

Gaussian Kernel GARCH Models

Xibin Zhang¹ and Maxwell L. King

Department of Econometrics and Business Statistics, Monash University, Australia

The following supplementary tables summarize detailed results obtained in Section 5. These tables present parameter estimates, their simulation inefficiency factor (SIF) values, log marginal likelihood and averaged log likelihood for each asset return series under the Gaussian kernel GARCH(1,1) and skewed Student t GARCH(1,1) models.

¹Address: 900 Dandenong Road, Caulfield East, Victoria 3145, Australia. Telephone: +61-3-99032130. Fax: +61-3-99032007. Email: xibin.zhang@monash.edu.

Table 5.1 Results from Bayesian estimation of the Gaussian kernel asymmetric GARCH(1,1) models of stock index returns. SIF values are given in parentheses. LL represents averaged log likelihood. LML stands for log marginal likelihood.

Parameter	DJIA	Nasdaq	NYSE	CAC	DAX	FTSE	AORD	HS	Nikkei
<u>Global bandwidth</u>									
σ_0^2	1.2004 (67.3)	0.9433 (9.9)	1.1224 (17.1)	0.5127 (5.4)	0.3719 (7.9)	0.4178 (5.8)	0.5349 (6.2)	1.3989 (4.9)	0.8966 (3.7)
α_1	0.0026 (66.4)	0.0049 (4.5)	0.0028 (8.6)	0.0028 (4.9)	0.0026 (36.9)	0.0045 (42.3)	0.0191 (47.2)	0.0238 (26.0)	0.0738 (44.2)
α_2	0.0897 (66.4)	0.0835 (13.3)	0.0673 (14.4)	0.1099 (15.1)	0.0881 (19.1)	0.0929 (15.1)	0.0963 (21.8)	0.0895 (24.5)	0.0826 (14.2)
β	0.9029 (68.3)	0.9052 (14.9)	0.9252 (15.1)	0.8795 (16.2)	0.9066 (20.0)	0.8993 (15.4)	0.8952 (21.0)	0.9003 (25.8)	0.9015 (24.7)
h	0.4005 (67.0)	0.2641 (7.0)	0.3067 (13.7)	0.2794 (5.0)	0.3699 (7.3)	0.3225 (6.4)	0.1410 (22.4)	0.2406 (8.6)	0.2305 (21.7)
LL	-2328.78	-2652.85	-2588.95	-2867.35	-2745.83	-2509.84	-2367.81	-2875.60	-2781.04
LML	-2334.70	-2660.02	-2596.59	-2875.54	-2754.23	-2517.56	-2376.49	-2881.85	-2787.74
<u>Localized bandwidths</u>									
σ_0^2	0.2643 (4.9)	0.6856 (3.7)	0.4446 (5.5)	0.5335 (4.2)	0.4134 (4.6)	0.4288 (7.0)	0.5612 (5.3)	1.3211 (4.1)	0.8150 (6.2)
α_1	0.0029 (5.0)	0.0046 (3.9)	0.0037 (43.8)	0.0025 (50.1)	0.0028 (4.1)	0.0036 (4.1)	0.0179 (10.4)	0.0237 (25.0)	0.0534 (25.8)
α_2	0.1055 (4.7)	0.1001 (9.5)	0.0844 (12.2)	0.1045 (12.3)	0.0834 (5.0)	0.0999 (7.3)	0.0954 (12.1)	0.0895 (15.2)	0.1011 (12.0)
β	0.8875 (4.7)	0.8901 (9.9)	0.9078 (12.4)	0.8860 (13.0)	0.9102 (5.1)	0.8909 (7.8)	0.8962 (13.0)	0.9013 (15.3)	0.8840 (12.9)
h	0.2311 (25.0)	0.2078 (19.6)	0.2079 (31.7)	0.2050 (20.5)	0.2078 (19.3)	0.2331 (18.7)	0.1566 (11.9)	0.2068 (15.6)	0.1848 (18.9)
h_ϵ	0.6371 (26.8)	0.4688 (19.3)	0.5682 (32.8)	0.2845 (25.3)	0.4883 (19.6)	0.3756 (15.6)	0.0869 (6.7)	0.3152 (18.4)	0.5907 (37.4)
LL	-2308.48	-2650.06	-2580.53	-2868.09	-2741.42	-2509.24	-2371.07	-2875.05	-2776.99
LML	-2317.66	-2657.14	-2588.32	-2875.68	-2749.22	-2516.86	-2379.79	-2881.63	-2782.82

Table 5.2 Results from Bayesian estimation of the asymmetric skewed Student t GARCH(1,1) models of stock index returns. SIF values are given in parentheses. LL represents averaged log likelihood. LML stands for log marginal likelihood.

Parameter	DJIA	Nasdaq	NYSE	CAC	DAX	FTSE	AORD	HS	Nikkei
Skew t with constant η and λ									
σ_0^2	0.3471 (4.4)	0.8112 (3.9)	0.5425 (4.8)	0.7375 (4.4)	0.6687 (4.6)	0.6250 (3.9)	0.9761 (4.7)	1.8308 (4.7)	1.0705 (4.6)
ω	0.0307 (32.7)	0.0625 (38.9)	0.0425 (32.5)	0.1001 (39.8)	0.0639 (41.6)	0.0581 (39.3)	0.0377 (38.9)	0.0460 (41.9)	0.0950 (43.2)
α_1	0.0404 (37.7)	0.0326 (41.5)	0.0391 (38.6)	0.0190 (53.8)	0.0246 (40.3)	0.0270 (51.3)	0.0506 (31.0)	0.0562 (25.7)	0.0564 (39.6)
α_2	0.1127 (25.8)	0.1215 (32.0)	0.1072 (24.9)	0.1292 (34.4)	0.1054 (31.6)	0.1251 (32.1)	0.1191 (30.9)	0.0943 (31.8)	0.1322 (32.9)
β	0.8856 (25.8)	0.8762 (30.7)	0.8905 (25.4)	0.8685 (33.7)	0.8924 (32.1)	0.8728 (32.3)	0.8784 (31.5)	0.9025 (31.7)	0.8620 (31.7)
η	8.1639 (6.8)	9.2185 (5.4)	8.7350 (5.5)	10.0403 (5.6)	9.1942 (4.9)	10.8769 (4.7)	12.5356 (4.5)	10.8843 (5.5)	11.4501 (4.9)
λ	-0.1360 (4.9)	-0.1233 (4.7)	-0.1764 (4.4)	-0.0762 (4.6)	-0.0919 (4.2)	-0.1098 (4.7)	-0.1607 (5.1)	-0.0831 (4.5)	-0.1253 (4.2)
LL	-2334.34	-2669.39	-2596.80	-2886.71	-2757.27	-2521.07	-2383.33	-2882.55	-2776.02
LML	-2348.43	-2681.99	-2610.04	-2899.04	-2769.90	-2534.33	-2397.52	-2895.55	-2788.20
Skew t with time-varying η and λ									
σ_0^2	0.3794 (5.0)	1.0458 (4.9)	0.5920 (4.4)	0.7217 (4.4)	0.6753 (4.6)	0.6355 (4.7)	0.9405 (4.1)	1.6345 (4.3)	1.1553 (4.7)
ω	0.0341 (25.8)	0.0605 (28.1)	0.0481 (34.4)	0.1040 (35.0)	0.0651 (33.7)	0.0565 (29.0)	0.0396 (33.3)	0.0555 (33.9)	0.1035 (35.7)
α_1	0.0667 (13.6)	0.0582 (14.1)	0.0656 (25.4)	0.0435 (23.1)	0.0466 (14.6)	0.0486 (16.2)	0.0507 (12.1)	0.0788 (13.9)	0.0747 (15.6)
α_2	0.1303 (24.2)	0.1248 (25.0)	0.1236 (37.2)	0.1449 (36.4)	0.1153 (30.0)	0.1346 (27.7)	0.1254 (28.7)	0.1104 (28.7)	0.1512 (29.5)
β	0.8677 (24.4)	0.8728 (25.6)	0.8741 (37.5)	0.8528 (36.9)	0.8824 (30.5)	0.8630 (28.5)	0.8721 (29.1)	0.8864 (29.5)	0.8430 (32.0)
η_a	-0.7827 (34.9)	-0.0052 (79.9)	-0.5530 (60.7)	2.1299 (81.5)	0.0634 (71.3)	3.0000 (60.9)	2.7450 (7.8)	0.8513 (49.5)	2.3983 (64.4)
η_b	-1.0060 (60.2)	-3.3487 (89.7)	-1.5338 (79.6)	-4.5492 (92.1)	-1.8986 (88.0)	-2.4581 (67.8)	-0.2634 (7.9)	-3.0407 (72.9)	-4.8419 (87.1)
η_c	-0.3754 (50.3)	0.6574 (82.7)	-0.1862 (72.0)	0.6575 (88.3)	-0.1312 (82.3)	-0.4650 (51.6)	2.0715 (9.6)	0.2357 (65.9)	1.1048 (80.0)
λ_a	-0.2801 (7.9)	-0.2948 (9.2)	-0.3923 (8.4)	-0.1335 (8.5)	-0.2122 (7.4)	-0.2465 (7.7)	-0.3383 (8.9)	-0.1366 (9.0)	-0.2139 (8.2)
λ_b	0.0572 (5.4)	-0.0127 (5.0)	0.0673 (6.5)	0.0023 (6.4)	-0.0169 (4.8)	-0.0571 (6.4)	0.0021 (7.8)	0.0366 (5.3)	0.0982 (8.1)
λ_c	-0.0106 (8.6)	0.0209 (10.3)	0.0201 (13.0)	-0.0027 (10.3)	0.0246 (8.1)	-0.0099 (8.8)	-0.0097 (11.7)	-0.0239 (10.5)	-0.0308 (9.7)
LL	-2321.77	-2655.93	-2588.23	-2875.05	-2743.47	-2508.95	-2381.78	-2869.19	-2766.25
LML	-2350.20	-2680.66	-2613.08	-2899.35	-2767.81	-2534.40	-2403.76	-2894.25	-2790.47

Table 5.3 Results from Bayesian estimation of the Gaussian kernel symmetric GARCH(1,1) models currency daily returns. The corresponding SIF values are given in parentheses. LL represents the averaged log likelihood, and LML refers to the log marginal likelihood.

Parameter	AUS	CAN	Denmark	Euro	Yen	Norway	NZ	Singapore	Swedish	Swiss	UK
<u>Global bandwidth</u>											
σ_0^2	0.7640 (4.6)	0.9688 (3.9)	1.4459 (3.28)	1.7604 (4.4)	1.0453 (3.9)	1.1008 (5.4)	0.8965 (4.2)	1.4211 (4.2)	1.5352 (5.7)	0.91058 (5.7)	1.7661 (5.7)
α	0.1747 (15.8)	0.2651 (9.7)	0.3259 (14.68)	0.0993 (13.2)	0.2604 (32.5)	0.1792 (11.1)	0.2452 (10.4)	0.5629 (10.5)	0.1687 (11.2)	0.26507 (11.6)	0.2717 (7.2)
β	0.7648 (13.1)	0.6272 (11.7)	0.5314 (15.79)	0.8449 (12.6)	0.6374 (25.2)	0.6851 (15.5)	0.4850 (10.0)	0.3250 (10.9)	0.7267 (17.0)	0.50746 (18.7)	0.5432 (12.2)
h	0.2561 (7.7)	0.2391 (4.6)	0.2634 (6.23)	0.2911 (8.7)	0.2553 (39.1)	0.4279 (7.1)	0.4118 (8.0)	0.2904 (6.3)	0.2965 (12.0)	0.43889 (13.9)	0.2110 (9.7)
LL	-1479.61	-1619.19	-1152.41	-811.58	-2723.63	-1583.32	-1074.26	-1674.91	-1634.76	-1738.82	-1930.36
LML	-1484.41	-1625.29	-1160.13	-816.63	-2729.61	-1589.50	-1083.50	-1687.74	-1640.37	-1747.35	-1937.21
<u>Localized bandwidths</u>											
σ_0^2	0.8523 (4.9)	0.9543 (4.4)	1.4942 (4.2)	1.9227 (4.4)	1.0815 (3.2)	1.1670 (5.8)	0.8047 (3.7)	1.4801 (5.1)	1.6749 (4.2)	0.9202 (4.1)	1.7975 (3.8)
α	0.1889 (11.5)	0.2693 (6.7)	0.3168 (10.8)	0.1280 (10.9)	0.2814 (9.4)	0.1893 (7.9)	0.1965 (12.5)	0.4204 (12.1)	0.1900 (11.5)	0.2652 (12.3)	0.2739 (8.0)
β	0.7313 (13.1)	0.6254 (10.2)	0.5356 (11.8)	0.7889 (13.0)	0.6471 (12.2)	0.6930 (10.3)	0.5910 (17.0)	0.4558 (8.9)	0.6788 (16.5)	0.5766 (15.6)	0.5525 (10.8)
h	0.1741 (14.5)	0.1823 (13.9)	0.1883 (19.5)	0.2108 (21.8)	0.2084 (17.1)	0.2070 (23.0)	0.1952 (30.3)	0.1756 (15.8)	0.1715 (18.9)	0.1996 (27.6)	0.1699 (14.8)
h_ϵ	0.5946 (19.5)	0.3597 (17.3)	0.3363 (25.1)	0.4974 (27.7)	0.5229 (20.7)	0.7233 (24.1)	0.7856 (35.5)	0.7251 (24.7)	0.5154 (22.4)	0.7824 (29.2)	0.3703 (22.4)
LL	-1475.00	-1618.35	-1151.96	-808.39	-2718.83	-1564.00	-1047.41	-1668.44	-1631.49	-1717.99	-1929.44
LML	-1480.22	-1624.62	-1159.29	-813.19	-2725.18	-1569.83	-1053.86	-1677.86	-1637.30	-1725.04	-1936.37

Table 5.4 Results from Bayesian estimation of the skewed Student t GARCH(1,1) models of currency daily returns. SIF values are given in parentheses. LL represents the averaged log likelihood, and LML stands for log marginal likelihood.

Parameter	AUS	CAN	Denmark	Euro	Yen	Norway	NZ	Singapore	Swedish	Swiss	UK
<i>Skewed t with constant η and λ</i>											
σ_0^2	0.8942 (4.7)	0.9433 (5.8)	1.4740 (3.9)	1.7761 (4.4)	1.0718 (4.0)	1.1301 (3.4)	0.6948 (3.5)	1.4530 (3.6)	1.7752 (4.0)	0.8708 (4.7)	1.9183 (4.7)
ω	0.0431 (50.4)	0.0593 (55.3)	0.0494 (43.6)	0.0190 (53.9)	0.1782 (35.9)	0.0503 (64.8)	0.0352 (67.4)	0.0861 (33.5)	0.0819 (70.3)	0.0728 (58.5)	0.1196 (41.9)
α	0.2324 (34.6)	0.2799 (33.2)	0.3273 (31.1)	0.1401 (30.5)	0.3181 (36.2)	0.2074 (55.5)	0.1737 (24.6)	0.3898 (32.4)	0.2276 (38.2)	0.2740 (34.9)	0.3068 (40.8)
β	0.6925 (34.0)	0.6277 (49.6)	0.5287 (38.9)	0.7592 (46.2)	0.6467 (23.6)	0.7020 (53.0)	0.6915 (57.1)	0.5069 (29.0)	0.6262 (66.8)	0.6219 (54.2)	0.5650 (31.4)
η	7.3934 (11.9)	12.3019 (5.8)	10.7077 (5.3)	7.2012 (9.5)	6.0385 (10.9)	9.3251 (7.6)	7.9978 (9.0)	9.7359 (9.4)	13.1328 (5.7)	10.9371 (7.5)	11.4958 (5.7)
λ	0.0457 (4.4)	0.1184 (5.1)	0.0361 (3.9)	0.0202 (3.7)	0.0218 (2.8)	0.1113 (4.1)	0.1075 (5.9)	0.1183 (4.6)	0.1040 (4.9)	0.0227 (6.5)	-0.0282 (4.7)
LL	-1474.57	-1615.61	-1147.57	-802.12	-2716.63	-1550.78	-1042.51	-1662.48	-1629.94	-1702.86	-1932.89
LML	-1485.65	-1628.36	-1161.25	-813.64	-2727.69	-1561.99	-1054.19	-1675.54	-1642.48	-1714.83	-1945.39
<i>Skewed t with time-varying η and λ</i>											
σ_0^2	1.0359 (5.0)	1.0820 (4.4)	1.5800 (4.5)	1.8236 (4.4)	1.1348 (4.5)	1.3881 (4.1)	0.7614 (4.9)	1.6052 (4.3)	2.4627 (4.8)	1.0637 (4.3)	2.4776 (5.1)
ω	0.0514 (53.5)	0.0896 (35.5)	0.0717 (37.8)	0.0241 (59.5)	0.3146 (37.8)	0.0711 (37.3)	0.0387 (49.0)	0.1683 (39.9)	0.1783 (65.6)	0.1475 (53.3)	0.2158 (48.3)
α	0.2819 (66.3)	0.4647 (30.4)	0.4923 (26.2)	0.2037 (54.7)	0.4036 (29.7)	0.3033 (21.5)	0.2251 (36.4)	0.5662 (28.0)	0.4003 (59.6)	0.3986 (44.0)	0.3505 (27.0)
β	0.6752 (64.8)	0.5006 (34.1)	0.4678 (28.4)	0.7208 (62.1)	0.5810 (31.7)	0.6624 (31.3)	0.6706 (47.9)	0.4221 (28.8)	0.5628 (67.6)	0.5862 (46.9)	0.6005 (35.1)
η_a	-2.4253 (48.6)	0.4884 (88.4)	-2.1037 (46.7)	-2.1024 (31.9)	-2.4935 (29.7)	-2.4837 (24.2)	-1.9517 (14.7)	-2.7208 (47.0)	-2.7934 (81.2)	-3.0609 (63.2)	-3.4865 (50.4)
η_b	-0.5094 (36.5)	-2.4385 (94.2)	0.1841 (42.9)	-0.0582 (32.7)	-0.4030 (11.1)	-0.2458 (14.7)	-0.3351 (11.4)	-0.2306 (23.8)	-0.1074 (40.0)	0.1539 (9.5)	-0.2070 (14.6)
η_c	1.8874 (63.4)	0.6165 (88.9)	0.3879 (48.1)	0.2679 (51.5)	0.1926 (23.9)	0.8933 (22.8)	1.2881 (29.8)	0.6690 (41.5)	0.6781 (73.1)	1.3708 (37.5)	3.3065 (18.3)
λ_a	0.0081 (9.1)	0.1570 (16.1)	0.0081 (9.9)	0.0355 (9.9)	0.0502 (6.8)	0.1673 (9.1)	0.2299 (8.8)	0.1896 (7.9)	0.0949 (10.6)	0.1666 (8.5)	-0.1236 (8.8)
λ_b	-0.3356 (26.0)	-0.6906 (21.2)	-0.6260 (31.8)	-0.2870 (20.2)	-0.3450 (19.9)	-0.6853 (18.9)	-0.3143 (23.2)	-0.8911 (26.2)	-1.1150 (72.2)	-0.9247 (56.3)	-0.7478 (50.7)
λ_c	0.0792 (17.8)	0.0187 (22.0)	0.0061 (10.5)	-0.0008 (11.7)	-0.0129 (10.2)	-0.0049 (10.5)	-0.0104 (9.6)	-0.1014 (11.2)	0.1078 (13.0)	-0.1702 (20.9)	0.1811 (22.0)
LL	-1464.11	-1578.73	-1118.26	-798.14	-2700.31	-1518.92	-1034.05	-1600.87	-1591.30	-1662.14	-1906.74
LML	-1486.79	-1602.56	-1146.18	-822.92	-2728.07	-1543.99	-1057.81	-1630.34	-1614.85	-1686.50	-1930.27

Table 5.5 Results from Bayesian estimation of the Gaussian kernel asymmetric GARCH(1,1) models of futures asset returns. SIF values are given in parentheses. LL represents averaged log likelihood, and LML refers to log marginal likelihood.

Parameter	Gold	Silver	Copper	Platinum	Palladium	Corn	Wheat	Soybean meal	Soybean oil	Soybeans	Oats
<u>Global bandwidth</u>											
σ_0^2	1.0710 (5.7)	1.3577 (7.5)	2.5781 (4.1)	1.3393 (8.0)	0.6983 (3.5)	2.1924 (3.8)	1.1507 (4.5)	2.7934 (15.8)	1.1618 (7.0)	0.1193 (31.7)	1.1644 (9.3)
α_1	0.0965 (26.5)	0.0647 (47.8)	0.0864 (31.1)	0.0614 (9.1)	0.0559 (15.5)	0.0292 (36.4)	0.0409 (19.8)	0.0215 (40.2)	0.0409 (20.8)	0.0269 (19.2)	0.0302 (19.4)
α_2	0.0783 (13.4)	0.0904 (73.8)	0.1559 (18.4)	0.0435 (22.0)	0.0609 (41.2)	0.0485 (8.8)	0.0585 (18.2)	0.0311 (74.1)	0.0588 (22.0)	0.0207 (29.5)	0.0744 (51.6)
β	0.8662 (23.8)	0.8686 (63.2)	0.8087 (28.5)	0.9329 (10.6)	0.9254 (22.6)	0.9446 (10.7)	0.9314 (13.5)	0.9669 (53.6)	0.9316 (12.0)	0.9673 (35.6)	0.9008 (50.3)
h	0.2788 (7.1)	0.2752 (39.2)	0.2583 (14.1)	0.2051 (7.0)	0.3170 (6.8)	0.3404 (10.1)	0.2771 (8.3)	0.3585 (12.8)	0.2759 (10.9)	0.2693 (7.1)	0.2588 (31.4)
LL	-2627.64	-3479.36	-3354.21	-2870.26	-3423.18	-3451.48	-2904.35	-3273.50	-2904.34	-3040.21	-3462.33
LML	-2635.92	-3483.21	-3361.96	-2877.51	-3429.40	-3459.11	-2910.35	-3282.20	-2910.34	-3051.38	-3467.54
<u>Localized bandwidths</u>											
σ_0^2	1.1222 (5.2)	1.4731 (5.2)	2.6115 (6.4)	1.3240 (6.1)	0.7027 (3.7)	2.1079 (6.4)	1.1809 (5.8)	2.3858 (5.9)	1.1366 (4.6)	0.2805 (6.5)	1.3454 (3.6)
α_1	0.0930 (9.9)	0.0517 (16.6)	0.0597 (44.0)	0.0623 (10.8)	0.0564 (11.9)	0.0329 (24.1)	0.0401 (16.0)	0.0321 (13.0)	0.0387 (17.6)	0.0419 (13.4)	0.0534 (6.9)
α_2	0.0621 (16.1)	0.0495 (26.5)	0.1425 (34.3)	0.0458 (24.9)	0.0586 (16.9)	0.0584 (22.1)	0.0587 (11.2)	0.0359 (31.9)	0.0585 (11.8)	0.0371 (26.0)	0.0864 (7.7)
β	0.8609 (13.7)	0.9271 (24.1)	0.8267 (38.1)	0.9313 (9.1)	0.9264 (9.4)	0.9290 (24.7)	0.9322 (10.4)	0.9463 (28.8)	0.9329 (9.7)	0.9471 (14.8)	0.8619 (12.2)
h	0.1742 (16.6)	0.1877 (17.8)	0.2175 (13.9)	0.1737 (16.3)	0.2024 (17.6)	0.2148 (18.3)	0.1713 (16.2)	0.2158 (29.1)	0.1722 (25.8)	0.1932 (12.5)	0.1595 (8.0)
h_c	0.6631 (18.4)	0.6058 (26.9)	0.3216 (47.9)	0.1811 (16.1)	0.5010 (16.8)	0.4416 (27.0)	0.4729 (20.3)	0.6739 (36.9)	0.4811 (25.5)	0.7279 (22.9)	0.6226 (5.8)
LL	-2619.86	-3473.27	-3354.71	-2871.17	-3419.37	-3450.04	-2901.28	-3258.84	-2901.24	-3037.78	-3457.84
LML	-2625.66	-3477.47	-3361.17	-2878.80	-3424.92	-3455.18	-2907.39	-3265.00	-2907.27	-3044.92	-3462.43

Table 5.6 Results from Bayesian estimation of the Gaussian kernel asymmetric GARCH(1,1) models of futures asset returns. SIF values are given in parentheses. LL represents averaged log likelihood, and LML refers to log marginal likelihood (continued from Table 5.5)

Parameter	Sugar	Coffee	Cocoa	Cotton	Orange juice	Live cattle	Lean hogs	Feeder cattle	Lumber	Heating oil
<u>Global bandwidth</u>										
σ_0^2	1.2235 (10.0)	0.7834 (75.8)	1.9009 (26.4)	0.4691 (6.0)	1.1455 (4.6)	0.7415 (16.4)	0.8907 (74.8)	0.1329 (85.5)	1.8302 (7.7)	3.2671 (4.4)
α_1	0.0309 (25.0)	0.0324 (58.1)	0.0296 (15.0)	0.0289 (9.0)	0.1216 (12.3)	0.0038 (25.8)	0.0050 (32.9)	0.0091 (15.9)	0.0027 (69.6)	0.0130 (24.5)
α_2	0.0288 (39.4)	0.0329 (75.1)	0.0197 (19.1)	0.0320 (26.3)	0.0451 (22.1)	0.0108 (28.0)	0.0001 (80.3)	0.0422 (21.8)	0.0293 (62.9)	0.0511 (10.4)
β	0.9644 (31.8)	0.9423 (67.6)	0.9628 (33.5)	0.9556 (21.0)	0.7217 (13.4)	0.9780 (28.6)	0.9894 (62.9)	0.9369 (24.7)	0.9687 (53.0)	0.9393 (11.3)
h	0.2243 (6.9)	0.2372 (22.3)	0.2055 (22.9)	0.3789 (24.6)	0.3576 (9.9)	0.4374 (40.8)	0.2491 (21.1)	0.2439 (17.1)	0.2833 (26.7)	0.4369 (13.6)
LL	-3550.04	-3264.06	-3325.29	-3377.39	-3523.98	-2157.82	-2894.64	-1848.35	-3491.89	-3262.10
LML	-3556.08	-3267.03	-3331.93	-3384.01	-3530.28	-2165.84	-2909.44	-1859.02	-3501.93	-3269.74
<u>Localized bandwidths</u>										
σ_0^2	1.3391 (5.6)	1.3100 (4.0)	1.5732 (8.1)	0.5359 (4.7)	1.1515 (4.1)	1.0571 (3.8)	1.6386 (8.9)	1.2302 (2.9)	2.0775 (8.2)	3.3172 (4.4)
α_1	0.0334 (16.8)	0.0277 (15.8)	0.0289 (20.8)	0.0472 (20.4)	0.1328 (8.9)	0.0068 (19.7)	0.0043 (6.3)	0.0100 (15.0)	0.0023 (30.9)	0.0180 (13.8)
α_2	0.0278 (30.8)	0.0183 (16.4)	0.0185 (13.3)	0.0608 (19.0)	0.0493 (12.8)	0.0257 (10.3)	0.0004 (4.5)	0.0485 (14.4)	0.0253 (15.8)	0.0438 (15.0)
β	0.9624 (16.3)	0.9460 (15.4)	0.9629 (20.5)	0.9269 (13.9)	0.7327 (11.2)	0.9463 (11.0)	0.9931 (11.4)	0.9310 (15.3)	0.9712 (16.6)	0.9512 (16.6)
h	0.1905 (14.1)	0.2063 (23.0)	0.2026 (17.4)	0.2000 (23.3)	0.2271 (14.4)	0.2572 (19.2)	0.1557 (9.4)	0.2129 (9.0)	0.1900 (27.7)	0.2178 (32.5)
h_e	0.4768 (25.9)	0.6063 (27.0)	0.4751 (18.2)	0.7131 (28.7)	0.6281 (18.0)	0.6931 (19.4)	0.6928 (9.9)	0.7010 (24.9)	0.3680 (38.2)	0.6160 (32.8)
LL	-3546.61	-3253.58	-3325.00	-3357.52	-3511.72	-2132.10	-2875.60	-1845.40	-3493.16	-3252.68
LML	-3552.27	-3258.19	-3330.89	-3363.74	-3516.78	-2138.03	-2887.15	-1851.48	-3502.05	-3259.51

Table 5.7 Results from Bayesian estimation of skewed Student t GARCH(1,1) models of futures asset returns. SIF values are given in parentheses. LL represents averaged log likelihood, and LML refers to log marginal likelihood.

Parameter	Gold	Silver	Copper	Platinum	Palladium	Corn	Wheat	Soybean meal	Soybean oil	Soybeans	Oats
<u>Skew t with constant η and λ</u>											
σ_0^2	1.4561 (4.8)	1.6319 (4.4)	4.0394 (6.3)	1.4919 (4.7)	0.7095 (4.6)	1.8309 (4.6)	1.3876 (4.4)	1.4918 (4.7)	1.3941 (4.3)	1.5934 (4.8)	1.3909 (4.4)
ω	0.1348 (61.2)	0.1251 (61.7)	0.1012 (50.8)	0.0437 (49.1)	0.0758 (46.5)	0.2085 (71.3)	0.0402 (53.0)	0.0964 (68.1)	0.0387 (47.2)	0.0534 (43.0)	0.3263 (71.4)
α_1	0.1092 (46.3)	0.0609 (42.3)	0.0413 (38.4)	0.0776 (35.7)	0.0643 (30.5)	0.0481 (40.3)	0.0485 (27.7)	0.0407 (44.7)	0.0483 (30.3)	0.0527 (35.3)	0.0726 (43.3)
α_2	0.0891 (48.2)	0.0551 (47.2)	0.0836 (45.0)	0.0692 (35.5)	0.0674 (35.5)	0.1061 (60.8)	0.0666 (38.9)	0.0460 (53.0)	0.0651 (37.4)	0.0528 (42.6)	0.0990 (46.1)
β	0.8454 (55.9)	0.9219 (54.3)	0.9119 (45.7)	0.9109 (39.8)	0.9198 (33.4)	0.8821 (63.3)	0.9267 (38.0)	0.9344 (59.0)	0.9282 (36.2)	0.9326 (31.7)	0.8665 (63.6)
η	3.7700 (28.4)	5.5625 (12.8)	6.9687 (8.2)	7.8610 (7.8)	7.1580 (8.4)	7.3955 (8.3)	10.6647 (4.9)	5.7441 (12.9)	10.6204 (5.0)	5.3805 (12.5)	4.3067 (22.5)
λ	-0.1100 (4.6)	-0.1907 (4.1)	-0.0284 (4.4)	-0.1882 (4.8)	-0.1061 (4.3)	-0.0204 (4.3)	-0.0004 (4.8)	-0.1270 (4.7)	-0.0007 (4.8)	-0.1132 (4.5)	0.0056 (4.1)
LL	-2622.51	-3465.51	-3350.76	-2875.24	-3412.64	-3444.06	-2904.51	-3249.10	-2904.48	-3032.14	-3459.72
LML	-2633.82	-3475.54	-3362.23	-2886.57	-3423.65	-3453.01	-2916.59	-3259.85	-2916.47	-3044.22	-3469.06
<u>Skew t with time-varying η and λ</u>											
σ_0^2	1.4246 (5.2)	1.1045 (5.6)	3.6708 (6.8)	1.4930 (4.4)	0.6854 (4.3)	1.6913 (5.6)	1.2852 (4.3)	1.5417 (5.1)	1.3083 (4.5)	1.2717 (4.4)	1.4877 (4.8)
ω	0.1535 (64.4)	0.1540 (61.0)	0.1012 (50.8)	0.0435 (39.4)	0.0714 (31.9)	0.2546 (69.2)	0.0362 (33.7)	0.1115 (68.5)	0.0374 (35.4)	0.0568 (37.6)	0.3032 (60.7)
α_1	0.1257 (49.1)	0.0714 (39.3)	0.0433 (28.9)	0.0740 (23.9)	0.0617 (13.1)	0.0473 (15.5)	0.0531 (14.5)	0.0427 (25.8)	0.0540 (15.5)	0.0614 (20.0)	0.0679 (20.4)
α_2	0.0858 (45.0)	0.0671 (45.0)	0.0844 (47.2)	0.0691 (26.5)	0.0670 (19.1)	0.1170 (55.3)	0.0677 (21.2)	0.0506 (47.6)	0.0689 (23.6)	0.0539 (21.6)	0.0955 (32.6)
β	0.8436 (62.3)	0.9080 (58.5)	0.9112 (49.6)	0.9117 (36.5)	0.9218 (25.4)	0.8701 (64.2)	0.9251 (25.9)	0.9287 (64.6)	0.9237 (28.0)	0.9273 (32.4)	0.8740 (57.8)
η_a	-3.0152 (29.5)	-1.8982 (17.5)	-1.5093 (22.0)	-1.0168 (39.3)	-1.6656 (9.5)	-1.3365 (57.6)	1.7171 (23.8)	-1.9967 (12.9)	1.6663 (23.8)	-1.6022 (44.5)	-2.6322 (15.3)
η_b	-0.2716 (40.1)	-0.1729 (24.2)	-0.2586 (26.2)	-1.1296 (64.3)	-0.3884 (11.2)	-0.9264 (80.6)	-1.5146 (28.6)	-0.0900 (9.6)	-1.4393 (28.6)	-1.5757 (71.7)	0.4481 (30.4)
η_c	0.0992 (55.0)	-0.1085 (29.0)	0.4230 (86.0)	0.8219 (62.4)	0.8763 (19.3)	0.6550 (78.5)	0.9290 (60.3)	0.0852 (18.4)	0.8117 (61.5)	0.4222 (69.3)	0.4425 (33.9)
λ_a	-0.1800 (5.8)	-0.3504 (8.6)	-0.1303 (7.5)	-0.4188 (7.4)	-0.2487 (7.3)	-0.0624 (10.6)	-0.0050 (8.2)	-0.2303 (6.9)	-0.0054 (8.7)	-0.1683 (6.9)	-0.0200 (6.7)
λ_b	0.1395 (8.3)	0.1179 (6.8)	0.1203 (21.9)	0.2519 (6.9)	0.2280 (7.7)	-0.0351 (9.9)	0.1273 (6.3)	0.0038 (6.9)	0.1290 (6.0)	0.1458 (5.4)	0.1040 (9.0)
λ_c	-0.0678 (10.0)	-0.0352 (15.5)	0.0467 (14.2)	0.0379 (9.2)	0.0224 (9.5)	0.0126 (15.6)	0.0015 (9.6)	-0.0376 (11.6)	0.0025 (10.1)	-0.0757 (10.5)	0.0324 (7.0)
LL	-2619.80	-3460.60	-3350.77	-2873.19	-3410.09	-3448.25	-2903.41	-3248.80	-2903.46	-3024.35	-3458.52
LML	-2645.50	-3486.24	-3375.94	-2895.73	-3433.89	-3468.56	-2925.23	-3274.35	-2925.25	-3049.39	-3481.58

Table 5.8 Results from Bayesian estimation of skewed Student t GARCH(1,1) models of futures asset returns. SIF values are given in parentheses. LL represents averaged log likelihood, and LML refers to log marginal likelihood (continued from Table 5.7).

Parameter	Sugar	Coffee	Cocoa	Cotton	Orange juice	Live cattle	Lean hogs	Feeder cattle	Lumber	Heating oil
<u>Skewed t with constant η and λ</u>										
σ_0^2	1.2568 (7.3)	1.5951 (7.8)	1.8918 (6.6)	0.5442 (4.3)	1.2225 (4.8)	1.2837 (5.5)	1.2647 (10.9)	1.6246 (4.2)	3.6128 (4.2)	4.1253 (5.7)
ω	0.0402 (40.3)	0.1183 (88.1)	0.0533 (72.1)	0.1007 (49.1)	0.7499 (70.7)	0.0435 (86.1)	0.7889 (76.3)	0.0451 (63.6)	0.1185 (60.2)	0.0288 (57.4)
α_1	0.0458 (28.5)	0.0391 (68.0)	0.0320 (57.6)	0.0765 (30.5)	0.1564 (44.3)	0.0085 (67.3)	0.0382 (50.5)	0.0238 (56.8)	0.0209 (36.6)	0.0352 (27.1)
α_2	0.0425 (25.2)	0.0161 (51.8)	0.0206 (35.5)	0.0901 (37.0)	0.0608 (34.5)	0.0420 (59.2)	0.0820 (72.4)	0.0822 (56.5)	0.0530 (48.4)	0.0460 (44.7)
β	0.9496 (29.3)	0.9416 (83.5)	0.9612 (59.4)	0.8977 (38.5)	0.7651 (59.9)	0.9315 (82.8)	0.8553 (87.3)	0.9042 (60.5)	0.9425 (52.8)	0.9510 (45.0)
η	6.0832 (8.3)	6.4309 (8.9)	6.6706 (9.2)	7.0889 (9.4)	4.7683 (17.0)	4.5202 (17.5)	2.2722 (86.1)	3.1961 (40.0)	5.6105 (12.4)	10.1901 (5.6)
λ	0.0227 (4.8)	-0.0286 (4.0)	-0.0425 (4.4)	-0.0287 (4.3)	-0.0523 (4.9)	-0.0185 (4.3)	0.0112 (4.8)	0.0358 (4.4)	0.1593 (4.5)	-0.0125 (4.0)
LL	-3545.58	-3248.58	-3324.11	-3352.16	-3506.98	-2120.16	-2881.94	-1846.28	-3512.64	-3242.36
LML	-3557.98	-3258.25	-3335.97	-3362.56	-3516.75	-2132.07	-2892.91	-1859.79	-3525.25	-3255.45
<u>Skewed t with time-varying η and λ</u>										
σ_0^2	1.1783 (7.2)	1.6607 (9.5)	1.8507 (6.1)	3.6287 (7.5)	1.2652 (4.3)	1.3704 (6.9)	1.5012 (5.2)	1.7012 (5.5)	3.8487 (5.3)	3.9887 (7.2)
ω	0.0394 (28.3)	0.1279 (92.1)	0.0505 (63.9)	0.1087 (48.9)	0.7632 (54.4)	0.0666 (94.0)	0.8035 (62.5)	0.0359 (70.9)	0.1586 (59.3)	0.0338 (44.0)
α_1	0.0447 (21.9)	0.0392 (62.8)	0.0324 (43.3)	0.0477 (31.1)	0.1617 (28.2)	0.0154 (42.1)	0.0399 (15.9)	0.0128 (20.4)	0.0249 (20.2)	0.0390 (19.8)
α_2	0.0415 (18.1)	0.0196 (30.8)	0.0216 (15.1)	0.0916 (47.2)	0.0424 (14.9)	0.0506 (55.4)	0.0403 (24.8)	0.0628 (51.8)	0.0616 (51.0)	0.0495 (35.8)
β	0.9510 (24.6)	0.9384 (90.7)	0.9614 (54.9)	0.9037 (49.1)	0.7721 (53.6)	0.9053 (93.3)	0.9295 (66.8)	0.9254 (66.2)	0.9337 (55.9)	0.9475 (37.5)
η_a	-1.6768 (17.7)	-1.5687 (56.2)	-1.6075 (36.8)	-1.5638 (22.2)	-2.5547 (22.4)	-2.4269 (18.1)	-8.1294 (55.3)	-3.4104 (30.0)	-2.5150 (17.0)	-0.2879 (43.0)
η_b	-0.5066 (30.7)	0.2025 (74.4)	0.1792 (43.7)	-0.2739 (18.1)	-0.5762 (23.8)	-0.1826 (7.5)	-2.8506 (57.1)	-0.0734 (7.2)	0.7042 (12.4)	-2.6205 (60.9)
η_c	0.0580 (28.2)	-0.0263 (68.7)	0.2112 (42.3)	0.3781 (81.2)	0.5480 (22.9)	-0.0605 (15.5)	0.5562 (31.4)	0.4971 (18.2)	0.6007 (18.3)	1.0286 (54.3)
λ_a	0.0789 (6.9)	-0.1217 (8.5)	-0.0805 (7.1)	-0.1219 (6.5)	-0.1286 (7.4)	-0.0191 (8.0)	0.0233 (4.9)	0.0734 (5.4)	0.2838 (12.3)	0.0143 (9.9)
λ_b	0.1755 (4.8)	0.1367 (5.2)	0.0672 (5.0)	0.1213 (19.8)	0.1464 (8.2)	-0.0232 (5.2)	-0.3620 (7.6)	0.0373 (8.6)	0.1160 (7.9)	0.0743 (4.5)
λ_c	-0.0368 (7.6)	0.0532 (10.8)	-0.0106 (6.7)	0.0460 (12.8)	-0.0263 (8.2)	-0.0194 (8.3)	0.0318 (6.9)	-0.0076 (7.9)	0.0648 (24.4)	-0.0390 (12.2)
LL	-3539.95	-3246.06	-3324.92	-3363.06	-3502.45	-2119.33	-2871.49	-1842.46	-3504.90	-3239.38
LML	-3567.25	-3269.87	-3350.84	-3388.34	-3526.07	-2146.24	-2893.60	-1869.98	-3531.37	-3263.29