Lie theory: frontiers, algorithms, and applications

Monash Prato Centre, Italy
10th - 13th January 2023

Lie Theory in Prato 2023

January 10-13, 2023

We welcome you to the 2023 workshop "Lie Theory: frontiers, algorithms, and applications", jointly organised by **Monash University** and the **University of Trento**. We aim to bring together leading experts and early career researchers in (computational) Lie theory, to discuss recent advances in the field and to foster new collaborations. The workshop will focus on theoretical and computational aspects, as well as applications in broad areas such as quantum information theory and theoretical physics. The workshop website

`https://users.monash.edu/~heikod/lietheory_prato_2023.html`

and this document provide some further information. We hope you will enjoy the workshop, and we encourage you to reach out if you have any questions.

We look forward to seeing you soon!

Heiko Dietrich (Monash University)

Willem A. de Graaf (University of Trento)

1 Venue

The workshop venue is the **Monash University Prato Centre**. The Monash Prato Centre is a teaching and research centre in Tuscany. It is located in an 18th-century palace, the Palazzo Vaj, in the historical centre of Prato. It was opened on 17 September 2001, as part of Monash University's internationalisation policy. It was established with the assistance of the local government of Prato, the region of Tuscany, and prominent Italian-Australians Rino and Diana Grollo. The Centre aims to develop Australia's links with Europe, through research collaboration, seminars and conferences, and as a centre of study for Australian students. It is now the largest Australian institution of its kind in Europe. Some general information about the Monash Prato Centre, Prato, and surroundings, can be found at

- `https://monash.it/`
- `https://www.pratoturismo.it/en/`
- `https://www.visittuscany.com/en/`
- `https://www.cittadiprato.it/EN/`

Most participants stay in the **Art Hotel Milano** `www.arthotel-milano.it/en/`, which is in walking distance to the Centre (see **Figure 1**). See **Figure 4** for the directions from the Prato train station (Prato Centrale) to the hotel.

2 Schedule

Days: The workshop will be Tuesday January 10 to Friday January 13, 2023. It is an OH&S requirement of the Centre that participants sign in on arrival each day. Tuesday we will have a brief welcome at 9:15. Thursday afternoon is free, and in the evening we will have our workshop dinner.

Talks: Each talk has a 50 minutes time slot: 40 minutes for the presentation (*please stick to this time*), 5 minutes for questions, and 5 minutes for setting up the next speaker. The default technology will be a projector and screen. There will only be small whiteboards (80cm x 120cm) available.

Rooms: All talks will be in **Sala Veneziana** (Room 6 in **Figure 2**); in addition, we have two breakout rooms available (Rooms 4 and 5 in Figure 2). The breakout rooms will also be available during the free Thursday afternoon. The conference will conclude with the lunch on Friday. Free WiFi (Eduroam) will be provided.

Meals: If we have organised accommodation for you in Art Hotel Milano, then breakfast will be included. Morning tea, lunch, and afternoon tea will be served at the Monash Prato Centre as indicated in the schedule below. The workshop dinner is on Thursday.

time	Tue	Wed	Thu	Fri
9:00 – 9:20	**registration/welcome**	**registration**	**registration**	–
9:20 – 10:00	Santi	Bocheński	Deré	**registration**
10:10 – 10:50	Marrani	Tralle	Gandini	Kunyavskii
11:00 – 11:30	**morning tea**	**morning tea**	**morning tea**	**morning tea**
11:30 – 12:10	Carnovale	Michel	Lê	Cantarini
12:20 – 13:00	Papi	Tsanov	Di Trani	Maffei
13:10 – 14:20	**lunch**†	**lunch**	**lunch**	**lunch**
14:20 – 15:00	Chavli	Jibladze‡	**free** or **breakout**	
15:10 – 15:50	Pfeiffer	**afternoon tea**	**free** or **breakout**	
16:00 – 16:30	**afternoon tea**	**breakout**	**free** or **breakout**	
16:30 – 17:30	**breakout**	**breakout**	**free** or **breakout**	
			[Dinner 20:00]	

† Just before the lunch break we will make a workshop photo.
‡ This talk was originally scheduled as a joint presentation of Elashvili/Jibladze; the talk duration has been extended to 50 minutes.

Workshop Dinner. Thursday evening we will have a workshop dinner at **Osteria Su Santa Trinita** (see `www.osteriasantatrinita.it`); the dinner will start at 20:00. The restaurant is 550m from the Monash Prato Centre, see **Figure 3**. The dinner will be a set menu that respects the dietary requirements you have disclosed to us. The set menu includes three courses, wine, water, and a café. The costs for the dinner are covered by us if you will present a talk. If you do not present a talk (which includes all "+1"s that you have registered), then you will have to cover the costs for the meal (EUR 50 each). You will also have to pay for anything you will order extra (outside the set menu).

3 Abstracts

Maciej Bocheński. *Non-virtually abelian discontinuous group actions vs. proper* $\mathrm{SL}(2,\mathbb{R})$*-actions on homogeneous spaces.*
Let G be a real semisimple Lie group and H a closed reductive subgroup of G such that G/H is non-compact. In this talk, I will discuss some results concerning the problem of existence of non-virtually abelian discrete subgroups of G/subgroups locally isomorphic to $\mathrm{SL}(2,\mathbb{R})$ acting properly on G/H. Also I will report on a recent joint work with Willem A. de Graaf, Piotr Jastrzębski, and Aleksy Tralle on algorithms and computer programs which verify the existence of such subgroups.

Nicoletta Cantarini. *Verma modules for infinite dimensional Lie superalgebras.*
In this talk we will describe the construction of the so-called finite Verma modules for $\mathbb{Z}$-graded Lie superalgebras and concentrate on some explicit examples. In particular, we will explain the differences between the linearly compact Lie superalgebras $E(5,10)$ and $E(4,4)$.

Giovanna Carnovale. *Sheets in reductive algebraic groups in arbitrary characteristic.*
The orbits for an action of a connected algebraic group can be collected into families, called sheets, consisting of orbits of fixed dimension. Such families retain information on the action of the group, so one can predict the behaviour of an orbit from knowing the behaviour of other orbits in the same sheet. The case of the adjoint action of a reductive group on its Lie algebra has been studied in the late 70's by Borho and Kraft in the characteristic zero case, and more recently by Premet and Stewart in the good characteristic situation. The case of the action of a reductive group on itself by conjugation, when the base field is of good characteristic, has been studied 10 years ago in a series of papers with F. Esposito, and a relation with Springer's correspondence has been unveiled by showing the role played by sheets in Lusztig's partition of a reductive algebraic group. Lusztig's partition relates data that depend on the characteristic of the base field k with irreducible representations of the Weyl group. Since the latter are independent of the characteristic of k, it is important to understand the behaviour of sheets also in bad characteristic. In a joint work with F. Ambrosio and F. Esposito, we provide a parametrization of sheets in arbitrary characteristic and discuss dependence of the characteristic of the base field of various data related to sheets and Lusztig's partition.

Eirini Chavli. *Complex Hecke algebras are real.*
Iwahori Hecke algebras associated with real reflection groups appear in the study of finite reductive groups. In 1998 Broué, Malle and Rouquier generalized in a natural way the definition of these algebras to complex case. However, some basic properties of the real case are also true for Hecke algebras in the complex case. In this talk we will talk about these properties and their state of the art.

Jonas Deré. *Lie algebras over subfields.*
Given a Lie algebra $\mathfrak{n}$ over a field E, there are two possible ways to consider it over a subfield $F \subset E$. (Take for example $\mathbb{R} \subset \mathbb{C}$ if you want to keep things more concrete.) One possibility is to define the underlying Lie algebra $\mathfrak{n}_F$, which one finds by restricting the scalar multiplication on $\mathfrak{n}$ to F. In this way, one finds a Lie algebra over the field F which has a higher dimension than the original Lie $\mathfrak{n}$ over the field E. A second possibility is to look for a so-called F-form of $\mathfrak{n}$, namely a subspace over the field F in $\mathfrak{n}$ which is closed under the Lie bracket. Of course,

such an F-form does not always exist, and a Lie algebra $\mathfrak{n}$ can contain several non-isomorphic F-forms. Both of these constructions have important applications in geometry, for example when studying Anosov diffeomorphisms on nilmanifolds or constructing quasi-Kähler Chern-flat manifolds. In this talk, I will start by motivating why one studies these constructions. Afterwards, I will explain how one can classify the F-forms in certain families of nilpotent Lie algebras and how the structure of the underlying Lie algebra $\mathfrak{n}_F$ sometimes determines the structure of the original Lie algebra $\mathfrak{n}$.

Sabino Di Trani. *A formula for tangent plane dimension at T-fixed points in flat linear degenerations of the flag variety.*
Linear degenerations of the flag variety arise as very natural generalizations of the complete flag variety. The talk will focus on flat degenerations, i.e. linear degenerations of flag variety that are equidimensional algebraic varieties of the same dimension as the complete flag variety. In some recent works of M. Lanini and A. Pütz it is proved that linear degenerations of the flag variety can be endowed with a structure of GKM variety, under the action of a suitable algebraic torus T. The aim of the talk is to present an effective method to compute dimension of tangent plane at a generic T fixed point. As a consequence, smoothness criteria can be achieved, generalizing some known results proved by G. Cerulli Irelli, E. Feigin, and M. Reineke for Feigin Degeneration.

Jacopo Gandini. *On the multiplication of spherical functions of reductive spherical pairs of type A.*
Let G be a complex simple algebraic group and let $X = G/K$ be an affine G-homogeneous variety whose coordinate ring $\mathbb{C}[X]$ is a multiplicity free G-module. Then X is called a multiplicity free G-variety, and (G, K) is called a reductive spherical pair. While the G-module structure of $\mathbb{C}[X]$ is well understood in terms of suitable combinatorial invariants associated to X, the G-algebra structure of $\mathbb{C}[X]$ is not so well understood. In the talk, I will consider the problem of decomposing the product of irreducible components in $\mathbb{C}[X]$ into irreducible summands. When the root system associated to X is of type A, I will propose a conjectural decomposition rule for the product of irreducible G-submodules of $\mathbb{C}[X]$, which relies on a conjecture of Stanley on the multiplication of Jack symmetric functions. With the exception of one case, I will also explain how this decomposition rule holds true whenever the root system associated to X is direct sum of subsystems of rank one. The talk is based on a joint work with Paolo Bravi.

Mamuka Jibladze (and Alexander Elashvili). *Some computational experience with Hesselink strata.*
In late seventies - early eighties, in the works of Kempf, Kirwan, Ness, Hesselink and others, certain stratifications of nullcones of representations of reductive groups have been investigated. These stratifications are especially important for understanding structure of the nilpotent orbits when there are infinitely many of them, as the number of strata is always finite. In 2003 Popov devised an algorithm for computing these strata. It has been implemented by A'Campo in PARI/GP for semisimple groups, but we have not been able to use it in some interesting cases, notably for spin(15). The reason the latter case is interesting is that this is the spin representation of smallest dimension with infinitely many nilpotent orbits. Recently Willem de Graaf kindly provided us with another implementation of the Popov algorithm in GAP, with some simplifications and improvements. Using it we successfully computed the stratification for spin(15)

and have been able to construct small support representatives of orbits of maximal dimensions in each of the 169 strata. In the talk, we will briefly recall definition of the stratification and the principal steps of the algorithm. We will also report on some of the peculiar aspects of our computations with the de Graaf program. If time permits, we will also describe statistics of behavior of strata in some series of representations, and share our thoughts on possible further improvements and simplifications of the algorithm.

Boris Kunyavskii. *Local-global invariants of groups, Lie algebras, and associative algebras.*
The Tate-Shafarevich set of a group G defined by Takashi Ono coincides, in the case where G is finite, with the group of outer class-preserving automorphisms of G introduced by Burnside. We consider analogues of this important group-theoretic object for Lie algebras and associative algebras and establish some new structure properties thereof. We also discuss open problems and eventual generalizations to other algebraic structures. Joint work with Vadim Ostapenko.

Hông Vân Lê. *Semisimple elements and the little Weyl group of real semisimple Z_m-graded Lie algebras.*
In my talk, I shall report on our recent joint results with Willem de Graaf on the Weyl groups of $\mathbb{Z}_m$-graded semisimple Lie algebras over $\mathbb{C}$ and $\mathbb{R}$. We prove that if m is a prime number or $\mathfrak{g}$ is a complex semisimple $\mathbb{Z}_m$-graded Lie algebra of maximal rank then the centralizer $Z_{\mathfrak{g}}(p)$ of a homogeneous semisimple element p and the stabilizer W_p of p under the Weyl group action define each other. As a result, we obtain a number of consequences on the conjugacy classes of homogeneous semisimple elements in $\mathfrak{g}$ and in its real forms. We also parameterize the conjugacy classes of Cartan subspaces in a real semisimple $\mathbb{Z}_m$-graded Lie algebra $\mathfrak{g}$ in terms of Galois cohomology.

Andrea Maffei. *Pavings Springer fibers of type E_7..*
We prove that Springer fibers of type E_7 have a paving by affine cells. (Joint work with Corrado De Concini).

Alessio Marrani. *The "magic star" projection: exceptional structures and Vinberg algebras.*
By exploiting the so-called "magic star" projection (also named "G_2 decomposition" by Mukai), I will unravel various physical applications of the novel mathematical framework of "exceptional periodicity", which generalizes exceptional Lie algebras, especially in relation to black hole entropy in supergravity theories, Lie superalgebras, higher-dimensional Yang-Mills theories, spin factors, Bott periodicity, triality, and Vinberg special T-algebras.

Jean Michel. *Computing representations of cyclotomic Hecke algebras.*
The problem of computing all irreducible representations of Hecke algebras of finite complex reflection groups is still not completely solved (for G_{31}, G_{32} and G_{34}). I will retrace the history of this problem, methods and remaining challenges.

Paolo Papi. *Unitary minimal W-algebras.*
We shall discuss the problem of unitarity of minimal W-algebras, proposing a classification of such vertex algebras. We shall then outline an approach to the classification of unitary representations. Joint work with Adamovic, Kac, Moseneder Frajria.

Götz Pfeiffer. *Parabolic Normalizers as Subdirect Products.*
Motivated by Serre's recent discussion of involution centralizers in finite Coxeter groups, we review and refine the description of normalizers of parabolic subgroups. This is joint work G. Roehrle and J. M. Douglass.

Andrea Santi. *Symmetry superalgebras in parabolic supergeometries.*
I will present an overview of recent results on Lie superalgebras of symmetries for geometric structures on supermanifolds related to nonholonomic distributions and their structure reductions. I will discuss a novel extension of Tanaka theory to the context of supermanifolds, giving an upper bound on the supersymmetry dimension, and some examples of distributions with flag supervarieties as models with maximum symmetry. In particular, I will show realizations of the exceptional simple Lie superalgebras $G(3)$ and $F(4)$ that generalize the first realizations by É. Cartan and F. Engel in 1893 of the simple Lie algebra $G(2)$.

Aleksy Tralle. *Stretched non-positive Weyl connections on solvable Lie groups.*
We are dealing with Weyl connections which describe Gaussian thermostats. These are used in creating interesting models in nonequilibrium statistical mechanics. One of the important problems in this area is finding Weyl connections of non-positive curvature. I will present results on describing solvable Lie groups admiiting invariant stretched non-positive Weyl connections and a complete classification of compact solvmanifolds with such connections. This is a joint work with Maciej Bochenski and Piotr Jastrzebski.

Valdemar Tsanov. *Constructive and algorithmic aspects of Hesselink-type stratifications.*
Let X be a real or complex projective G-manifold, for a linear reductive, real or complex, connected Lie group G. There is a G-equivariant stratification of X, which is highly relevant in the study of the orbit structure and invariant theory in the above setting, developed by Kirwan-Ness in the complex case and Heinzner-Schwarz-Stötzel in the real case. The strata can be indexed by certain one-parameter subgroups (OPS) of G. The conditions for a given OPS to occur are nontrivial, and will be in the focus of this talk. For a complex projective space $X = \mathbb{CP}^n$, an algorithm due to Popov allows to determine all relevant OPS by a combinatorial procedure. I will describe a generalization of Popov's method and discuss its computability in case X is homogeneous under a larger reductive group.

4 Participants

Here is the list of participants and their attendance; unfortunately, we had a number of last minute cancellations due to sickness.

Maciej Bocheński	University of Warmia and Mazury	Tue – Fri
Nicoletta Cantarini	University of Bologna	Fri
Giovanna Carnovale	University of Padova	Tue – Wed
Eirini Chavli	University of Stuttgart	Tue – Fri
Jonas Deré	KU Leuven Kulak	Tue – Fri
Sabino Di Trani	University of Rome “La Sapienza”	Tue – Fri
Heiko Dietrich	Monash University	Tue – Fri
Bogdan Adrian Dina	Tel Aviv University	Tue – Fri
Victor Fagundes	Monash University	Tue – Fri
Jacopo Gandini	University of Bologna	Tue – Fri
Willem de Graaf	University of Trento	Tue – Fri
Piotr Jastrzębski	University of Warmia and Mazury	Tue – Fri
Mamuka Jibladze	Tbilisi State University	Tue – Fri
Boris Kunyavskii	Bar-Ilan University	Tue – Fri
Hông Vân Lê	Czech Academy of Sciences	Tue – Fri
Andrea Maffei	University of Pisa	Thu – Fri
Alessio Marrani	University of Murcia	Tue
Jean Michel	Paris Cité University (Denis Diderot - Paris 7)	Tue – Fri
Paolo Papi	University of Rome “La sapienza”	Tue – Fri
Mattia Pirani	University of Pisa	Tue – Fri
Götz Pfeiffer	University of Galway	Tue – Fri
Andrea Santi	University of Rome “Tor Vergata”	Tue – Fri
Alexy Tralle	University of Warmia and Mazury	Tue – Fri
Valdemar Tsanov	Bulgarian Academy of Sciences	Tue – Fri

5 Acknowledgements

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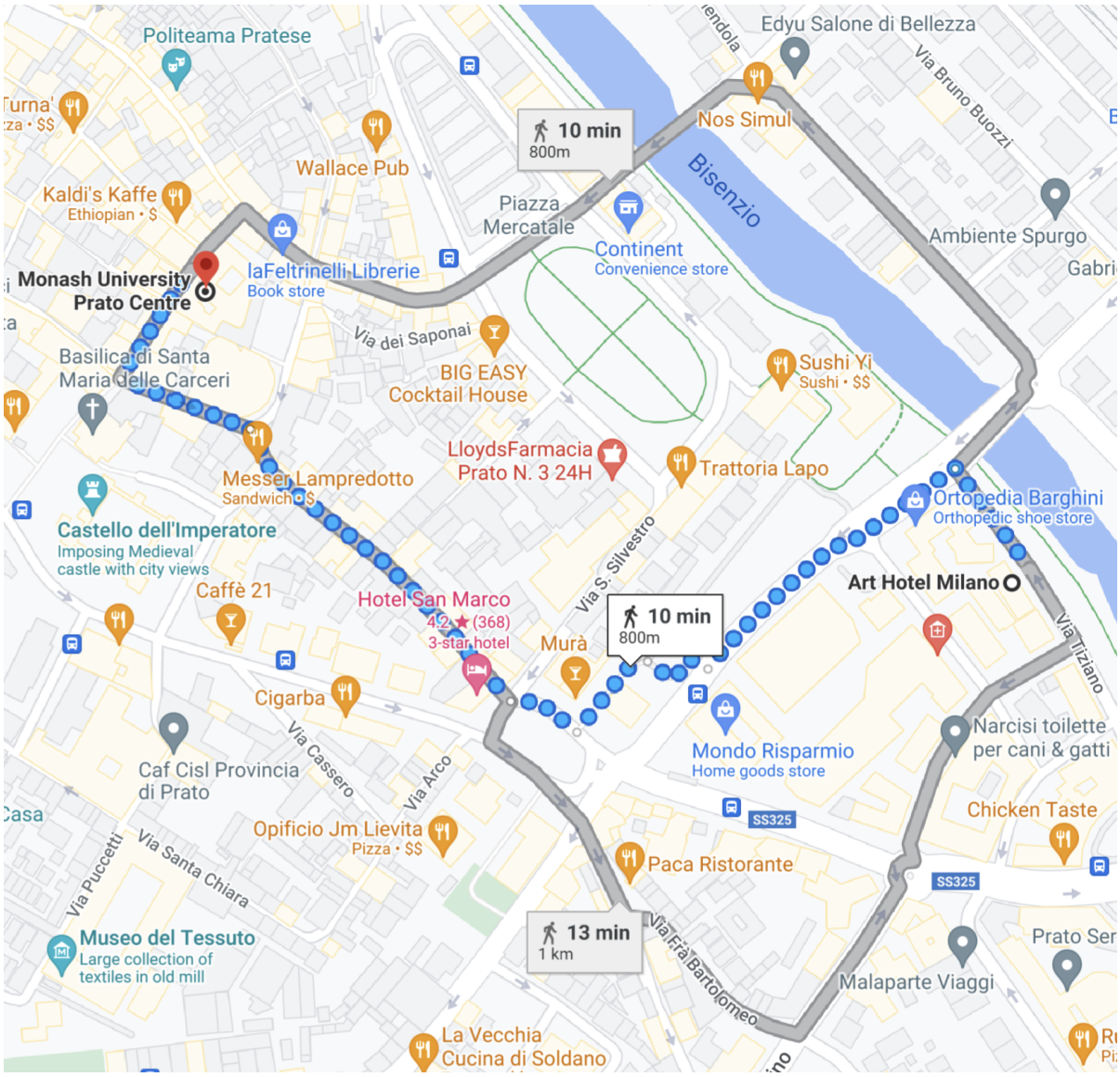

Figure 1: Monash Prato Centre and Art Hotel Milano

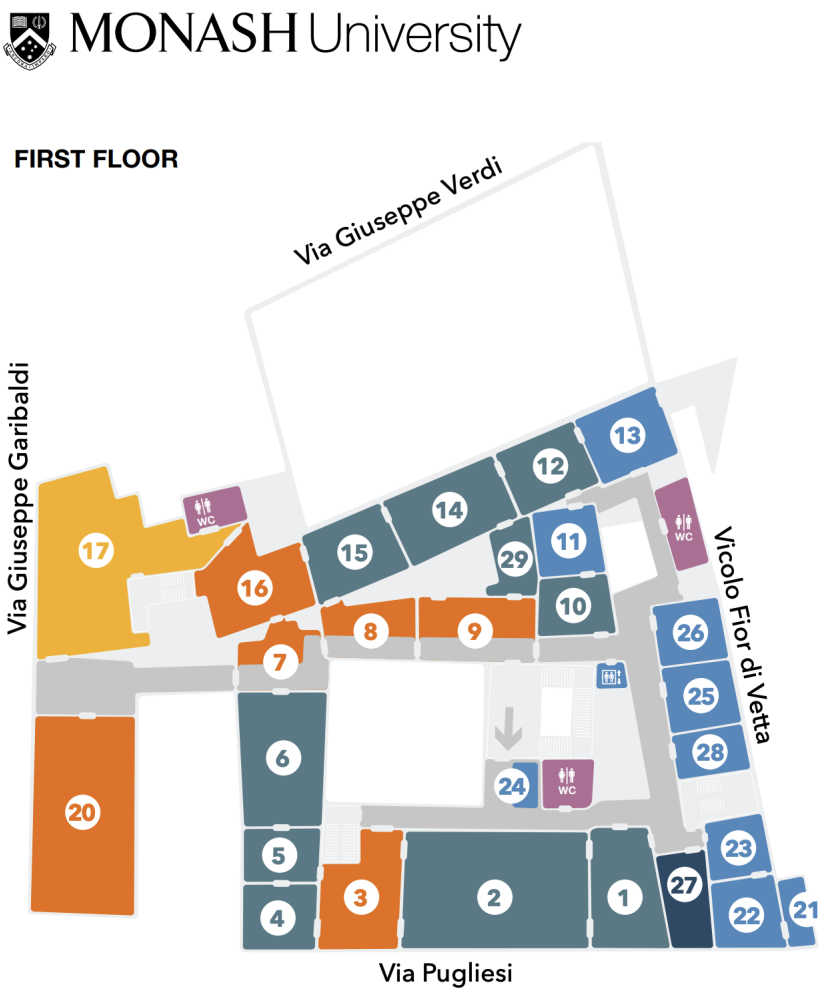

Figure 2: Sala Veneziana (Room 6) and two breakout rooms (Rooms 4 and 5)

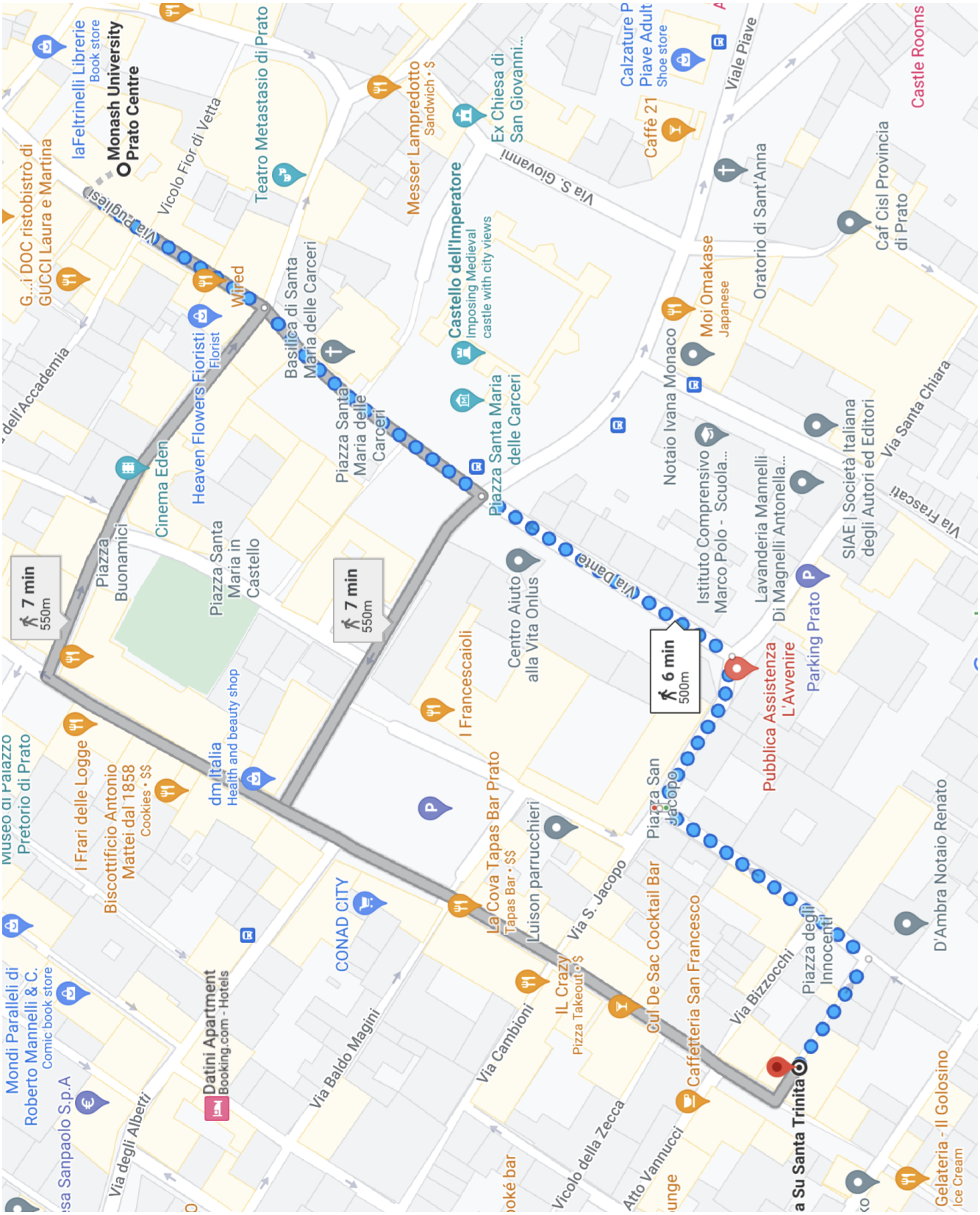

Figure 3: Monash Prato Centre and Osteria Su Santa Trinita

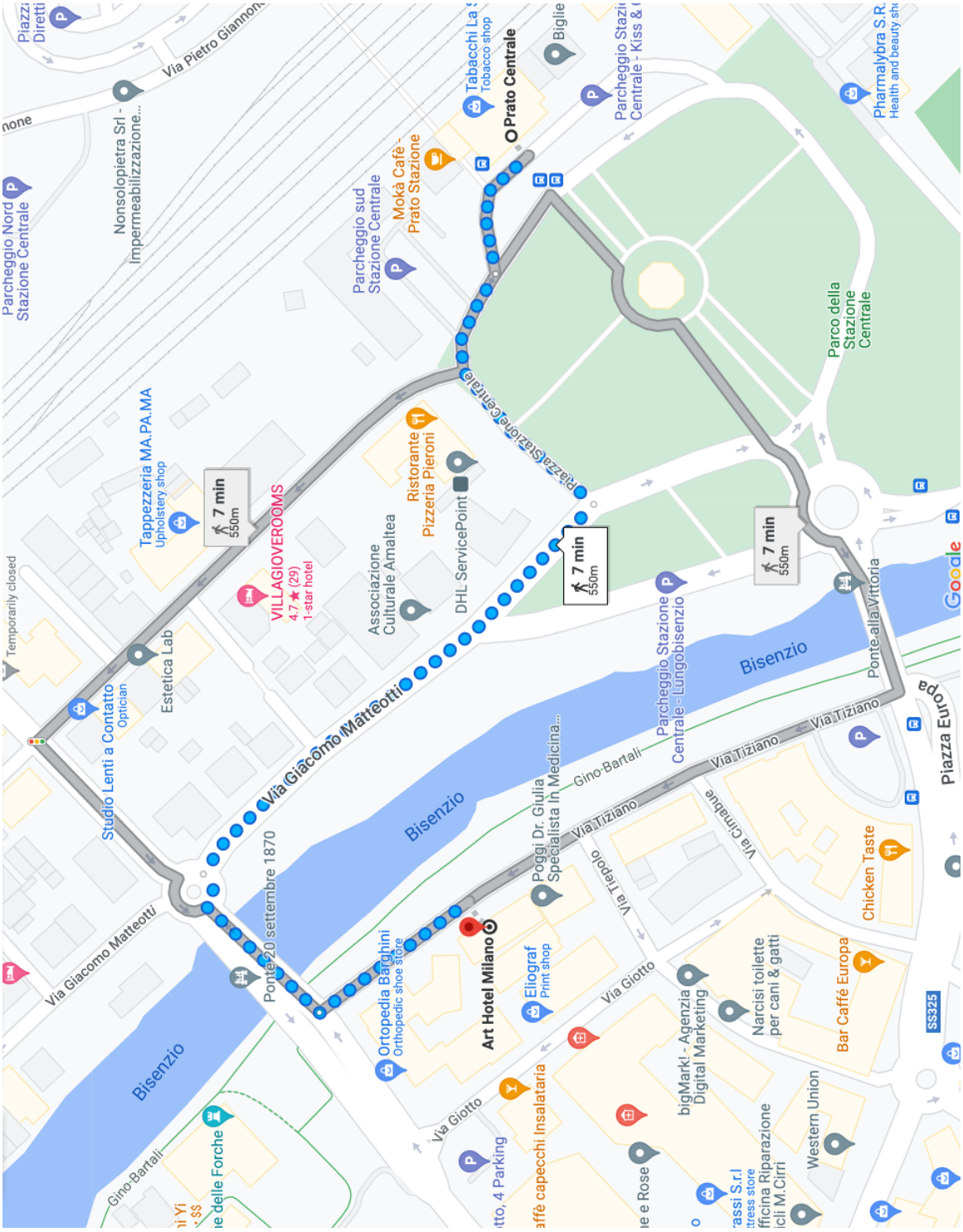

Figure 4: Prato Centrale train station and Art Hotel Milano