

**Zhang, X., and King, M.L.** (2005), “Influence Diagnostics in Generalized Autoregressive Conditional Heteroscedasticity Processes,” *Journal of Business and Economic Statistics*, 23, 118-129.

Unfortunately we used an incorrect form for  $\partial^2 L(\theta|\omega)/\partial\theta\partial\theta'$  in deriving the influence diagnostics for the GARCH( $p, q$ ) model under each of the three perturbation schemes discussed in Sections 4.1, 4.2.1 and 4.2.2. The correct form of  $\partial^2 L(\theta|\omega)/\partial\theta\partial\theta'$  is

$$\frac{\partial^2 L(\theta|\omega)}{\partial\theta\partial\theta'} = \sum_{t=1}^n \left[ \frac{1}{2h_t^2} - \frac{z_t^2}{h_t^3} \right] \frac{\partial h_t}{\partial\theta} \frac{\partial h_t}{\partial\theta'} + \left[ \frac{z_t^2}{2h_t^2} - \frac{1}{2h_t} \right] \frac{\partial^2 h_t}{\partial\theta\partial\theta'}.$$

This error did not affect any of our numerical results because we used the numerical derivatives.

For the data perturbation given in Section 4.1, the correct form of  $\partial^2 L(\theta|\omega)/\partial\omega\partial\omega'$  is

$$\begin{aligned} \frac{\partial^2 L(\theta|\omega)}{\partial\omega\partial\omega'} &= \sum_{t=1}^n \left[ \frac{1}{2h_t^2} - \frac{z_t^2}{h_t^3} \right] \frac{\partial h_t}{\partial\omega} \frac{\partial h_t}{\partial\omega'} + \sum_{t=1}^n \left[ \frac{z_t^2}{2h_t^2} - \frac{1}{2h_t} \right] \frac{\partial^2 h_t}{\partial\omega\partial\omega'} \\ &+ \sum_{t=1}^n \frac{z_t}{h_t^2} \left( \frac{\partial\omega_t}{\partial\omega} \frac{\partial h_t}{\partial\omega'} + \frac{\partial h_t}{\partial\omega} \frac{\partial\omega_t}{\partial\omega'} \right) - \sum_{t=1}^n \frac{1}{h_t} \frac{\partial\omega_t}{\partial\omega} \frac{\partial\omega_t}{\partial\omega'}. \end{aligned}$$

For the additive model perturbation given in Section 4.2.2, the correct form of  $\partial^2 L(\theta|\omega)/\partial\omega\partial\omega'$  is

$$\begin{aligned} \frac{\partial^2 L(\theta|\omega)}{\partial\omega\partial\omega'} &= \sum_{t=1}^n \left[ \frac{1}{2h_t^2} - \frac{z_t^2}{h_t^3} \right] \frac{\partial h_t}{\partial\omega} \frac{\partial h_t}{\partial\omega'} + \sum_{t=1}^n \left[ \frac{z_t^2}{2h_t^2} - \frac{1}{2h_t} \right] \frac{\partial^2 h_t}{\partial\omega\partial\omega'} \\ &+ \sum_{t=1}^n \frac{z_t}{2h_t^{3/2}} \left( \frac{\partial h_t}{\partial\omega} \frac{\partial\omega_t}{\partial\omega'} + \frac{\partial\omega_t}{\partial\omega} \frac{\partial h_t}{\partial\omega'} \right) - I. \end{aligned}$$

As these two errors did affect our numerical results, we have recalculated the Monte Carlo simulations for the data perturbation and additive model perturbation and updated the last four columns of Table 1 to Table 3, respectively. We have also recalculated the diagnostics in Section 5 for the above-mentioned two

perturbation schemes. The updated results are given in the last four columns of Table 4. The empirical analysis given in Section 5 does not include the 167th observation, which was identified influential in the updated calculation. We are grateful to Luiz K. Hotta for drawing these errors to our attention.

Table 1. Estimated Percentiles of the Maximum Curvature and Slope

Sample size	Percentiles (%)	Model perturbation				Data perturbation	
		Innovative		Additive			
		Slope	Curvature	Slope	Curvature	Slope	Curvature
500	99.0	1285.76	121.09	2255.39	152.60	2736.21	158.03
	97.5	1222.43	93.13	2231.50	135.76	2579.85	127.53
	95.0	1167.14	77.45	2211.87	119.91	2499.16	103.33
	90.0	1117.82	64.18	2185.64	110.26	2387.45	82.50
1000	99.0	2393.92	149.70	4448.84	217.84	5060.98	212.38
	97.5	2317.23	121.07	4409.17	181.43	4776.82	163.98
	95.0	2241.90	107.91	4378.79	163.50	4639.44	134.46
	90.0	2177.29	90.09	4345.84	151.75	4474.88	111.59
1500	99.0	3456.09	155.39	6601.16	207.25	7140.86	162.96
	97.5	3356.90	135.72	6574.91	197.19	6969.53	150.96
	95.0	3287.10	122.69	6532.34	186.80	6849.39	133.39
	90.0	3211.11	108.42	6500.36	179.38	6636.53	119.42

Table 2. Estimated Percentiles of the Largest Component in the Slope- and Curvature-Based Diagnostic Vectors

Sample size	Percentiles (%)	Model perturbation				Data perturbation	
		Innovative		Additive			
		Slope	Curvature	Slope	Curvature	Slope	Curvature
500	99.0	0.4831	0.6399	0.1988	0.3743	0.2137	0.4090
	97.5	0.4421	0.5935	0.1899	0.3454	0.2047	0.3605
	95.0	0.4050	0.5529	0.1825	0.3188	0.1955	0.3289
	90.0	0.3849	0.5019	0.1736	0.2943	0.1874	0.2969
1000	99.0	0.3728	0.5710	0.1458	0.3030	0.1652	0.2889
	97.5	0.3429	0.5010	0.1390	0.2793	0.1574	0.2698
	95.0	0.3249	0.4675	0.1345	0.2605	0.1496	0.2481
	90.0	0.3009	0.4262	0.1286	0.2368	0.1430	0.2271
1500	99.0	0.3176	0.5378	0.1222	0.2590	0.1360	0.2493
	97.5	0.2971	0.4713	0.1170	0.2389	0.1294	0.2249
	95.0	0.2810	0.4229	0.1128	0.2214	0.1247	0.2087
	90.0	0.2630	0.3877	0.1085	0.2043	0.1192	0.1934

Table 3. *Distributional Properties of the Largest Component of Significant Diagnostic Vectors Identified via Table 1*

Sample size	Percentiles	Model perturbation				Data perturbation	
		Innovative		Additive		Slope	Curvature
		Slope	Curvature	Slope	Curvature		
500	Maximum	0.5441	0.6624	0.2009	0.4198	0.2591	0.4781
	Minimum	0.2720	0.2606	0.1282	0.1629	0.1364	0.1763
	5% quantile	0.2799	0.3114	0.1321	0.1702	0.1392	0.1767
	Mean	0.3950	0.4080	0.1598	0.2522	0.1626	0.2482
	Standard deviation	0.0695	0.0852	0.0163	0.0565	0.0202	0.0614
1000	Maximum	0.5122	0.7885	0.1694	0.3458	0.1763	0.2751
	Minimum	0.1920	0.2266	0.0999	0.1282	0.0944	0.1336
	5% quantile	0.1988	0.2301	0.1030	0.1286	0.1048	0.1410
	Mean	0.2892	0.3776	0.1204	0.2002	0.1257	0.1915
	Standard deviation	0.0598	0.1228	0.0127	0.0438	0.0151	0.0345
1500	Maximum	0.3301	0.6975	0.1208	0.3175	0.1344	0.3038
	Minimum	0.1459	0.1921	0.0862	0.1192	0.0876	0.1152
	5% quantile	0.1674	0.2097	0.0877	0.1232	0.0904	0.1247
	Mean	0.2212	0.3168	0.1000	0.1773	0.1043	0.1650
	Standard deviation	0.0402	0.1035	0.0090	0.0410	0.0097	0.0326

Table 4. Summary of the Slope- and Curvature-Based Diagnostics

Mate	Serial no.	Model perturbation				Data perturbation	
		Innovative		Additive		Slope	Curvature
		Slope	Curvature	Slope	Curvature		
02/09/97	167	-0.0955	0.1351	-0.0914	-0.0450	<b>-0.1000</b>	-0.0595
20/10/97	201	-0.0076	-0.0082	0.0192	<b>0.1373</b>	0.0217	-0.0234
21/10/97	202	-0.0236	-0.0199	0.0377	<b>0.2410</b>	0.0405	0.0052
23/10/97	204	-0.0370	-0.0256	-0.0892	<b>-0.4094</b>	-0.0907	<b>-0.1801</b>
24/10/97	205	0.0071	-0.0134	-0.0610	<b>-0.1402</b>	-0.0545	<b>-0.1438</b>
27/10/97	206	<b>-0.5247</b>	<b>-0.4008</b>	<b>0.1757</b>	-0.0866	<b>0.1618</b>	<b>-0.2100</b>
26/08/98	415	-0.0009	0.0180	-0.0263	<b>-0.1356</b>	-0.0260	<b>-0.1682</b>
27/08/98	416	<b>-0.1948</b>	<b>0.2580</b>	0.0409	<b>-0.1561</b>	0.0402	0.0987
31/08/98	418	<b>-0.2104</b>	<b>-0.3320</b>	<b>0.1032</b>	<b>-0.1712</b>	0.0660	-0.1182
15/10/98	450	-0.1045	0.1741	<b>-0.1042</b>	-0.0375	-0.0795	-0.0371
28/10/99	711	-0.0946	0.0391	<b>-0.1014</b>	-0.0261	-0.0869	0.0312
04/01/00	757	-0.1519	-0.1265	0.0944	-0.0569	<b>0.0988</b>	-0.0965
16/03/00	807	-0.1283	-0.1475	<b>-0.1030</b>	0.0772	-0.0701	0.0411
12/04/00	826	-0.0043	0.0364	-0.0260	-0.1135	-0.0269	<b>-0.1796</b>
13/04/00	827	-0.0194	0.0489	-0.0486	<b>-0.1738</b>	-0.0492	<b>-0.3569</b>
14/04/00	828	<b>-0.3308</b>	0.2030	<b>0.1408</b>	-0.0231	<b>0.1326</b>	-0.0036
16/05/01	1101	-0.0942	<b>0.2293</b>	-0.0902	-0.0823	<b>-0.1064</b>	-0.1175
07/09/01	1180	-0.0335	-0.0452	-0.0127	<b>-0.2247</b>	-0.0120	-0.1000
17/09/01	1182	<b>-0.2247</b>	-0.1803	<b>0.1085</b>	-0.1222	0.0963	-0.1112
Maximum		5247.96	95.83	5695.82	241.96	5728.46	117.60
$T_0$		2774.95	115.45	5477.10	175.38	5766.51	133.91
$T_1$		0.1828	0.2197	0.0952	0.1259	0.0975	0.1327
$T_2$		0.3025	0.4448	0.1234	0.2406	0.1369	0.2280