Notes by Eric Thrane

The attached hand-written notes describe how stochastic variables like sensInt, ccspec and ccvar are related to STAMP estimators map.y and map.sigma.

formulas relevant for parsing Emma Robinson's VSR1 - S5 data

see also T1000354

broadband sigma on Ω_3 | fref

$$\sigma = \left[\sum \delta f * (\mathrm{SensInt}) \right]^{-1/2}$$

narrowband estimate for Ω_3 given fref

$$Y(f) = \frac{\mathrm{real}(\mathrm{PtEstInt})}{\sigma^2 \sqrt{\delta f}\,(\mathrm{SensInt})}$$

narrowband sigma on $Y(f)$ given fref

$$\sigma(f) = \frac{1}{\sqrt{\delta f * (\mathrm{SensInt})}}$$

sanity checks

$$\sigma = \left(\sum_i \sigma_{f_i}^{-2} \right)^{-1/2}$$

$$\mathrm{std}\left(\frac{Y(f)}{\sigma(f)} \right) \cong 1$$

$$i_{R(f)} = \frac{\cdots}{I(f)\,df\ \sigma^2}$$ ← radiometer

$$\sigma_{R(f)} = \frac{1}{\sqrt{I(f)\,df}}$$

↳ broadband radiometer

σ^2 = ccvar = "theoretical variance"

$I(f)$ = sensInt

$\sigma(f)$ = stochastic sigma

in normalization.m << calOptimalFilter.m

$$I(f) = \frac{1}{T}\left(\cancel{\frac{3H_0^2}{10\pi^2}}\right)^2 \frac{\left(\overline{w^2}\right)^2}{\left(\overline{w^4}\right)} \frac{2\,|\gamma(f)|^2\ \bar{H}(f)^2}{P_1(f)\ P_2(f)}$$

$$\sigma^2 = T\left(\cancel{\frac{10\pi^2}{3H_0^2}}\right)^2 \frac{\left(\overline{w^4}\right)}{\left(\overline{w^2}\right)^2}\left[2\int_{f_{low}}^{f_{high}} \delta f\ \frac{|\gamma(f)|^2\ H(f)^2}{P_1(f)\ P_2(f)}\right]^{-1}$$

these factors are unity for radiometer

i.e., $$\boxed{\sigma^2 \equiv \left[\int df\ I(f)\right]^{-1}}$$

$$\frac{(f/f_{ref})^{\alpha}}{f^3} \longrightarrow H(f)$$ for radiometer

$$\text{norm} \equiv \left(\cancel{\frac{10\pi^2}{3H_0}}\right) \frac{2}{\left(\overline{w^2}\right)}\left[2\int_{f_{low}}^{f_{high}} \delta f\ \frac{|\gamma(f)|^2\ \bar{H}(f)^2}{P_1(f)\ P_2(f)}\right]^{-1}$$

in calOptimalFilter.m

$$Q(f) \equiv \frac{\text{norm}\cdot\gamma(f)\ \bar{H}(f)}{P_1(f)\ P_2(f)}$$

in processCSD.m

$$\text{ccspec} = \text{CSD}(f)\ Q(f)$$

in ccSpecReadout.m

$$\hat{y}(f) = \text{Re}\left[\frac{2}{\sigma^2\gamma_0}\ e^{i\phi}\ \frac{\text{ccspec}}{I(f)}\right]$$

ignore [illegible] phase [illegible]
" to focus on normalizations

$$\hat{y}(f) = \frac{2}{\sigma^2 \gamma_0} \frac{CSD(f)\,Q(f)}{I(f)}$$

but

$$\boxed{Q(f) = \sigma^2 \frac{2}{T} \left(\cancel{\frac{3H_0}{10\pi^2}}\right)\left(\frac{\overline{w^2}}{\overline{w^4}}\right) \frac{\gamma(f)\,\overline{H}(f)}{P_1(f)\,P_2(f)}}$$

Thus,

$$\hat{y}(f) = \frac{\frac{2}{\gamma_0}\,\frac{2}{T}\left(\cancel{\frac{3H_0}{10\pi^2}}\right)\left(\frac{\overline{w^2}}{\overline{w^4}}\right)\frac{\gamma(f)\,\overline{H}(f)}{P_1(f)\,P_2(f)}\,CSD(f)}{\frac{1}{T}\left(\cancel{\frac{3H_0^2}{10\pi^2}}\right)\frac{\overline{w^2}^{(2)}}{\overline{w^4}}\,\frac{2\,|\gamma(f)|^2\,\cancel{\;\;}\,\overline{H}(f)^2}{P_1(f)\,P_2(f)}}$$

STAMP stat

$$= \frac{2}{\gamma_0}\,\cancel{\overline{w^2}\,\gamma(f)\,\overline{H}(f)}\left(\frac{CSD(f)}{\overline{w^2}\;\gamma(f)\;\overline{H}(f)}\right)$$

antenna factor

just an array of ones

$$= \boxed{\frac{2}{\gamma_0}\frac{1}{\overline{w^2}}\,CSD(f) = \hat{g}(f)}$$

$$\hat{\sigma}(f) = \frac{1}{|\gamma_0|}\frac{1}{\sqrt{I(f)\,\delta f}} = \frac{\sqrt{\overline{w^4}}}{|\gamma_0|\;\overline{w^2}}\;\frac{\sqrt{P_1(f)\;P_2(f)}}{\sqrt{2}\,\underbrace{|\gamma(f)|\;\overline{H}(f)}_{\text{arrays of ones}}}\sqrt{\frac{T}{\delta f}}$$

STAMP uncertainty

$$\boxed{\hat{\sigma}(f) = \frac{\sqrt{\overline{w^4}}}{|\gamma_0|\;\overline{w^2}}\;\frac{\sqrt{P_1(f)\,P_2(f)}}{\sqrt{2}}\sqrt{\frac{T}{\delta f}}}$$