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TEARING UP THE DISC: HOW BLACK HOLES ACCRETE

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ABSTRACT

We show that in realistic cases of accretion in active galactic nuclei or stellar–mass X–ray binaries, the Lense–Thirring effect breaks the central regions of tilted accretion discs around spinning black holes into a set of distinct planes with only tenuous flows connecting them. If the original misalignment of the outer disc to the spin axis of the hole is $45^\circ \lesssim \theta \lesssim 135^\circ$, as in $\sim 70\%$ of randomly oriented accretion events, the continued precession of these discs sets up partially counter–rotating gas flows. This drives rapid infall as angular momentum is cancelled and gas attempts to circularize at smaller radii. Disc breaking close to the black hole leads to direct dynamical accretion, while breaking further out can drive gas down to scales where it can accrete rapidly. For smaller tilt angles breaking can still occur, and may lead to other observable phenomena such as QPOs. For such effects not to appear, the black hole spin must in practice be negligibly small, or be almost precisely aligned with the disc. Qualitatively similar results hold for any accretion disc subject to a forced differential precession, such as an external disc around a misaligned black hole binary.

Subject headings: accretion, accretion disks — black hole physics — hydrodynamics — galaxies: active — stars: neutron

1. INTRODUCTION

Accretion discs are common in astrophysics on all scales from protostars to AGN (see e.g. Pringle 1981; Frank et al. 2002). Many treatments assume that the disc is aligned with the symmetry axis of the central object, although there is often no *a priori* reason for this. The first widely–studied case relaxing this restriction was the evolution of tilted discs around spinning black holes. Until recently the standard picture of tilted disc evolution was that, in the regime where viscosity acts diffusively (technically, $\alpha > H/R$), the inner disc would align or counteralign rapidly with the hole's spin, with a smooth warp to the still misaligned outer parts. This is often called the Bardeen–Petterson effect (Bardeen & Petterson 1975, but note their equations do not conserve angular momentum; see Papaloizou & Pringle 1983; Pringle 1992; Ogilvie 1999 for detailed discussions of the correct equations; and King et al. 2005 for the possibility of counteralignment).

In a recent paper, Nixon & King (2012) showed that this evolution can be very different for large inclinations of the disc and spin, and/or low values of the dimensionless viscosity coefficient α (Shakura & Sunyaev 1973). Enforcing the connections imposed by conservation laws between the various components ('radial', and 'vertical') of viscosity (Ogilvie 1999), Nixon & King (2012) showed that the viscous torques in the disc may be unable to communicate the Lense–Thirring precession efficiently enough to produce a smooth warp. Instead one expects a sharp break in the disc plane between the aligned inner parts and the misaligned outer parts, connected only by tenuous rings of gas with inclinations changing rapidly across the break. We shall see from our Eqn. 7 below, that even if the viscosity coefficients remained constant with warp amplitude, the disc would still break for realistic parameters. Lodato & Price (2010) show that an assumed break of this type remains stable in 3D simulations of such discs. Nixon et al. (2012) show that inclined, partially counter–rotating gas orbits within an accretion disc lead to cancellation of angular momentum and thus subsequent accretion. The mass flow rate through the disc can, for a time, be increased by large factors up to $\sim 10^4$ times that of the corresponding disc with zero inclination.

These results suggest that sufficiently inclined discs might break, and that if the precession rate of the inner and outer disc differs enough, discs rotating in opposed senses might interact and produce dynamical mass infall. We consider these questions in this Letter. We ask
1) for realistic parameters, can the Lense–Thirring torques exceed the local viscous torques in the disc?
2) If they can, can we arrange the broken disc such that disc orbits counter–rotate?

2. WHERE DOES THE DISC BREAK?

Here we estimate analytically the radius at which the disc is likely to break. This will give us an idea of the parameters which lead to breaking, and whether it is a common event. To break the disc, the torque resulting from the Lense–Thirring effect must overcome the local viscous torques, or equivalently, the orbits in the disc must precess faster than the viscosity can communicate the precession. To illustrate this point, let us imagine the two extremes. If the viscosity is dominant, the precession is communicated throughout the disc instantaneously, and the whole disc precesses rigidly. At the other extreme where viscosity is negligible, orbits at different radii precess at different rates and the disc must break into many distinct rings. The nonlinear connection between effective viscosity coefficients, enforced by conservation laws (Ogilvie 1999), tell us that once a disc

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starts to break in this way, the viscosity evolves so as to reinforce the tendency to break (Nixon & King 2012).

To calculate the breaking radius for a given viscosity we can assume that the disc has no initial warp. Then we can consider the usual viscous torque, and to a good approximation neglect the more complicated physics of a warped disc. The azimuthal viscous force per unit area in the disc is proportional to the rate of shear $R\mathrm{d}\Omega/\mathrm{d}R$ (where R is the radial coordinate and Ω is the disc angular velocity) and so can be written as

$$f_\nu = \mu R \frac{\mathrm{d}\Omega}{\mathrm{d}R}. \quad (1)$$

where μ is the dynamical viscosity. The area of an interface in the disc is $2\pi RH$, where H is the disc vertical thickness. So the viscous force acting in the azimuthal direction is given by

$$F_\nu = 2\pi RH\mu R \frac{\mathrm{d}\Omega}{\mathrm{d}R} = 2\pi R\nu\Sigma R \frac{\mathrm{d}\Omega}{\mathrm{d}R}, \quad (2)$$

where we have substituted the dynamic viscosity for the kinematic viscosity ($\mu = \rho\nu$), and used $\Sigma = \rho H$. The magnitude of the viscous torque G_ν is given by

$$G_\nu = |\boldsymbol{R} \times \boldsymbol{F}_\nu| = 2\pi R\nu\Sigma R^2 \left(-\Omega'\right), \quad (3)$$

where the prime denotes the radial derivative. This is the usual viscous torque at the interface of two annuli in an accretion disc (see Lynden-Bell & Pringle 1974; Frank et al. 2002). For near–Keplerian rotation ($\Omega^2 \approx GM/R^3$) it becomes

$$G_\nu = 3\pi\nu\Sigma \left(GMR\right)^{1/2}. \quad (4)$$

The Lense–Thirring precession induces a torque with magnitude

$$G_{\rm LT} = 2\pi RH \left|\boldsymbol{\Omega}_{\rm p} \times \boldsymbol{L}\right| = 2\pi RH\Omega_{\rm p}\Sigma R^2 \Omega \left|\sin\theta\right|, \quad (5)$$

where the Lense–Thirring frequency $\boldsymbol{\Omega}_{\mathbf{p}} = 2G\boldsymbol{J}_{\rm h}/c^2R^3$, the disc angular momentum density $|\boldsymbol{L}| = \Sigma R^2\Omega$ and θ is the angle between the angular momentum of the black hole and the disc. We assume the disc breaks when the Lense–Thirring torque tears gas off the disc faster than viscosity can make it spiral inwards, which requires

$$G_{\rm LT} \gtrsim G_\nu. \quad (6)$$

Making the standard assumption of a thin Keplerian α–disc (Shakura & Sunyaev 1973) and using $J_{\rm h} = aGM^2/c$, the radius at which we expect the disc to break is given by

$$R_{\rm break} \lesssim \left(\frac{4}{3}\left|\sin\theta\right| \frac{a}{\alpha}\frac{R}{H}\right)^{2/3} R_{\rm g}, \quad (7)$$

where $R_g = GM/c^2$ is the gravitational radius of the black hole.

Equation (7) looks plausible; it makes sense that at large viscosity, low spin or small inclination angles the disc cannot be broken, i.e. $R_{\rm break} < R_{\rm g}$. Conversely, for low viscosity, high spin and/or large inclinations we expect the disc to break into distinct rings at some radius $R_{\rm break} > R_g$.

The typical radius at which the disc breaks is given by

$$R_{\rm break} \lesssim 350 R_{\rm g} \left|\sin\theta\right|^{2/3} \left(\frac{a}{0.5}\right)^{2/3} \left(\frac{\alpha}{0.1}\right)^{-2/3} \left(\frac{H/R}{10^{-3}}\right)^{-2/3} \quad (8)$$

where we have parameterized using quantities typical for AGN discs. This radius falls within typical discs, suggesting that inclined discs near spinning black holes are quite susceptible to breaking. We caution that at extreme parameters this simple argument may not suffice to predict the behaviour of the system, although we expect the general behaviour to hold. In the next section we confirm the breaking of the disc with numerical simulations. These simulations are a preliminary investigation into this problem and we intend to follow up in more detail in future publications.

3. COUNTER–ROTATION

Let us consider an inclined disc that breaks under the action of a strong differential precession. Its inner and outer regions precess almost independently at different rates. The precession timescale is much shorter than the alignment timescale, which must wait for precession to induce dissipation. So both the inner and outer discs retain their inclinations to the black hole spin. The outer disc remains almost unmoved, while the inner disc (typically a ring of radial width $\sim H$) precesses rapidly about the spin axis. If the angle θ between the outer disc and the hole spin lies between $\sim 45 - 135^\circ$ the inner ring must form an angle $2\theta > 90^\circ$ with respect to the outer disc after half a precession period. The rotational velocities are now partially opposed.

This configuration is similar to those adopted in the counter–rotating disc simulations in Nixon et al. (2012) and so must result in rapid accretion. We note the probability that a randomly oriented accretion event lies in the critical range of inclinations is given by the fractional solid angle as $\cos\left(\pi/4\right)$ i.e. $\approx 70\%$. In other words, disc breaking and dynamical infall from counter–rotating accretion flows must be common in active galactic nuclei. It is also common in stellar–mass X–ray binaries if the spin of the black hole (or neutron star) accretor is sufficiently misaligned with the binary plane.

We report two simulations of an inclined disc around a spinning black hole using the Smoothed Particle Hydrodynamics (SPH) code PHANTOM (see e.g. Price & Federrath 2010; Lodato & Price 2010; Nixon et al. 2012; Nixon 2012). SPH performs well in modelling warped discs (Lodato & Price 2010) finding excellent agreement with the analytical treatment of Ogilvie (1999). This is to be expected as both treatments solve the Navier Stokes equations with an isotropic viscosity. The connections between the viscosity coefficients derived by Ogilvie (1999) therefore naturally holds in our numerical treatment. In the simulations reported here we implemented the Lense–Thirring effect, following Nelson & Papaloizou (2000). The simulations use a disc viscosity with Shakura & Sunyaev $\alpha \simeq 0.1$ (cf. Lodato & Price 2010), a disc angular semi–thickness of $H/R \simeq 0.01$ and the black hole has a spin $a = 1$. Initially the disc has no warp and extends from an inner radius of $50R_{\rm g}$ to an outer radius of $250R_{\rm g}$, with a surface density profile $\Sigma = \Sigma_0(R/R_0)^{-p}$ and locally isothermal sound speed profile $c_{\rm s} = c_{{\rm s},0}(R/R_0)^{-q}$ where we have chosen $p = 3/2$ and $q = 3/4$ to achieve a uniformly resolved disc (Lodato & Pringle 2007). The disc is initially composed of 2 million particles, which for this setup gives $\left\langle h\right\rangle/H \approx 0.8$ (cf. Lodato & Price 2010). The two simulations differ only by the relative inclination angle to the black hole.

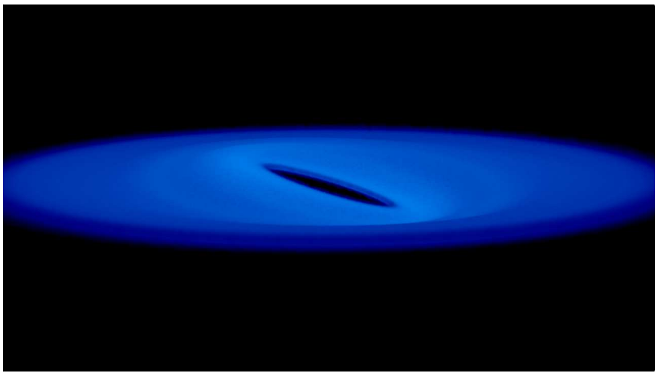

FIG. 1.— Full 3D surface rendering of the small inclination simulation. The whole disc was initially inclined at 10° to the hole with no warp. This snapshot is after approximately 500 dynamical times at the inner edge of the disc ($50R_g$).

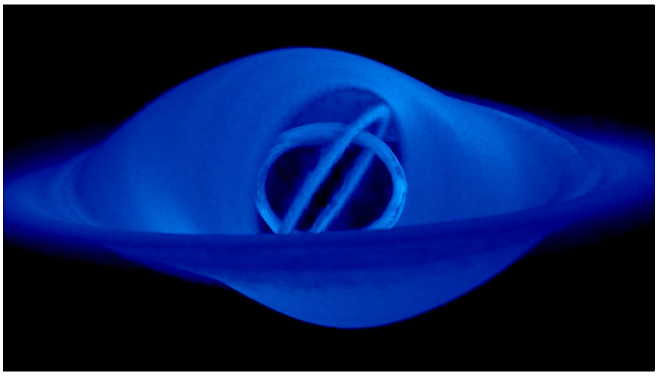

FIG. 2.— Full 3D surface rendering of the large inclination simulation. The whole disc was initially inclined at 60° to the hole with no warp. This snapshot is after approximately 500 dynamical times at the inner edge of the disc ($50R_g$).

The simulation shown in Fig. 1 has an initial inclination of 10°, and thus $R_{\rm break} \approx 40R_g$, i.e. at a radius inside the disc's inner boundary, and so we do not expect the disc to break. This agrees with the simulation, which shows the usual (Bardeen–Petterson) evolution with a smooth warp. In contrast, the simulation shown in Fig. 2 has an inclination of 60° and therefore $R_{\rm break} = 110R_g$, i.e. we expect the disc to break. The simulated disc does indeed break, producing multiple distinct rings of gas with large relative inclinations. This leads to phases of strong accretion when the rings are highly inclined, and quieter phases when they are not[4]. The accretion rates for the two cases are shown in Fig. 3.

We note that this evolution is more extreme than the 1D simulations reported in Nixon & King (2012). However the numerical method used there assumed the gas always remained on circular orbits, evolving purely by viscous diffusion. This was appropriate for the problem studied there, namely, whether such a relatively orderly disc could break at all, given the nonlinear evolution of the viscosity predicted by Ogilvie (1999). The current paper studies a dynamical problem, where the inclination of disc orbits can change so rapidly that viscous diffusion of gas in circular rings is no longer an adequate approximation. Interestingly, Nixon & King (2012) did remark that their simulated disc rings appeared to be trying to break into more than one distinct plane.

[4] Movies of the simulations in this paper are available at http://www.astro.le.ac.uk/users/cjn12/tearing.shtml

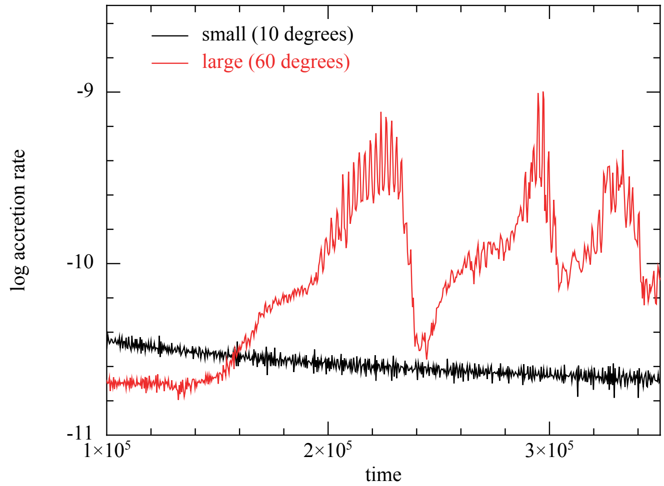

FIG. 3.— The accretion rates with time for the small (black) and large (red) inclination simulations. The accretion rate is in arbitrary units. The time unit is the dynamical time at $R = R_g$, i.e. the dynamical time at the inner edge of the discs is ~ 350. Note this is the mass flow rate through $R = 50R_g$ and not the final accretion rate on to the black hole.

4. DISCUSSION

We first discuss the possible arguments against this picture. The main unknown in this work is the nature of the viscosity controlling angular momentum transport in the disc. In this paper we have assumed that this can be modelled as an isotropic Shakura & Sunyaev α viscosity. There is a strong basis for assuming that the radial transport of angular momentum (governed by the azimuthal viscosity) is limited to $\alpha \sim 0.1$ (King et al. 2007) and we expect discs around black holes to be very thin away from the immediate vicinity of the strongly accreting hole (e.g. King & Pringle 2007, and references therein). However, the nature of the viscosity is unknown. In reality the local viscosity is likely to result from MHD effects (Balbus & Hawley 1991), and may well be anisotropic. The azimuthal shear is likely to be secular, with gas parcels continually moving away from each other, whereas the vertical shear is probably oscillatory (Pringle 1992). This is suggestive of a favourable anisotropy where the vertical viscosity does not strongly oppose breaking, but the result is simply not known. The consistency requirements worked out by Ogilvie (1999) for a locally isotropic viscosity show that in a strong warp, the viscosity trying to hold the disc together is likely to weaken. There appears no reason to suggest this differs for an anisotropic viscosity.

Another possible complication is the thermal evolution of the gas. As the disc orbits do not all lie in the equatorial plane, they must shock and heat up. In the simulations above we have assumed an isothermal equation of state, so this extra heat is assumed to be radiated away instantly. This is reasonable, as the densities in black hole discs are high and cooling is likely to be efficient. However if the disc cannot cool on the local precession timescale, it may heat up significantly so that Eq. 7 is no longer satisfied. In this case the disc may rapidly thicken, perhaps even becoming thermally unstable, and switch to a different mode of both accretion and warp propagation. On the other hand, Nixon et al. (2012) found, in counter–rotating disc simulations using an adiabatic equation of state, that although the gas dynamics can be strongly modified by gas heating, the net result in terms of rapid accretion is similar.

Otherwise there appears to be no obvious reason why this behaviour should be suppressed. We therefore expect this to be generic to most cases of accretion on to black holes (or neutron stars, since the Lense–Thirring effect applies here too), and more generally for the evolution of gas discs in the presence of a strong precession.

5. CONCLUSIONS

We have shown that for realistic parameters a randomly oriented accretion event on to a spinning black hole is likely to form a disc which is susceptible to breaking at a radius close to the hole. If the angle between the disc and the hole lies between $\sim 45 - 135°$ the interaction of partially opposed gas motions is likely, and leads to cancellation of angular momentum and rapid infall.

This quasi–dynamical form of accretion, which appears to be a generic consequence of randomly oriented accretion on to a black hole, significantly alters the standard picture of slow viscous accretion. There is nothing to prevent a succession of events where rings break off the inner disc edge and then precess independently of those inside or further out (see Fig. 2). It is reasonable to think of this process as tearing up the disc in a chaotic way.

We shall consider possible consequences of this picture in subsequent papers, but note several points here.

1. Tearing the disc can lead to rapid gas infall, but the long–term rate of central accretion is ultimately controlled by the outer disc.

2. In a stellar–mass binary system this means that tearing *modulates* a quasi–steady mass transfer rate. The modulation might have large amplitude if the disc/spin inclination is high. Even if the inclination is modest, there are likely to be observable effects which could include quasiperiodic behaviour such as quasiperiodic oscillations (QPOs). Still more effects can occur if the infalling rings shadow the central X–ray source.

3. By contrast, in active galactic nuclei, the viscous timescale of the outer disc may easily exceed a Hubble time, and no steady state is ever set up. Thus tearing of a significantly inclined AGN disc may promote significant accretion when the central black hole would otherwise not gain mass at all.

4. The torques we have considered here are all internal to the disc – black hole system. They cannot affect any conclusions concerning the global conservation of angular momentum or mass of this system. In particular, considerations of the long–term evolution of black hole spin through accretion remain unchanged, whether the accretion is assumed to be coherent (e.g. Volonteri & Rees 2005; Berti & Volonteri 2008) or chaotic (King & Pringle 2006, 2007; King et al. 2008).

5. We can expect qualitatively (and sometimes quantitatively) similar effects for other cases where an accretion disc is subject to a forced external differential precession. Most obviously, Nixon et al. (2011b) have shown that the effective potential experienced by a disc accreting on to a misaligned binary, as is thought to occur when supermassive black holes are close to coalescence, is extremely similar to that caused by the Lense–Thirring effect. For initial inclinations near to co– or counter–rotation the disc respectively coaligns or counteraligns (for the subsequent evolution of prograde circumbinary discs see Cuadra et al. 2009; Lodato et al. 2009, and for retrograde circumbinary discs see Nixon et al. 2011a). However, disc tearing changes this picture and may well bring gas into the close vicinity of the holes on near–dynamical timescales, and thus help with the last parsec problem (Begelman et al. 1980) as well as feeding problems.

6. In general any black hole has some spin, and in general any accretion disc plane may be inclined to this spin. To prevent any of the effects we have discussed here from appearing, the misalignment must satisfy

$$|\sin\theta| \lesssim \frac{3\alpha}{4a}\frac{H}{R}, \tag{9}$$

which is extremely small for realistic parameters. For example, a moderately thick disc with $H/R = 0.1$, $\alpha = 0.1$ and a low spin $a = 0.1$ must be inclined by less than $4°$ to avoid this process. We suggest that disc tearing, particularly in the inner disc, probably occurs in many if not most cases of black hole or non-magnetic neutron star accretion.

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