Riemman normal coordinates

There is nothing new in this notebook, it serves only to display the transformation from one set of Riemann normal coordinates x^a to another y^a.

```
::KeepHistory(false).
::PostDefaultRules( @@collect_terms!(%), @@sumflatten!(%) ).

{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,t,u#,v#}::Indices.

\nabla_{#}::PartialDerivative.

R_{a b c d}::RiemannTensor.

# --- imported from metric.lib ------------------------------------------------------------

metric:="import metric.lib metric":
@run(metric){"/Users/leo/local/sh/cdbfile"}:

# --- imported from geodesic-bvp.lib ------------------------------------------------------

y05:="import geodesic-bvp.lib y05Riemann":
@run(y05){"/Users/leo/local/sh/cdbfile"}:

# ============================================================================================
#   in an effort to tidy up the equations we will expand each term0* in powers of s and then Dx^a
# ============================================================================================

poly:=@(y05):

Dx^{a}::Weight(label=Dxterms,value=1).

term00:=@(poly): @keep_weight!(term00){Dxterms}{0}:
term01:=@(poly): @keep_weight!(term01){Dxterms}{1}:
term02:=@(poly): @keep_weight!(term02){Dxterms}{2}:
term03:=@(poly): @keep_weight!(term03){Dxterms}{3}:
term04:=@(poly): @keep_weight!(term04){Dxterms}{4}:
term05:=@(poly): @keep_weight!(term05){Dxterms}{5}:

{Dx^{a},x^{a},R^{a}_{b c d}}::SortOrder.

@prodsort!(term01): @rename_dummies!(%): @factor_out!!(%){Dx^{a}}:
```

```
@prodsort!(term02): @rename_dummies!(%): @factor_out!!(%){Dx^{a}}:
@prodsort!(term03): @rename_dummies!(%): @factor_out!!(%){Dx^{a}}:
@prodsort!(term04): @rename_dummies!(%): @factor_out!!(%){Dx^{a}}:
@prodsort!(term05): @rename_dummies!(%): @factor_out!!(%){Dx^{a}}:

y05:=@(term00) + @(term01) + @(term02) + @(term03) + @(term04) + @(term05):

@substitute!(y05)(Dx^{a} -> \Delta{x}^{a}); "y05.del"
```

```
# ================================================================================================
#   transforming from old to new RNC coordinates
# ================================================================================================

@print["\Btag{01}y^a="~@(y05)~"+\BigO{\eps^6}\Etag{01}"];
```

Let P and Q be two points with RNC coordinates x^a and $x^a + \Delta x^a$ relative to a third point O. At P we can construct a new set of RNC coordinates, y^a. We will rotate the y^a frame so that the x^a and y^a coordinate axes are aligned. Then the RNC coordinates of Q relative to P are

$$
\begin{aligned}
y^a = \Bigg(& \Delta x^a + \Delta x^b \Delta x^c \left(\frac{1}{3} x^d R^a{}_{bdc} + \frac{1}{12} x^d x^e \nabla_b R^a{}_{dec} + \frac{1}{6} x^d x^e \nabla_d R^a{}_{bec} + \frac{1}{24} x^d x^e \nabla^a R_{dbec} + \frac{4}{45} x^d x^e x^f R^a{}_{dge} R_{gbfc} - \frac{2}{45} x^d x^e x^f R^a{}_{bgd} R_{gefc} - \frac{1}{45} x^d x^e x^f R^a{}_{dgb} R_{gefc} \right. \\
& + \frac{1}{20} x^d x^e x^f \nabla_{db} R^a{}_{efc} + \frac{1}{20} x^d x^e x^f \nabla_{de} R^a{}_{bfc} + \frac{1}{45} x^d x^e x^f R^a{}_{gdb} R_{gefc} + \frac{1}{40} x^d x^e x^f \nabla^a{}_d R_{ebfc} + \frac{2}{45} x^d x^e x^f x^g R_{hbdc} \nabla_e R^a{}_{fhg} + \frac{1}{60} x^d x^e x^f x^g R^a{}_{dhe} \nabla_b R_{hfgc} \\
& + \frac{2}{45} x^d x^e x^f x^g R^a{}_{dhe} \nabla_f R_{hbgc} - \frac{1}{45} x^d x^e x^f x^g R^a{}_{bhd} \nabla_e R_{hfgc} - \frac{1}{90} x^d x^e x^f x^g R^a{}_{dhb} \nabla_e R_{hfgc} - \frac{1}{90} x^d x^e x^f x^g R_{hdeb} \nabla_c R^a{}_{fhg} - \frac{1}{45} x^d x^e x^f x^g R_{hdeb} \nabla_f R^a{}_{chg} \\
& - \frac{1}{90} x^d x^e x^f x^g R_{hdeb} \nabla_f R^a{}_{ghc} + \frac{1}{60} x^d x^e x^f x^g \nabla_{deb} R^a{}_{fgc} + \frac{1}{90} x^d x^e x^f x^g \nabla_{def} R^a{}_{bgc} + \frac{1}{72} x^d x^e x^f x^g R^a{}_{dhe} \nabla_h R_{fbgc} + \frac{1}{90} x^d x^e x^f x^g R^a{}_{hdb} \nabla_e R_{hfgc} \\
& \left. - \frac{1}{90} x^d x^e x^f x^g R_{hdeb} \nabla^a R_{hfgc} + \frac{1}{90} x^d x^e x^f x^g R_{hdeb} \nabla_f R^a{}_{hgc} + \frac{1}{120} x^d x^e x^f x^g \nabla^a{}_{de} R_{fbgc} \right) + \Delta x^b \Delta x^c \Delta x^d \left(\frac{1}{12} x^e \nabla_b R^a{}_{ced} + \frac{1}{45} x^e x^f R^a{}_{egb} R_{gcfd} \right. \\
& + \frac{4}{45} x^e x^f R^a{}_{bge} R_{gcfd} - \frac{1}{45} x^e x^f R^a{}_{bgc} R_{gefd} + \frac{1}{20} x^e x^f \nabla_{eb} R^a{}_{cfd} + \frac{1}{60} x^e x^f \nabla_{bc} R^a{}_{efd} + \frac{7}{45} x^e x^f R^a{}_{geb} R_{gcfd} + \frac{1}{120} x^e x^f \nabla^a{}_b R_{ecfd} + \frac{1}{54} x^e x^f x^g R_{hbec} \nabla_d R^a{}_{fhg} \\
& + \frac{1}{27} x^e x^f x^g R_{hbec} \nabla_h R^a{}_{fgd} + \frac{1}{27} x^e x^f x^g R_{hbec} \nabla_f R^a{}_{dhg} + \frac{2}{27} x^e x^f x^g R_{hbec} \nabla_f R^a{}_{hgd} - \frac{1}{108} x^e x^f x^g R_{hbec} \nabla^a R_{hfgd} + \frac{1}{54} x^e x^f x^g R^a{}_{bhe} \nabla_c R_{hfgd} \\
& \left. + \frac{1}{27} x^e x^f x^g R^a{}_{bhe} \nabla_f R_{hcgd} + \frac{1}{108} x^e x^f x^g R^a{}_{bhe} \nabla_h R_{fcgd} + \frac{1}{27} x^e x^f x^g R^a{}_{heb} \nabla_c R_{hfgd} + \frac{2}{27} x^e x^f x^g R^a{}_{heb} \nabla_f R_{hcgd} + \frac{1}{54} x^e x^f x^g R^a{}_{heb} \nabla_h R_{fcgd} \right) \\
& + \Delta x^b \Delta x^c \Delta x^d \Delta x^e \left(\frac{1}{54} x^f R^a{}_{bgc} R_{gdfe} + \frac{1}{216} x^f x^g R^a{}_{bhc} \nabla_d R_{hfge} + \frac{1}{108} x^f x^g R^a{}_{bhc} \nabla_f R_{hdge} + \frac{1}{432} x^f x^g R^a{}_{bhc} \nabla_h R_{fdge} + \frac{1}{216} x^f x^g R_{hbfc} \nabla_d R^a{}_{ghe} \right. \\
& - \frac{5}{216} x^f x^g R_{hbfc} \nabla_d R^a{}_{hge} + \frac{1}{108} x^f x^g R_{hbfc} \nabla_g R^a{}_{dhe} + \frac{7}{108} x^f x^g R_{hbfc} \nabla_h R^a{}_{dge} + \frac{1}{216} x^f x^g R_{hbfc} \nabla^a R_{hdge} + \frac{1}{36} x^f x^g R_{hbfc} \nabla_d R^a{}_{ehg} + \frac{1}{72} x^f x^g R^a{}_{bhf} \nabla_c R_{hdge} \\
& \left. + \frac{1}{24} x^f x^g R^a{}_{hfb} \nabla_c R_{hdge} \right) + \Delta x^b \Delta x^c \Delta x^d \Delta x^e \Delta x^f \left(\frac{1}{180} x^g R^a{}_{bhc} \nabla_d R_{hegf} + \frac{1}{120} x^g R_{hbgc} \nabla_d R^a{}_{ehf} \right) \Bigg) + \mathcal{O}\left(\epsilon^6\right)
\end{aligned}
$$

```
# ================================================================================================
#   the metric at P, the origin of the new RNC coordinates
# ================================================================================================
```

At P the coordinate axes are aligned. Thus we have $\delta^a_b = \partial x^a/\partial y^b$ at P and thus $g_{ab}(x) = g_{ab}(y)$ at P. The metric at the point P in the RNC y^a frame is given by

```
@print["g_{ab}(P)="~@(metric)~"+\BigO{\eps^6}"];
```

$$g_{ab}\left(P\right) = \left(g_{ab} - \frac{1}{3}\,R_{acbd}x^c x^d - \frac{1}{6}\,\nabla_c R_{adbe}x^c x^d x^e + \frac{2}{45}\,R_{acde}R_{bfdg}x^c x^e x^f x^g - \frac{1}{20}\,\nabla_{cd}R_{aebf}x^c x^d x^e x^f + \frac{1}{45}\,R_{acde}\nabla_f R_{bgdh}x^c x^e x^f x^g x^h + \frac{1}{45}\,R_{bcde}\nabla_f R_{agdh}x^c x^e x^f x^g x^h - \frac{1}{90}\,\nabla_{cde}R_{afbg}x^c x^d x^e x^f x^g\right) + \mathcal{O}\left(\epsilon^6\right)$$

```
# ================================================================================================
#   export y05 for use by truncate.cdbp
# ================================================================================================

com:="open geodesic-rnc.lib":
@run(com){"/Users/leo/local/sh/cdbfile"}:

com:="export geodesic-rnc.lib y05.del":
@run(com){"/Users/leo/local/sh/cdbfile"}:
```