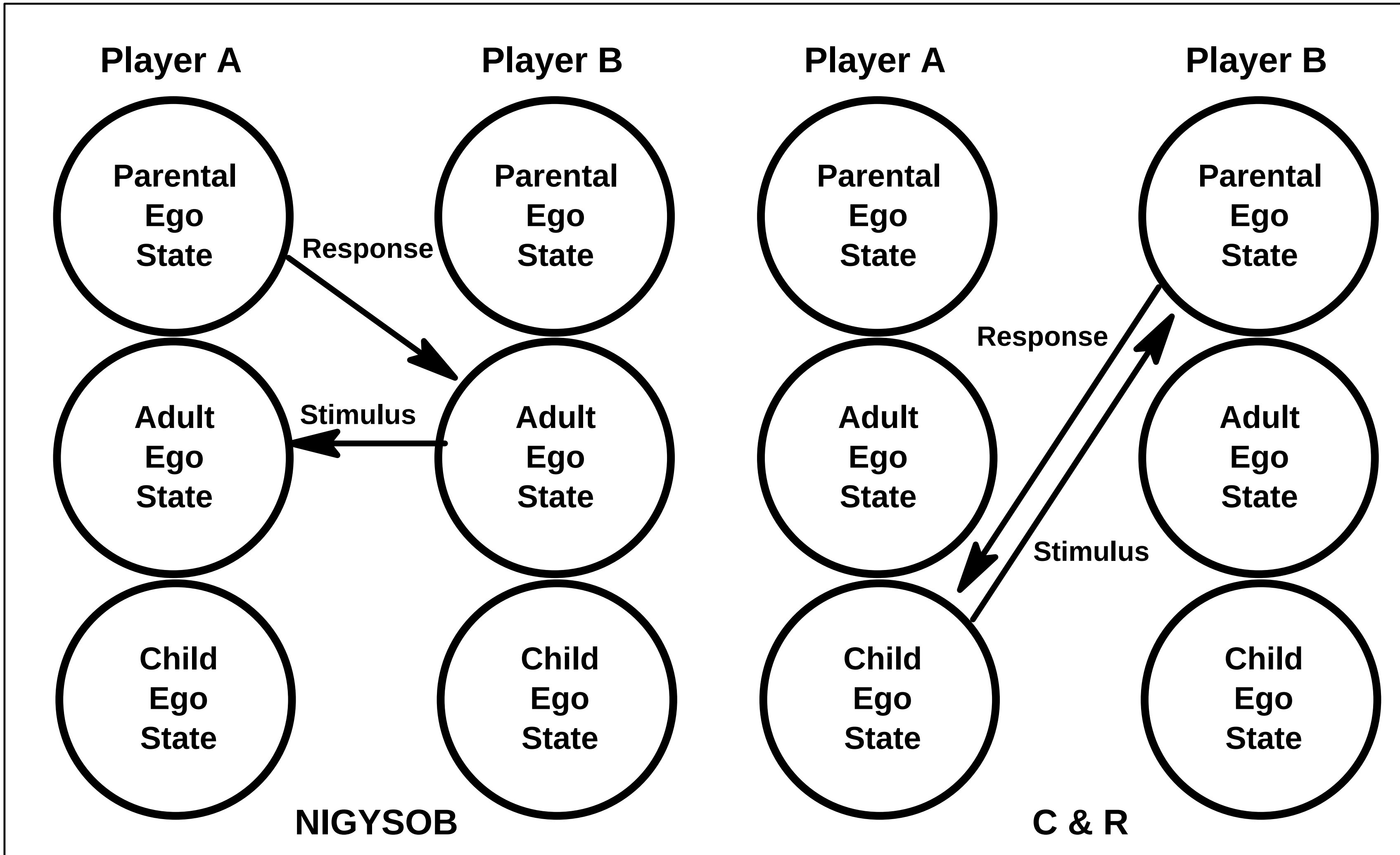
C&R Game (Berne)

- 'Cops and Robbers' game is often seen in criminal offenders, but also in individuals challenging authority.

1. Player A commits some transgression or offence, and attempts to outwit Player B.
2. Player B goes along with the game pretending to have been outwitted.
3. Player B then 'catches' Player A and 'wins' the game.

- Psychological payoffs – Player A 'see if you can catch me', Player B 'gotcha!'
- Transactions – Player A is in a Child state, Player B in a Parent state.
- Berne observes that criminals may or may not be C&R players, as many professional criminals are not interested in the excitement derived from playing C&R.

NIGYSOB Game vs C&R Game State Diagram



Tutorial

- Q&A on conceptual issues.
- PD and ‘The Doctor from Kharkov’ scenario (Grossman).
- Chicken vs the ‘absent steering wheel’.
- Berne’s Games