ASP2062 Introduction to Astrophysics

Planet formation III — Daniel Price

Key revision points

1. The two main models for planet formation in discs are ‘core accretion’ and ‘gravitational instability’
2. Gravitational instability requires massive discs that satisfy the Toomre criterion and a sufficiently fast cooling rate
3. Core accretion proceeds in stages from dust to planetesimals, to terrestrial planets and eventually gas giants. Several ‘growth barriers’ must be overcome.
4. Both models predict a phase where planets interact strongly with the disc

3 Planet formation

There are two main competing models[1] of how to form planets in accretion discs:

1. The *core accretion model* (Pollack et al., 1996) — dust coagulates into planetesimals which combine to form rocky cores. These terrestrial ‘cores’ either become terrestrial planets or later accrete gas from the disc to form gas giants.
2. *Gravitational instability* (Boss, 1997) — the disc becomes unstable to its own self-gravity and fragments, similar to the process of star formation in molecular clouds.

We’ll discuss gravitational instability first, since it is similar to the star formation process. For further reading on both, see Armitage (2010), which we follow closely.

3.1 Gravitational instability

The conditions required for a Keplerian disc to be unstable to its own gravity are that

$$\frac{M_{\rm disc}}{M_{\rm star}} \gtrsim \frac{H}{R}, \tag{1}$$

[1]It is important to note that planet formation is not a solved problem, and plenty of alternative models exist, including our very own Prentice (1978).

The minimum mass solar nebula

The 'minimum mass solar nebula' refers to the minimum surface density of material needed to form our own Solar System. The idea is to use the observed mass of heavy elements in the solar system planets and add enough Hydrogen and Helium to reach a solar composition. The mass required for each planet is then spread over the area between planets. This gives $\Sigma \propto R^{-3/2}$ out to Neptune. The most commonly quoted value is from Hayashi (1981),

$$\Sigma(R) = 1.7 \times 10^3 \left(\frac{R}{1\mathrm{AU}}\right)^{-3/2} \mathrm{g/cm^2}. \tag{2}$$

This provides a lower limit on the surface density of the accretion disc that formed the solar system.

and

$$Q \equiv \frac{c_\mathrm{s}\Omega}{\pi G \Sigma} < 1. \tag{3}$$

The latter is known as the *Toomre criterion* after the stability analysis (equivalent to the Jeans instability, but for a rotating disc) performed by Toomre (1964), and hence Q is known as the *Toomre Q parameter*. The two conditions are related — the first is the *global* condition for the disc to be unstable *anywhere*, whereas the second is the *local* condition for a disc to be unstable *at a particular point*.

The first condition implies that disc masses $\gtrsim 0.1 M_\odot$ are needed for fragmentation to occur (e.g. given a typical aspect ratio $H/R \sim 0.05$ around a typical $0.5 M_\odot$ star). More specifically, for the Toomre Q criterion to be satisfied we need

$$\Sigma \geq \frac{c_\mathrm{s}\Omega}{\pi G} = \frac{H\Omega^2}{\pi G} = \frac{H}{\pi G}\frac{GM}{R^3} = \frac{1}{\pi}\frac{H}{R} M_\mathrm{star} R^{-2}, \tag{4}$$

or

$$\Sigma \geq 1.4 \times 10^5 \left(\frac{H/R}{0.05}\right) \left(\frac{M_\mathrm{star}}{M_\odot}\right) \left(\frac{R}{1\mathrm{AU}}\right)^{-2} \mathrm{g/cm^2}. \tag{5}$$

This is two orders of magnitude higher than the 'minimum mass solar nebula' required to form the solar system, indicating a massive disc is needed. These are possible only in the very earliest stages of star formation when accretion rates from the collapsing cloud are high.

What are typical masses of planets formed by gravitational instability?

As with the star formation process, we can define a Jeans length and mass for fragmentation in discs, according to

$$L_{\rm J} \sim \frac{2c_{\rm s}^2}{G\Sigma}; \qquad M_{\rm J} \sim L_J^2\Sigma. \tag{6}$$

This gives a typical planet mass, using $Q = 1$ to define the surface density, as

$$M_{\rm J} \approx \frac{4c_{\rm s}^4}{G^2\Sigma} = \frac{4c_{\rm s}^4}{G}\frac{\pi G}{c_{\rm s}\Omega} = 4\pi M_{\rm star}\left(\frac{H}{R}\right)^3. \tag{7}$$

So we expect to form objects of $\approx 2M_{\rm Jupiter}$ in typical discs with $H/R = 0.05$.

3.1.1 Self-regulation and the cooling criterion

In practice things are not so simple, since Q depends on temperature (recall $c_{\rm s} \propto \sqrt{T}$). As the disc becomes unstable ($Q < 1$) it generates spiral shocks. These heat the disc, causing Q to increase and the disc to *stabilise* ($Q > 1$). This is known as *self-regulation* and leads to a disc where the surface density and temperature hover around $Q \approx 1$.

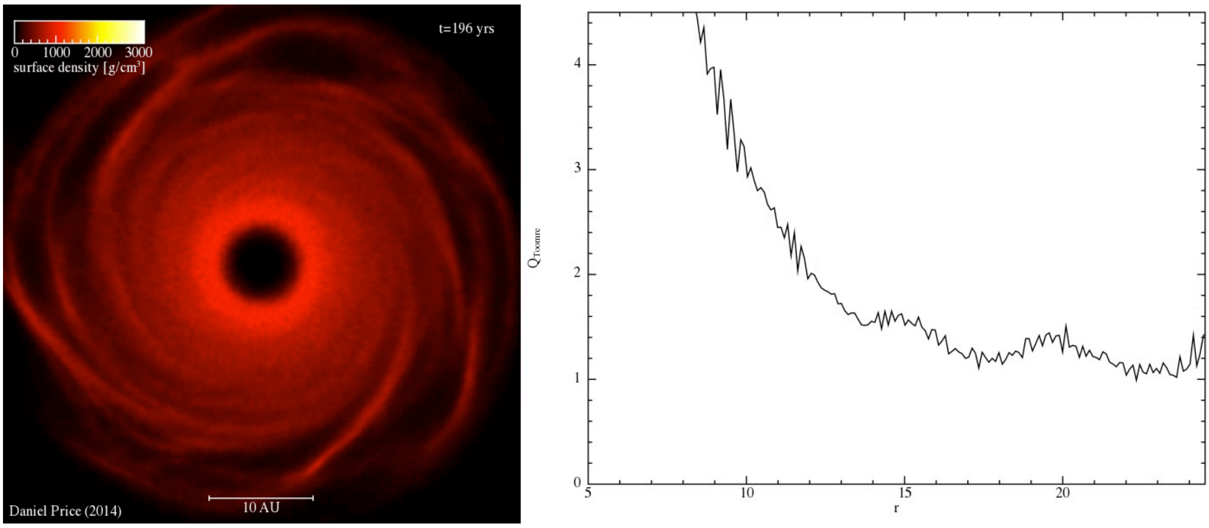

Figure 1: Self-regulated gravitational instability in an accretion disc, showing surface density (left) and Toomre Q as a function of radius (right). Click for video.

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An additional criterion is thus necessary for the disc to fragment, i.e. that not only $Q < 1$ but also that the cooling of the disc is sufficiently fast to prevent self-regulation. Gammie (2001) derived a critical cooling time in terms of the orbital timescale according to

$$t_{\rm cool} < t_{\rm cool,crit} \approx \frac{1}{3\Omega}. \tag{8}$$

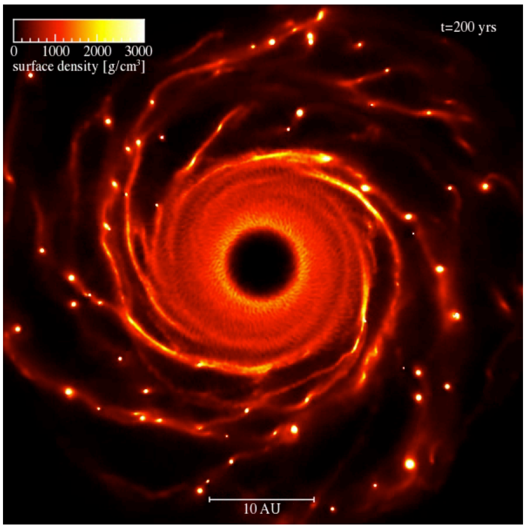

Figure 2: Fragmentation occurring in a gravitationally unstable disc, once all three conditions for gravitational instability are satisfied. Click for video.

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In other words, if the disc cools on a timescale comparable to the orbital timescale it cannot self-regulate. Fragmentation occurs if $Q < 1$ and $t_{\rm cool} \lesssim 1/(3\Omega)$ (see Figure 2).

3.2 The core accretion model

According to the core accretion model, planet formation proceeds in three key stages:

1. Coagulation of dust to form km-sized planetesimals
2. Formation of terrestrial planets from planetesimals
3. Giant planet formation and migration

Each of these steps involves a huge range of scales and is difficult to model, but the overall process is slow compared to gravitational instability. There are also several 'barriers' to growth that occur at particular scales.

3.2.1 Stage I — from dust to planetesimals

The idea is to start with a gaseous accretion disc with a small amount (typically 1%) of dust in small micron-sized grains. Gas and solid particles in the disc interact by *aerodynamic drag* forces. The drag force on a grain is proportional to its area ($\propto \pi s^2$, for a spherical grain of size s), giving an acceleration that depends on size according to

$$\text{Acceleration} = \frac{\text{Force}}{\text{Mass}} \propto \frac{\pi s^2}{\frac{4}{3}\pi s^3 \rho_{\rm grain}} \propto \frac{1}{s}. \tag{9}$$

That is, acceleration is inversely proportional to grain size. Hence small grains are strongly accelerated by, and hence "stuck to", the gas, whereas larger grains are less well coupled.

The main effects of drag are that solid particles *settle* quickly (in $\sim 10^5$ yr) to the disc midplane, because dust grains do not feel the pressure of the gas. The grains also *drift radially* towards the star, because they orbit faster than the gas (the gas orbits at sub-Keplerian speed because of pressure) and feel a headwind. The timescale for radial drift depends on the coupling to the gas and is therefore a strong function of the grain size.

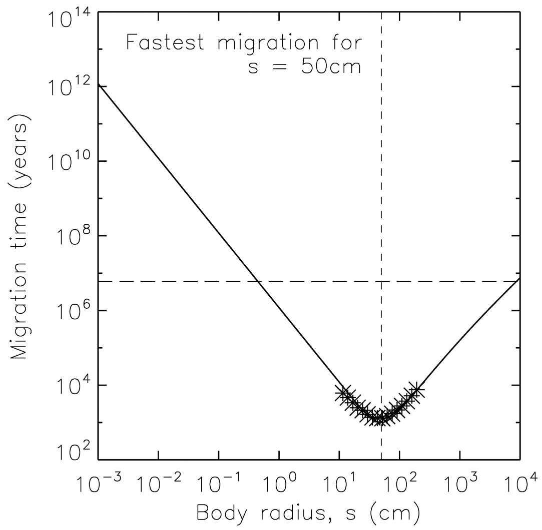

Figure 3: Time for grains to be swept onto the star via radial drift as a function of the grain size, assuming a gas surface density of 75 g/cm^2. Figure from Ayliffe et al. (2012), showing the analytic solution (solid line) from Takeuchi and Lin (2002).

For grain sizes between $\sim$ 1cm and 100 m, the migration time on to the star can be extremely short (10^3–10^4 yrs; see Figure 3). Therefore all of the dust grains that reach this size would quickly be swept into the star. This is known as the 'metre-sized' or more accurately the 'radial drift' barrier. Several solutions are possible (see thinkbox).

How to avoid the radial drift barrier?

Two possible solutions to the radial drift, or 'metre-sized' barrier are:

1. Grains grow quickly, and so avoid the radial drift barrier by growing big enough to decouple from the gas before being swept onto the star
2. Grains are trapped by pressure gradients in the disc, e.g. from the presence of existing planets, vortices, or spiral structure.

While the radial drift barrier is a problem for our theoretical understanding of planet formation, Nature clearly solves this since we observe both large grains and planetesimal-sized bodies in protoplanetary discs and in the Solar System.

3.2.2 Stage II — From planetesimals to terrestrial planets

Grains coagulate to eventually form km-sized planetesimals. Bodies of this size are largely decoupled from the gas in the disc. The main physics at play is the gravitational interaction between the large number ($\sim 10^9$) of planetesimals in the disc. Growth of these to form planets occurs in two phases:

1. *Runaway growth* — a small fraction of planetesimals grow by collision and interaction with others, forming 10^2–10^3 large bodies of 0.01–0.1 times the mass of Earth.
2. *Oligarchic growth* — A few larger 'Oligarchs' grow at similar rates by feeding from a local 'pool' of nearby, smaller, planetesimals

Eventually the larger bodies scatter the remaining planetesimals out of the disc.

3.2.3 Stage III — Core accretion and migration

One protoplanetary 'cores' are formed, these again interact strongly with the disc, by accreting gas from it. The relative amount of gas accreted determines whether the planet ends up as a terrestrial planet or as a gas giant. Importantly, interaction with the disc leads to *migration* of the planet in radius (you will study this in the labs next week). Migration occurs in two different types:

1. **Type I** migration occurs for low mass planets that have a weak interaction with the disc which does not affect the disc structure. That is, the planet is fully embedded in the disc.
2. **Type II** migration occurs once giant planets are formed — they have a strong interaction with the disc and the torques from the planet clear a 'gap', with gas accreting onto the planet across the gap.

3.2.4 Gap opening

An estimate for the critical planet mass required to open a gap is given by

$$\left(\frac{M_{\rm planet}}{M_{\rm star}}\right)_{\rm crit} \gtrsim \sqrt{\frac{27\pi}{8}} \left(\frac{H}{R}\right)^{5/2} \alpha^{1/2}, \tag{10}$$

where α is the disc viscosity (gap opening is a competition between the torque from the planet and the disc viscosity). For typical parameters ($H/R \approx 0.05$, $\alpha \approx 0.1$, $M_{\rm star} = 0.5{\rm M}_{\odot}$), planets with ${\rm M}_{\rm planet} \gtrsim 0.3\ {\rm M}_{\rm Jupiter}$ are required to open a gap.

When does planet formation stop?

Planet formation ceases once the gas disc is dispersed, which is observed to occur on a timescale of $\sim$ few Myr. The leading mechanisms thought to drive disc dispersal are either photoevaporation of the disc due to X-ray/UV radiation from the star, or due to the disc mass being either made into planets or accreted onto the star.

3.3 Recent progress

What is utterly amazing about the new ALMA image of the protoplanetary disc around HL Tauri (left panel in Figure 4) is that the disc is riddled with gaps! These kind of gaps should only be present at the very latest stages of planet formation (i.e. during Type II migration), yet they appear to be present in a very young system ($\lesssim$ 1 Myr). Having apparently such massive planets so early in the picture is challenging our ideas about the planet formation process.

How teaching benefits research

Kickstarted by my putting together the new ASP2062 planet formation lab in 2014, and together with summer research student Kieran Hirsh who completed ASP2062 last year, we recently published a nice explanation of how the gaps in HL Tau can be explained by the presence of planets (Dipierro et al. 2015; Figure 4; see video). Key to understanding the image is that it is much easier for planets to carving gaps in the thin dust disc (which is what is seen by ALMA) compared to carving gaps in the gas disc (see right panel of Figure 4).

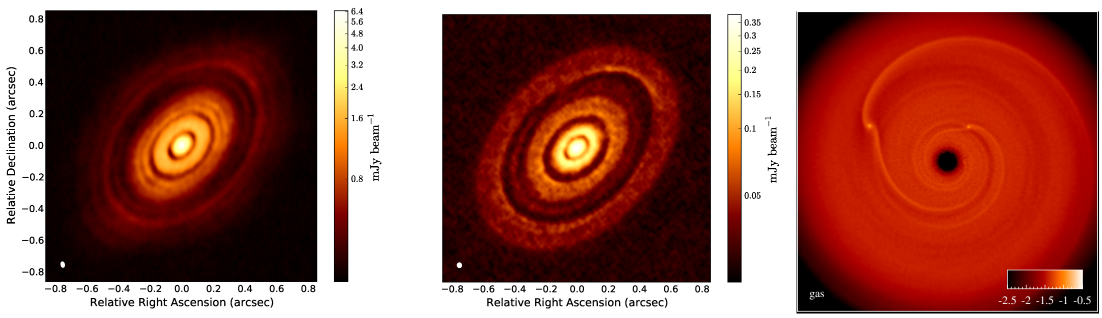

Figure 4: The protoplanetary disc in HL Tau imaged at mm wavelengths with ALMA (left) compared to our simulation of the system using 3 planets of masses 0.2, 0.27 and 0.5 times the mass of Jupiter (middle; showing our simulated observations of the dust disc). The right panel shows surface density in the gas disc in our simulations.

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