Received: 24 January 2017 | Revised: 2 October 2018 | Accepted: 29 October 2018

DOI: 10.1002/jgt.22430

ARTICLE

WILEY

Nonrepetitive colorings of graphs excluding a fixed immersion or topological minor

Paul Wollan[1] | **David R. Wood**[2]

[1]Department of Computer Science, University of Rome "La Sapienza," Rome, Italy

[2]School of Mathematical Sciences, Monash University, Melbourne, Victoria, Australia

Correspondence
David R. Wood, School of Mathematical Sciences, Monash University, Melbourne, Victoria 3800, Australia.
Email: david.wood@monash.edu

Funding information
Australian Research Council; European Research Council, Grant/Award Number: 279558

Abstract

We prove that graphs excluding a fixed immersion have bounded nonrepetitive chromatic number. More generally, we prove that if H is a fixed planar graph that has a planar embedding with all the vertices with degree at least 4 on a single face, then graphs excluding H as a topological minor have bounded nonrepetitive chromatic number. This is the largest class of graphs known to have bounded nonrepetitive chromatic number.

KEYWORDS

graph coloring, immersion, nonrepetitive coloring, topological minor

1 | INTRODUCTION

A vertex coloring of a graph is *nonrepetitive* if there is no path for which the first half of the path is assigned the same sequence of colors as the second half. More precisely, a *k-coloring* of a graph G is a function ψ that assigns one of k colors to each vertex of G. A path $(v_1, v_2, \ldots, v_{2t})$ of even order in G is *repetitively* colored by ψ if $\psi(v_i) = \psi(v_{t+i})$ for $i \in \{1, \ldots, t\}$. A coloring ψ of G is *nonrepetitive* if no path of G of even order is repetitively colored by ψ. Observe that a nonrepetitive coloring is *proper*, in the sense that adjacent vertices are colored differently. The *nonrepetitive chromatic number* $\pi(G)$ is the minimum integer k such that G admits a nonrepetitive k-coloring. We only consider simple graphs with no loops or parallel edges.

The seminal result in this area is by Thue [41], who in 1906 proved that every path is nonrepetitively 3-colorable. Thue expressed his result in terms of strings over an alphabet of three characters—Alon et al [3] introduced the generalization to graphs in 2002. Nonrepetitive graph colorings have since been widely studied [2–12,21,25–33,35,37–39]. The principle result of Alon et al [3] was that graphs with maximum degree Δ are nonrepetitively $\mathcal{O}(\Delta^2)$-colorable. Several subsequent papers improved the constant [16,26,30]. The best-known bound is due to Dujmović et al [16].

Theorem 1 (Dujmović et al [16]). *Every graph with maximum degree* Δ *is nonrepetitively* $(1 + o(1))\Delta^2$*-colorable.*

A number of other graph classes are known to have bounded nonrepetitive chromatic number. In particular, trees are nonrepetitively 4-colorable [8,33], outerplanar graphs are nonrepetitively 12-colorable [5,33], and graphs with bounded treewidth have bounded nonrepetitive chromatic number [5,33]. (See Section 2 for the definition of treewidth.) The best-known bound is due to Kündgen and Pelsmajer [33].

Theorem 2 (Kündgen and Pelsmajer [33]). *Every graph with treewidth* k *is nonrepetitively* 4^k*-colorable.*

The primary contribution of this paper is to provide a qualitative generalizations of Theorems 1 and 2 via the notion of graph immersions and excluded topological minors.

A graph G contains a graph H as an *immersion* if the vertices of H can be mapped to distinct vertices of G, and the edges of H can be mapped to pairwise edge-disjoint paths in G, such that each edge vw of H is mapped to a path in G whose endpoints are the images of v and w. The image in G of each vertex in H is called a *branch vertex*. Structural and coloring properties of graphs excluding a fixed immersion have been widely studied [1,13,14,18–20,22–24,34,36,40,42]. We prove that graphs excluding a fixed immersion have bounded nonrepetitive chromatic number.

Theorem 3. *For every graph* H *with* t *vertices, every graph that does not contain* H *as an immersion is nonrepetitively* $4^{t^4+O(t^2)}$*-colorable.*

Since a graph with maximum degree Δ contains no star with $\Delta + 1$ leaves as an immersion, Theorem 3 implies that graphs with bounded degree have bounded nonrepetitive chromatic number (as in Theorem 1).

We strengthen Theorem 3 as follows (although without explicit bounds). A graph G contains a graph H as a *strong immersion* if G contains H as an immersion, such that for each edge vw of H, no internal vertex of the path in G corresponding to vw is a branch vertex.

Theorem 4. *For every fixed graph* H*, there exists a constant* k*, such that every graph* G *that does not contain* H *as a strong immersion is nonrepetitively* k*-colorable.*

Note that planar graphs with n vertices are nonrepetitively $\mathcal{O}(\log n)$-colorable [15], and the same is true for graphs excluding a fixed graph as a minor or topological minor [17]. It is unknown whether any of these classes have bounded nonrepetitive chromatic number. Our final result shows that excluding a special type of topological minor gives bounded nonrepetitive chromatic number. A *subdivision* of a graph H is a graph obtained from H by replacing each edge vw of H by a path with endpoints v and w. A graph G contains H as a *topological minor* if a subdivision of H is a subgraph of G. Vertices with degree at least 4 are important for topological minors since it is easily seen and well known that for a graph H with maximum degree 3, a graph G contains H as a topological minor if and only if G contains H as a minor.

Theorem 5. *Let* H *be a fixed planar graph that has a planar embedding with all the vertices of* H *with degree at least* 4 *on a single face. Then there exists a constant* k*, such that every graph* G *that does not contain* H *as a topological minor is nonrepetitively* k*-colorable.*

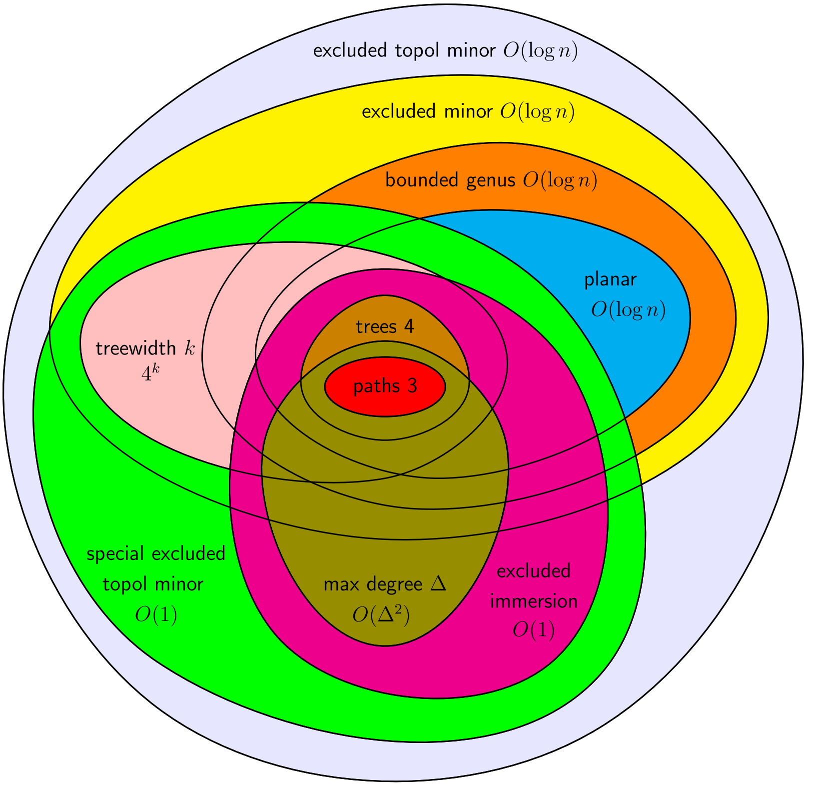

FIGURE 1 Upper bounds on the nonrepetitive chromatic number of various graph classes. "Special" refers to the condition in Theorem 5 [Color figure can be viewed at wileyonlinelibrary.com]

Graphs with bounded treewidth exclude fixed walls as topological minors. Since walls are planar graphs with maximum degree 3, Theorem 5 implies that the graphs of bounded treewidth have bounded nonrepetitive chromatic number (as in Theorem 2). Similarly, for every graph H with t vertices, the "fat star" graph (which is the 1-subdivision of the t-leaf star with edge multiplicity t) contains H as a strong immersion. Since fat stars embed in the plane with all vertices of degree at least 4 on a single face, Theorem 5 implies that graphs excluding a fixed graph as a strong immersion have bounded nonrepetitive chromatic number (as in Theorem 4). In this sense, Theorem 5 generalizes all of Theorems 1 to 4.

The results of this paper, in relation to the best-known bounds on the nonrepetitive chromatic number, are summarized in Figure 1.

Finally, note that several papers study nonrepetitive choosability. In particular, all of the $O(\Delta^2)$ upper bounds mentioned above hold for nonrepetitive choosability. Contrarily, Fiorenzi et al [21] showed that trees have unbounded nonrepetitive choosability. It follows that for all of the above graph classes with unbounded degree, the nonrepetitive choosability is unbounded.

2 | TREE DECOMPOSITIONS

For a graph G and tree T, a *tree decomposition* or T*-decomposition* of G consists of a collection $(T_x \subseteq V(G) : x \in V(T))$ of sets of vertices of G, called *bags*, indexed by the nodes of T, such that for each vertex $v \in V(G)$ the set $\{x \in V(T) : v \in T_x\}$ induces a connected subtree of T, and for each edge vw of G there is a node $x \in V(T)$ such that $v, w \in T_x$. The *width* of a T-decomposition is the maximum, taken over the nodes $x \in V(T)$, of $|T_x| - 1$. The *treewidth* of a graph G is the minimum width of a tree decomposition of G. The *a*dhesion of a tree decomposition $(T_x : x \in V(T))$ is $\max\{|T_x \cap T_y| : xy \in E(T)\}$. The *torso* of each node $x \in V(T)$ is the graph obtained from the induced subgraph $G[T_x]$ by adding a clique on $T_x \cap T_y$ for each edge $xy \in E(T)$ incident to x. Dujmovic et al [17] generalized Theorem 2 as follows.

Lemma 6 (Dujmovic et al [17]). *If a graph G has a tree decomposition with adhesion k such that every torso is nonrepetitively c-colorable, then G is nonrepetitively $c\ 4^k$-colorable.*

For integers $c, d \geq 0$ a graph G has (c, d)*-bounded degree* if G contains at most c vertices with degree greater than d.

Lemma 7. *Every graph with (c, d)-bounded degree is nonrepetitively $c + (1 + o(1))d^2$-colorable.*

Proof. Assign a distinct color to each vertex of degree at least d, and color the remaining graph by Theorem 1. For each vertex v of degree at least d, no other vertex is assigned the same color as v. Thus v is in no repetitively colored path. The result then follows from Theorem 1. □

Dvořák [18] proved the following structure theorem for graphs excluding a strong immersion.

Theorem 8 (Dvořák [18]). *For every fixed graph H, there exists a constant k, such that every graph G that does not contain H as a strong immersion has a tree decomposition such that each torso is (k, k)-bounded degree.*

Lemmas 7 and 6 and Theorem 8 imply Theorem 4.

3 | WEAK IMMERSIONS

The proof of Theorem 4 gives no explicit bound on the constant k. In this section, we prove an explicit bound on the nonrepetitive chromatic number of graphs excluding a weak immersion. Theorem 3 follows from Lemma 6 and the following structure theorem of independent interest.

Theorem 9. *For every graph H with t vertices, every graph that does not contain H as a weak immersion has a tree decomposition with adhesion at most t^2 such that every torso has $(t, t^4 + 2t^2)$-bounded degree.*

The starting point for the proof of Theorem 9 is the following structure theorem of Wollan [42]. For a tree T and graph G, a T*-partition* of G is a partition $(T_x \subseteq V(G) : x \in V(T))$ of $V(G)$ indexed by the nodes of T. Each set T_x is called a *bag*. Note that a bag may be empty. For each edge xy of a tree T, let $T(xy)$ and $T(yx)$ be the components of $T - xy$ where x is in $T(xy)$ and y is in $T(yx)$. For each edge $xy \in E(T)$, let $G(T, xy) := \bigcup\{T_z : z \in V(T(xy))\}$ and

$G(T, yx) := \bigcup\{T_z : z \in V(T(yx))\}$. Let $E(T, xy)$ be the set of edges in G between $G(T, xy)$ and $G(T, yx)$. The *adhesion* of a T-partition $(T_x : x \in V(T))$ is the maximum, taken over all edges xy of T, of $|E(T, xy)|$. For each node x of T, the *torso* of x (with respect to a T-partition) is the graph obtained from G by identifying $G(T, yx)$ into a single vertex for each edge xy incident to x (deleting resulting parallel edges and loops).

Theorem 10 (Wollan [42]). *For every graph H with t vertices, for every graph G that does not contain H as a weak immersion, there is a T-partition of G with adhesion at most t^2 such that each torso has (t, t^2)-bounded degree.*

Proof of Theorem 9. Let G be a graph that does not contain H as a weak immersion. Consider the T-partition $(T_x : x \in V(T))$ of G from Theorem 10.

Let T' be obtained from T by orienting each edge towards some root vertex. We now define a tree decomposition $(T_x^* : x \in V(T))$ of G. Initialize $T_x^* := T_x$ for each node $x \in V(T)$. For each edge vw of G, if $v \in T_x$ and $w \in T_y$ and z is the least common ancestor of x and y in T', then add v to T_α^* for each node α on the $\overrightarrow{xz}$ path in T', and add w to T_α^* for each node α on the $\overrightarrow{yz}$ path in T'. Thus each vertex $v \in T_x$ is in a sequence of bags that correspond to a directed path from x to some ancestor of x in T'. By construction, the endpoints of each edge are in a common bag. Thus $(T_x^* : x \in V(T))$ is a tree decomposition of G.

Consider a vertex $v \in T_x^* \cap T_y^*$ for some edge $\vec{xy}$ of T'. Then v is in $G(T, xy)$ and v has a neighbor w in $G(T, yx)$, implying $vw \in E(T, xy)$. Thus $|T_x^* \cap T_y^*| \leq |E(T, xy)| \leq t^2$. That is, the tree decomposition $(T_x^* : x \in V(T))$ has adhesion at most t^2.

Let G_x^+ be the torso of each node $x \in V(T)$ with respect to the tree decomposition $(T_x^* : x \in V(T))$. That is, G^+ is obtained from $G[T_x^*]$ by adding a clique on $T_x^* \cap T_y^*$ for each edge xy of T. Our goal is to prove that G_x^+ has $(t, t^4 + 2t^2)$-bounded degree.

Consider a vertex v of G_x^+. Then v is in the bag corresponding to at most one child node z of x, as otherwise v would belong to a set of bags that do not correspond to a directed path in T'. Since $(T_x^*: x \in V(T))$ has adhesion at most t^2, v has at most t^2 neighbors in $T_x^* \cap T_z^*$. For the same reason, if p is the parent of x, then v has at most t^2 neighbors in $T_x^* \cap T_p^*$. Thus the degree of v in G_x^+ is at most the degree of v in $G[T_x^*]$ plus $2t^2$. Call this property ($\star$).

First consider the case that $v \notin T_x$. Let z be the node of T for which $v \in T_z$. Since $v \in T_x^*$, by construction, x is an ancestor of z. Let y be the node immediately before x on the $\overrightarrow{zx}$ path in T'. We now bound the number of neighbors of v in T_x^*. Say $w \in N_G(v) \cap T_x^*$. If w is in $G(T, xy)$ then let e_w be the edge vw. Otherwise, w is in $G(T, yx)$ and thus w has a neighbor u in $G(T, xy)$ since $w \in T_x^*$; let e_w be the edge wu. Observe that $\{e_w : w \in N_G(v) \cap T_x^*\} \subseteq E(T, xy)$, and thus $|\{e_w : w \in N_G(v) \cap T_x^*\}| \leq t^2$. Since $e_u \neq e_w$ for distinct $u, w \in N_G(v) \cap T_x^*$, we have $|N_G(v) \cap T_x^*| \leq t^2$. By ($\star$), the degree of v in G_x^+ is at most $3t^2$.

Now consider the case that $v \in T_x$. Suppose further that v is not one of the at most t vertices of degree greater than t^2 in the torso Q of x with respect to the given T-partition. Suppose that in Q, v has d_1 neighbors in T_x and d_2 neighbors not in T_x (the identified vertices). So $d_1 + d_2 \leq t^2$. Consider a neighbor w of v in $G[T_x^*]$ with $w \notin T_x$. Then $w \in G(T, yx)$ for some child y of x. For at most d_2 children y of x, there is a neighbor of v in $G(T, yx)$. Furthermore, for each child y of x, v has at most t^2 neighbors in $G(T, yx)$ since the T-partition has adhesion at most t^2. Thus v has degree at most $d_1 + d_2 t^2 \leq t^4$ in $G[T_x^*]$. By ($\star$), v has degree at most $2t^2 + t^4$ in G_x^+.

Since $3t^2 \leq t^4 + 2t^2$, the torso G_x^+ has $(t, t^4 + 2t^2)$-bounded degree. □

4 | EXCLUDING A TOPOLOGICAL MINOR

Theorem 5 is an immediate corollary of Lemma 6 and the following structure theorem of Dvořák [18] that extends Theorem 8.

Theorem 11 (Dvořák [18]). *Let H be a fixed planar graph that has a planar embedding with all the vertices of H with degree at least 4 on a single face. Then there exists a constant k, such that every graph G that does not contain H as a topological minor has a tree decomposition such that each torso has (k, k)-bounded degree.*

While Theorem 11 is not explicitly stated in Dvořák [18], we now explain that it is in fact a special case of Theorem 3 in Dvořák [18]. This result provides a structural description of graphs excluding a given topological minor in terms of the following definition. For a graph H and surface Σ, let $\mathrm{mf}(H, \Sigma)$ be the minimum, over all possible embeddings of H in Σ, of the minimum number of faces such that every vertex of degree at least 4 is incident with one of these faces. By assumption, for our graph H and for every surface Σ, we have $\mathrm{mf}(H, \Sigma) = 1$. In this case, Theorem 3 of Dvořák [18] says that for some integer $k = k(H)$, every graph G that does not contain H as a topological minor is a clique sum of (k, k)-bounded degree graphs. It immediately follows that G has the desired tree decomposition. See Corollary 1.4 in Liu and Thomas [34] for a closely related structure theorem.

The following natural open problem arises from This study: Do graphs excluding a fixed planar graph as a topological minor have bounded nonrepetitive chromatic number? And what is the structure of such graphs?

ACKNOWLEDGMENTS

This study was initiated at the *Workshop on New Trends in Graph Coloring* held at the Banff International Research Station in October 2016. Authors thank the organizers and thank Chun-Hung Liu and Zdeněk Dvořák for stimulating conversations. Supported by the European Research Council under the European Union's Seventh Framework Program (FP7/2007-2013)/ERC grant agreement no. 279558. Research supported by the Australian Research Council.

ORCID

David R. Wood http://orcid.org/0000-0001-8866-3041

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How to cite this article: Wollan P, Wood DR. Nonrepetitive colorings of graphs excluding a fixed immersion or topological minor. *J Graph Theory*. 2019;91:259-266. https://doi.org/10.1002/jgt.22430