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::KeepHistory(false).
::PostDefaultRules( @@collect_terms!(%), @@sumflatten!(%) ).

{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,t,u#,v#}::Indices.

\nabla{#}::PartialDerivative.
\partial{#}::PartialDerivative.

{g_{a b},x^{a},\Delta{x}^{a},R_{a b c d},
 \nabla_{a}{R_{p q r s}},\nabla_{a b}{R_{p q r s}},\nabla_{a b c}{R_{p q r s}},
 \partial_{a}{R_{p q r s}},\partial_{a b}{R_{p q r s}},\partial_{a b c}{R_{p q r s}}>::SortOrder.

# === metric =====

metric:="import metric.lib metric":
@run(metric){"/Users/leo/local/sh/cdbfile"}:

poly:=@(metric):

x^{a}::Weight(label=myterms,value=1).
\Delta{x}^{a}::Weight(label=myterms,value=1).

term00:=@(poly): @keep_weight!(term00){myterms}{0}:
term01:=@(poly): @keep_weight!(term01){myterms}{1}:
term02:=@(poly): @keep_weight!(term02){myterms}{2}:
term03:=@(poly): @keep_weight!(term03){myterms}{3}:

metric:=@(term00) + @(term01) + @(term02) + @(term03):
@prodsort!(%): @rename_dummies!(%):
@print["\Btag{01a}g_{ab}(x)=\killL"~@(metric)~"\killR+\Big0{\eps^4}\Etag{01a}"];

@substitute!(metric)(\nabla_{a}{R_{b c d e}} -> \partial_{a}{R_{b c d e}}):
@print["\Btag{01b}g_{ab}(x)=\killL"~@(metric)~"\killR+\Big0{\eps^4}\Etag{01b}"];

```

$$g_{ab}(x) = g_{ab} - \frac{1}{3} x^c x^d R_{acbd} - \frac{1}{6} x^c x^d x^e \nabla_c R_{adbe} + \mathcal{O}(\epsilon^4)$$

$$g_{ab}(x) = g_{ab} - \frac{1}{3} x^c x^d R_{acbd} - \frac{1}{6} x^c x^d x^e \partial_c R_{adbe} + \mathcal{O}(\epsilon^4)$$

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# === inverse metric =====

metric:="import metric-inv.lib metric":
@run(metric){"/Users/leo/local/sh/cdbfile"}:

poly:=@(metric):

x^{a}::Weight(label=myterms,value=1).
\Delta{x}^{a}::Weight(label=myterms,value=1).

term00:=@(poly): @keep_weight!(term00){myterms}{0}:
term01:=@(poly): @keep_weight!(term01){myterms}{1}:
term02:=@(poly): @keep_weight!(term02){myterms}{2}:
term03:=@(poly): @keep_weight!(term03){myterms}{3}:

metric:=@(term00) + @(term01) + @(term02) + @(term03):
@prodsort!(%): @rename_dummies!(%):
@print["\Btag{02a}g^{ab}(x)=\killL"~@(metric)~"\killR+\Big0{\eps^4}\Etag{02a}"];

@substitute!(metric)(\nabla_{a}\{R^{b}_{c}{}^{d}_{e}\} -> \partial_{a}\{R^{b}_{c}{}^{d}_{e}\}):
@print["\Btag{02b}g^{ab}(x)=\killL"~@(metric)~"\killR+\Big0{\eps^4}\Etag{02b}"];

```

$$g^{ab}(x) = g^{ab} + \frac{1}{3} x^c x^d R^a{}_c{}^b{}_d + \frac{1}{6} x^c x^d x^e \nabla_c R^a{}_d{}^b{}_e + \mathcal{O}(\epsilon^4)$$

$$g^{ab}(x) = g^{ab} + \frac{1}{3} x^c x^d R^a{}_c{}^b{}_d + \frac{1}{6} x^c x^d x^e \partial_c R^a{}_d{}^b{}_e + \mathcal{O}(\epsilon^4)$$

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# === generalised gamma in rnc =====

genx2rnc:="import gen-gamma.lib genx2rnc":
@run(genx2rnc){"/Users/leo/local/sh/cdbfile"}:

poly:=@(genx2rnc):

x^{a}::Weight(label=myterms,value=1).
\Delta{x}^{a}::Weight(label=myterms,value=1).

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term00:=@(poly): @keep_weight!(term00){myterms}{0}:
term01:=@(poly): @keep_weight!(term01){myterms}{1}:
term02:=@(poly): @keep_weight!(term02){myterms}{2}:
term03:=@(poly): @keep_weight!(term03){myterms}{3}:

tmp:= 6 @(term03):
@factor_out!!(tmp)(\Delta{x}^{a}):
term03:=@(tmp):

genx2rnc:=@(term00) + @(term01) + @(term02) + (1/6) @(term03):
@print["\Btag{03}y^a(x)=\"~@(genx2rnc)~"+\BigO{\eps^4}\Etag{03}"];

```

$$y^a(x) = \left(\Delta x^a + \frac{1}{2} \Delta x^b \Delta x^c \Gamma^a_{bc} + \Delta x^b \Delta x^c \Delta x^d \left(\frac{1}{6} \Gamma^a_{be} \Gamma^e_{cd} + \frac{1}{6} \partial_b \Gamma^a_{cd} \right) \right) + \mathcal{O}(\epsilon^4)$$

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genGamma02:="import gen-gamma.lib genGamma02":
@run(genGamma02){"/Users/leo/local/sh/cdbfile"}:

poly:=@(genGamma02):

x^{a}::Weight(label=myterms,value=1).
\Delta{x}^{a}::Weight(label=myterms,value=1).

term00:=@(poly): @keep_weight!(term00){myterms}{0}:
term01:=@(poly): @keep_weight!(term01){myterms}{1}:
term02:=@(poly): @keep_weight!(term02){myterms}{2}:

tmp:= 12 @(term02):
@factor_out!!(tmp)(x^{a}):
term02:=@(tmp):

genGamma02:=@(term00) + @(term01) + (1/12) @(term02):
@print["\Btag{04a}\genGammaA=\killL"~@(genGamma02)~"\killR+\BigO{\eps^4}\Etag{04a}"];

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genGamma03:="import gen-gamma.lib genGamma03":
@run(genGamma03){"/Users/leo/local/sh/cdbfile"}:

poly:=@(genGamma03):

x^{a}::Weight(label=myterms,value=1).
\Delta{x}^{a}::Weight(label=myterms,value=1).

term00:=@(poly): @keep_weight!(term00){myterms}{0}:
term01:=@(poly): @keep_weight!(term01){myterms}{1}:

genGamma03:=@(term00) + @(term01):
@print["\Btag{04b}\genGammaB=\killL"~@(genGamma03)~"\killR+\BigO{\eps^4}\Etag{04b}"];

@print["\Btag{04c}\genGammaC=\BigO{\eps^4}\Etag{04c}"];

```

$$\Gamma_{bc}^a(x) = \frac{2}{3} x^d R^a{}_{bdc} + \frac{1}{12} x^d x^e (2 \nabla_b R^a{}_{dec} + 4 \nabla_d R^a{}_{bec} + \nabla^a R_{dbec}) + \mathcal{O}(\epsilon^4)$$

$$\Gamma_{bcd}^a(x) = \frac{1}{2} x^e \nabla_b R^a{}_{ced} + \mathcal{O}(\epsilon^4)$$

$$\Gamma_{bcde}^a(x) = \mathcal{O}(\epsilon^4)$$

```

# === geodesic ivp =====

ivp:="import geodesic-ivp.lib sol":
@run(ivp){"/Users/leo/local/sh/cdbfile"}:

poly:=@(ivp):
@distribute!(%):

x^{a}::Weight(label=myterms,value=1).
\thetodot{x}^{a}::Weight(label=myterms,value=1).

term00:=@(poly): @keep_weight!(term00){myterms}{0}:

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term01:=@(poly): @keep_weight!(term01){myterms}{1}:
term02:=@(poly): @keep_weight!(term02){myterms}{2}:
term03:=@(poly): @keep_weight!(term03){myterms}{3}:
term04:=@(poly): @keep_weight!(term04){myterms}{4}:

ivp:=@(term00) + @(term01) + @(term02) + @(term03) + @(term04):
@print["\Btag{06}x^{a}(s)="\~@(ivp)~"+\Big0{\eps^4}\Etag{06}"];

poly:=@(ivp):
@distribute!(%):
@substitute!(%)(s -> 1):

\thetdotx^{a}::Weight(label=sterms,value=1).

term00:=@(poly): @keep_weight!(term00){sterms}{0}:
term01:=@(poly): @keep_weight!(term01){sterms}{1}:
term02:=@(poly): @keep_weight!(term02){sterms}{2}:
term03:=@(poly): @keep_weight!(term03){sterms}{3}:

tmp:= -24 @(term02):
@factor_out!!(tmp)(\thetdotx^{a}):
term02:=@(tmp):

ivp:=@(term00) + s @(term01) - (1/24) s**2 @(term02) + s**3 @(term03):
@print["\Btag{07}x^{a}(s)=\killL"~@(ivp)~"\killR+\Big0{\eps^4}\Etag{07}"];

```

$$x^a(s) = \left(x^a + s\dot{x}^a - \frac{1}{3} s^2 \dot{x}^b \dot{x}^c x^d R^a{}_{bdc} - \frac{1}{12} s^2 \dot{x}^b \dot{x}^c x^d x^e \nabla_b R^a{}_{dec} - \frac{1}{6} s^2 \dot{x}^b \dot{x}^c x^d x^e \nabla_d R^a{}_{bec} - \frac{1}{24} s^2 \dot{x}^b \dot{x}^c x^d x^e \nabla^a R_{dbec} - \frac{1}{12} s^3 \dot{x}^b \dot{x}^c \dot{x}^d x^e \nabla_b R^a{}_{ced} \right) + \mathcal{O}(\epsilon^4)$$

$$x^a(s) = x^a + s\dot{x}^a - \frac{1}{24} s^2 \dot{x}^b \dot{x}^c \left(8 x^d R^a{}_{bdc} + 2 x^d x^e \nabla_b R^a{}_{dec} + 4 x^d x^e \nabla_d R^a{}_{bec} + x^d x^e \nabla^a R_{dbec} \right) - \frac{1}{12} s^3 \dot{x}^b \dot{x}^c \dot{x}^d x^e \nabla_b R^a{}_{ced} + \mathcal{O}(\epsilon^4)$$

```

# === geodesic bvp =====
@print["\Btag{08a}x^a(s) = x^a + s x_1^a + s^2 x_2^a + s^3 x_3^a + \Big0{\eps^4}\Btag{08a}"];
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x1:="import geodesic-bvp.lib term01":
@run(x1){"/Users/leo/local/sh/cdbfile"}:

poly:=@(x1):
@distributed!(%):

x^{a}::Weight(label=myterms,value=1).
\Delta{x}^{a}::Weight(label=myterms,value=1).

term00:=@(poly): @keep_weight!(term00){myterms}{0}:
term01:=@(poly): @keep_weight!(term01){myterms}{1}:
term02:=@(poly): @keep_weight!(term02){myterms}{2}:
term03:=@(poly): @keep_weight!(term03){myterms}{3}:
term04:=@(poly): @keep_weight!(term04){myterms}{4}:

x1:=@(term00) + @(term01) + @(term02) + @(term03) + @(term04):
@prodsort!(%): @rename_dummies!(%):
@print["\Btag{08b}x_1^a(s)=\killL"~@(x1)~"\killR\Etag{08b}"];

# -----

x2:="import geodesic-bvp.lib term02":
@run(x2){"/Users/leo/local/sh/cdbfile"}:

poly:=@(x2):
@distributed!(%):

x^{a}::Weight(label=myterms,value=1).
\Delta{x}^{a}::Weight(label=myterms,value=1).

term00:=@(poly): @keep_weight!(term00){myterms}{0}:
term01:=@(poly): @keep_weight!(term01){myterms}{1}:
term02:=@(poly): @keep_weight!(term02){myterms}{2}:
term03:=@(poly): @keep_weight!(term03){myterms}{3}:
term04:=@(poly): @keep_weight!(term04){myterms}{4}:

x2:=@(term00) + @(term01) + @(term02) + @(term03) + @(term04):
@prodsort!(%): @rename_dummies!(%):
@print["\Btag{08c}x_2^a(s)=\killL"~@(x2)~"\killR\Etag{08c}"];

```

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x3:="import geodesic-bvp.lib term03":
@run(x3){"/Users/leo/local/sh/cdbfile"}:

poly:=@(x3):
@distributed!(%):

x^{a}::Weight(label=myterms,value=1).
\Delta{x}^{a}::Weight(label=myterms,value=1).

term00:=@(poly): @keep_weight!(term00){myterms}{0}:
term01:=@(poly): @keep_weight!(term01){myterms}{1}:
term02:=@(poly): @keep_weight!(term02){myterms}{2}:
term03:=@(poly): @keep_weight!(term03){myterms}{3}:
term04:=@(poly): @keep_weight!(term04){myterms}{4}:

x3:=@(term00) + @(term01) + @(term02) + @(term03) + @(term04):
@prodsort!(%): @rename_dummies!(%):
@print["\Btag{08d}x_3^a(s)="\~@(x3)\~"\Etag{08d}"];

```

$$x^a(s) = x^a + sx_1^a + s^2x_2^a + s^3x_3^a + \mathcal{O}(\epsilon^4)$$

$$x_1^a(s) = \Delta x^a + \frac{1}{3}x^b\Delta x^c\Delta x^dR^a{}_{cbd} + \frac{1}{12}x^bx^c\Delta x^d\Delta x^e\nabla_dR^a{}_{bce} + \frac{1}{6}x^bx^c\Delta x^d\Delta x^e\nabla_bR^a{}_{dce} + \frac{1}{24}x^bx^c\Delta x^d\Delta x^e\nabla^aR_{bdce} + \frac{1}{12}x^b\Delta x^c\Delta x^d\Delta x^e\nabla_cR^a{}_{dbe}$$

$$x_2^a(s) = -\frac{1}{3}x^b\Delta x^c\Delta x^dR^a{}_{cbd} - \frac{1}{12}x^bx^c\Delta x^d\Delta x^e\nabla_dR^a{}_{bce} - \frac{1}{6}x^bx^c\Delta x^d\Delta x^e\nabla_bR^a{}_{dce} - \frac{1}{24}x^bx^c\Delta x^d\Delta x^e\nabla^aR_{bdce}$$

$$x_3^a(s) = \left(\frac{-1}{12}\right)x^b\Delta x^c\Delta x^d\Delta x^e\nabla_cR^a{}_{dbe}$$

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# === translated rnc =====

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y05:="import geodesic-rnc.lib y05":
@run(y05){"/Users/leo/local/sh/cdbfile"}:

poly:=(y05):
@distribute!(%):

x^{a}::Weight(label=myterms,value=1).
\Delta{x}^{a}::Weight(label=myterms,value=1).

term00:=(poly): @keep_weight!(term00){myterms}{0}:
term01:=(poly): @keep_weight!(term01){myterms}{1}:
term02:=(poly): @keep_weight!(term02){myterms}{2}:
term03:=(poly): @keep_weight!(term03){myterms}{3}:
term04:=(poly): @keep_weight!(term04){myterms}{4}:

y05:=(term00) + @(term01) + @(term02) + @(term03) + @(term04):
@prodsort!(%): @rename_dummies!(%):
@print["\Btag{12}y^a(s)="\~@(y05)\~"+\Big0{\eps^4}\Etag{12}"];

```

$$y^a(s) = \left(\Delta x^a + \frac{1}{3} x^b \Delta x^c \Delta x^d R^a{}_{cbd} + \frac{1}{12} x^b x^c \Delta x^d \Delta x^e \nabla_d R^a{}_{bce} + \frac{1}{6} x^b x^c \Delta x^d \Delta x^e \nabla_b R^a{}_{dce} + \frac{1}{24} x^b x^c \Delta x^d \Delta x^e \nabla^a R_{bdce} + \frac{1}{12} x^b \Delta x^c \Delta x^d \Delta x^e \nabla_c R^a{}_{dbe} \right) + \mathcal{O}(\epsilon^4)$$

```

# === geodesic lsq =====

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```

lsq:="import geodesic-lsq.lib lsq":
@run(lsq){"/Users/leo/local/sh/cdbfile"}:

```

```

poly:=(lsq):

```

```

x^{a}::Weight(label=myterms,value=1).
\Delta{x}^{a}::Weight(label=myterms,value=1).

```

```

term00:=(poly): @keep_weight!(term00){myterms}{0}:
term01:=(poly): @keep_weight!(term01){myterms}{1}:
term02:=(poly): @keep_weight!(term02){myterms}{2}:
term03:=(poly): @keep_weight!(term03){myterms}{3}:
term04:=(poly): @keep_weight!(term04){myterms}{4}:
term05:=(poly): @keep_weight!(term05){myterms}{5}:

```



```

lsq:=@(term00) + @(term01) + @(term02) + @(term03) + @(term04) + @(term05):
@prodsort!(%): @rename_dummies!(%):
@print["\Btag{13}L^2_{PQ}=\killL"~"@{lsq}~"\killR+\Big0{\eps^4}\Etag{13}"];

```

$$L_{PQ}^2 = g_{ab} \Delta x^a \Delta x^b - \frac{1}{3} x^a x^b \Delta x^c \Delta x^d R_{acbd} - \frac{1}{12} x^a x^b \Delta x^c \Delta x^d \Delta x^e \nabla_c R_{adbe} - \frac{1}{6} x^a x^b x^c \Delta x^d \Delta x^e \nabla_a R_{bdce} + \mathcal{O}(\epsilon^4)$$